Ch. 1  Introduction

Basic example: Falling object

Two forces acting = gravity

\[ \sum F = Ma = M \frac{dv}{dt} \]

\[ M \frac{dv}{dt} = Mg - c v \]

let \( M = 10 \text{kg}, c = 2 \text{kg/s} \)

\[ \Rightarrow \frac{dv}{dt} = g, \frac{v}{c} \]

gives acceleration at any velocity

We can plot a direction (slope) field

\[ \frac{dv}{dt} = \frac{v}{c} \]

We haven't found a solution \( v(t) \)

so we can't say anything about \( v \)

just \( \frac{dv}{dt} \)

When is \( \frac{dv}{dt} = 0 ? \)

\[ v = 49 \text{ m/s} \]

Terminal Velocity

Forces are equal so \( v \) is constant

"equilibrium solution"
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drug \rightarrow \text{patient}

- Fluid $5\text{mg/cm}^3$ of drug given at $100\text{cm}^3/h$
- Drug absorbed or leaves in blood at rate proportional to amount present with rate constant, $0.4\text{h}^{-1}$

$M(t)$ is amount of drug at any time

$$\frac{dM(t)}{dt} = \frac{5\text{mg}}{\text{cm}^3} \left(\frac{100\text{cm}^3}{\text{h}}\right) - 0.4M(t) \left[\text{mg/h}\right]$$

$$\frac{dM}{dt} = 500 - 0.4M$$

What is the equilibrium amount after a long time?

$$\frac{dM}{dt} = 0 = 500 - 0.4M$$

$$M = 1250\text{mg}$$
Solutions to some Diff. Eq.

\[ m \frac{dv}{dt} = mg - \kappa v \] falling object

\[ \frac{dy}{dt} = a y - b \] Basic Form

\[ \text{const. coeff.} \]

Separating variables and integrating

\[ \frac{dy}{dt} = a \]

\[ \frac{y - \frac{b}{a}}{y - a} = e^a \]

\[ \ln |y - \frac{b}{a}| = at + c \]

\[ y = \frac{b}{a} + ce^{at} \] general solution

If we have an initial condition we can find the unique solution

\[ y(0) = y_0 \Rightarrow y = \left( \frac{b}{a} \right) + [y_0 - \frac{b}{a}]e^{at} \]
Back to falling object...

\[ \frac{du}{dt} = 9.8 - \frac{v}{5} \]

dropped from 300 m and \( u(0) = 0 \)

\[ \frac{du}{dt} = \frac{-1}{5} \rightarrow v = 100 + Ce^{-\frac{t}{5}} \]

apply IC \( v = 100 (1 - e^{-\frac{t}{5}}) \)

\[ \text{as } t \rightarrow 00 \quad v = 100 \text{ terminal velocity} \]

For position use integrate velocity

\[ \frac{dx}{dt} = 100 (1 - e^{-\frac{t}{5}}) \]

\[ x = 100t + 245 e^{-\frac{t}{5}} + C \]

at \( t = 0 \quad x = 0 \)

\[ x = 100t + 245 e^{-\frac{t}{5}} - 245 \]

hits ground at \( x = 300 \) \( a + t = T \)

\[ 300 = 100T + 245 e^{-\frac{T}{5}} - 245 \text{ need numerical process} \]
Classification of Diff-Eq:

- ordinary - dependent variable fn of only one independent variable
  \[ L \frac{d^2 Q(t)}{dt^2} + R \frac{dQ(t)}{dt} + \frac{1}{C} Q(t) = E(t) \]

- partial - \[ \alpha^2 \frac{\partial^2 u(x,t)}{\partial x^2} = \frac{\partial u(x,t)}{\partial t} \]

For ordinary DE:

\[ F(t, y, y', \ldots, y^{(n)}) = 0 \]

Order = \( n \) - highest derivative

Linear if linear fn of derivatives

\( \alpha^2 \) ok. \( y^2 \), \( \sin y' \), \( yy'' \) is nonlinear!