# A Bi-Criterion Model for Human Arm Posture Prediction

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*Abstract*— **The human arm may be considered to be a redundant mechanism given a pointing task. As a result, multiple arm configurations can be used to complete a pointing task in which the tip of the index finger is brought to a preselected point in space. A kinematic model of the human arm with four degrees of freedom (DOF) and the synthesis of two criteria were developed as an analytical tool for studying position tasks. The two criteria were: (1) minimizing angular joint displacement (Minimal Angular Displacement (MAD)) and (2) averaging limits of the shoulder joint range (Joint Range Availability (JRA)). Joint angles predicted by a weighted model synthesizing the MAD and JRA criteria was linearly correlated (slope=0.97; $r^2$=0.81) with experimental data compared to individual criteria (MAD slope=0.76; $r^2$=0.67 or JRA slope=1; $r^2$=0.56). The partial contributions to the synthesized criterion were 70% MAD and 30% JRA. Solving the inverse kinematics problem of articulated redundant serials mechanism such as the human or robotic arm has applications in fields of human-robot interaction and wearable robotics, ergonomics, and computer graphics animation**.

*Keywords*— **human arm, redundancy, pointing task, kinematics, optimization.**

## I. INTRODUCTION

Pointing with the fingertip to a preselected point in space is a task that involves three degrees of freedom (DOF), which are the X,Y and Z coordinates. Whereas the human arm includes seven DOF excluding scapular motion. When the wrist joint is fixed, four DOF ($\theta, \eta, \zeta, \phi$ - Fig. 1a) remain active. Because the number of DOF of the arm is greater than the number of DOF required for the task, the arm is considered a redundant manipulator. As such, a specific pointing task can be accomplished by infinite arm configurations. As a result, there is not a unique solution for the inverse kinematics (IK) problem involved in defining the joint angles of the human arm given a pointing task.

Despite the human arm redundancy, it has been shown experimentally that a small range of unique solutions for the joint angles are selected by human subjects in pointing tasks, a result consistent within and across multiple participants [1-4]. It has also been shown experimentally that the final arm configuration depends on its initial posture [2-5].

One approach for solving the under-determined IK problem of the redundant human arm is by adding additional kinematics, dynamics, or energy-based criteria, formulated as a cost function. As part of the solution the cost function is either minimized or maximized to provide a unique solution to the IK problem when applied to points along the trajectory of the human arm end effector (i.e. the finger tip for a point task)9 [3-7, 9-11].

The majority of criteria, when studied individually, and validated experimentally, have demonstrated limited capabilities for solving the IK problem of a redundant human arm and predicting arm configuration. In order to overcome the limited capabilities of individual criteria, it was suggested [3-5, 13-15] that two or more criteria should be synthesized with weighted factors.

The objective of this research effort is to develop a model that synthesis two criteria for solving the IK of a redundant human arm given a pointing task. The leading hypothesis is that the human arm adopts a configuration that takes into account energy expenditure (MAD model, as an implicit expression of work) and comfortable posture given limited joint range (JRA model).

The contribution of the reported research is an improved version of the JRA model including realistic description of the shoulder joint, which in conjunction with the MAD model forms the cost function.

## II. METHODS

### A. Kinematic Arm Model

The human arm may be modeled as a serial kinematic chain. For the purposes of this study it is modeled as a four DOF kinematic linkage, consisting of two links (upper arm and forearm along with the hand) and two joints (shoulder joint and elbow joint, with a fixed wrist joint). The shoulder joint is simplified as a ball and socket joint with 3 DOFs and the elbow is simplified as a revolute joint with 1 DOF.

The forward kinematic equations of a 4 DOF human arm model depicted in Fig. 1a are defined by (1), (2) and (3) for which θ is the pitch angle, η is the yaw angle, ζ is the torsion angle of the upper arm (shoulder joint) and φ is the flexion/exertion angle of the elbow joint.

$$\begin{bmatrix} {}^{0}\overline{P} \\ 1 \end{bmatrix} = \begin{bmatrix} {}^{0}H_{sh} \end{bmatrix} \cdot \begin{bmatrix} {}^{sh}H_{el} \end{bmatrix} \cdot \begin{bmatrix} {}^{el}\overline{P} \\ 1 \end{bmatrix} \quad (1)$$

Where ${}^{0}\overline{P}$ is the hand's position vector in a shoulder fixed frame, ${}^{el}\overline{P} = (0,0,-L_2)^T$ is the hand's position vector in an elbow frame, $L_2$ is the distance from the elbow to the hand, ${}^{0}H_{sh}$ and ${}^{sh}H_{el}$ are the 4x4 homogenous coordinate transformations for the shoulder and elbow joints respectively as defined by (2) and (3).

$$ {}^{0}H_{sh} = R_z(\zeta) \cdot R_x(\theta) \cdot R_z(\eta) \quad (2)$$

$$ {}^{sh}H_{el} = R_x(\phi) \cdot T(0,0,-L_1) \quad (3)$$

Where $R_x$ and $R_z$ are 4x4 coordinate rotations, $T$ is a 4x4 coordinate translation, and $L_1$ is the distance from the shoulder to the hand.

Once the position of the hand is fixed at a specific target in space, the elbow joint may swivel around a virtual line connecting the shoulder joint and the location of the hand with an angle defined as the swivel angle α which constitutes the redundancy of the human arm (see Fig. 1b).

The IK of the human arm model can be derived by solving (2) for ${}^{0}H_{sh}$ and specifying one of the shoulder angles. For the purposes of this study, we used an IK procedure formulated in 18, which finds the configuration of the arm for a given point is space and a swivel angle (see (4)).

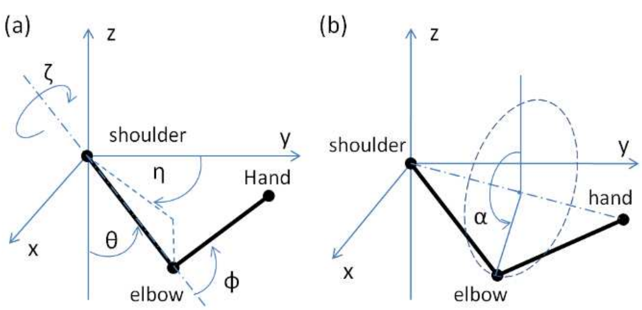


Fig. 1 A 4 DOF model of the human arm (a) Definition of arm parameters – Shoulder joint pitch angle θ, yaw angle η, torsion angle ζ and elbow joint flexion/extension angle φ. (b) The swivel angle α.

$$\overline{\Theta} = f(\overline{P}_{hand}, \alpha) \quad (4)$$

Where $\overline{\Theta}$ is the vector containing the 4 DOF angles, α is the swivel angle, and $\overline{P}_{hand}$ is the hand's position vector.

Note that this algorithm does not provide a solution to the IK problem of kinematically redundant mechanisms, because the swivel angle has to be specified as input to this algorithm.

*B. Shoulder and elbow joints motion ranges*

In a previous research efforts [16,17], investigators conducted a series of experiments rendering mathematical models for the range of motion of the shoulder complex. These models define the shoulder sinus cone which restricts the angular motion of the shoulder joint's pitch and yaw angles, and the humeral torsion motion range which was found to be dependent on the former two angles as in Fig. 2.

The motion range of the elbow joint (flexion/extension angle) is bounded by a minimal and a maximal value and defined by $\phi_{\min} < \phi < \phi_{\max}$ . Since the elbow flexion/extension angle is uniquely defined by the distance between the center of the shoulder joint and the hand, it can be calculated directly according to the hand's position and the segments' lengths.

*C. Joint range availability (JRA) criterion*

The JRA criterion is based on the idea that the human arm tends to adopt postures with joint angles that are as close as possible to their mid-range values and as far as possible from their joint limits. As the elbow joint swivels, and (theoretically) provides an infinite number of possible arm postures, the pitch, yaw and torsion angles of the shoulder joint are adjusted appropriately to maintain the hand position. A valid anatomical posture is achieved if all three angles of the shoulder joint are within their anatomical ranges of motion as in (5).

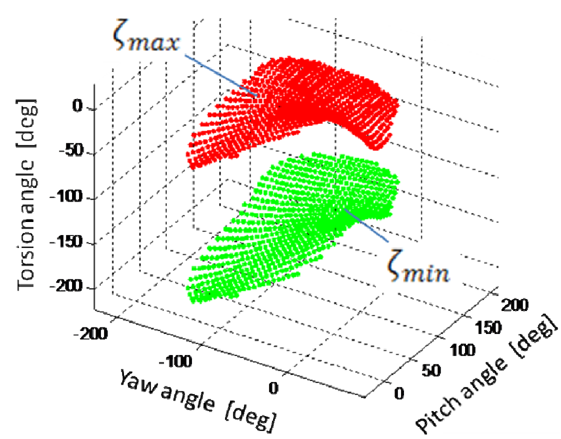


Fig. 2 Shoulder joints' limits and dependencies. The distance between the two surfaces defines the torsion motion range.

$$\begin{cases} \theta_{\min}(\eta) < \theta < \theta_{\max}(\eta) \\ \eta_{\min}(\theta) < \eta < \eta_{\max}(\theta) \\ \zeta_{\min}(\theta,\eta) < \zeta < \zeta_{\max}(\theta,\eta) \end{cases} \quad (5)$$

There is a continuous subset of valid arm postures with a swivel angle in the range of $\alpha_{\min} < \alpha < \alpha_{\max}$. Based on the JRA criterion, the optimal posture is achieved by a mean value of the swivel angle limits defined by

$$\alpha_{mean} = \tfrac{1}{2}\left(\alpha_{\min} + \alpha_{\max}\right). \quad (6)$$

Once the mean swivel angle is defined, the corresponding torsion angle ζ is determined by the IK algorithm. The input for the IK algorithm is the target point in space (hand position) along with the swivel angle, and its output is the four angles of the DOF θ,η,ζ and φ.

### *D. Min. angular displacement (MAD) criterion*

The minimal angular displacement (MAD) criterion minimizes the sum of the difference of the various joint angles, between their initial and final values.

In other words, the final arm posture defined by this criterion yields the shortest distance between the initial and the final value of the joint angles in joint space. This criterion can be formulated as an optimization problem using the following cost function

$$\begin{aligned} &\text{minimize}: f(\overline{\Theta}) = \sum_{i=1}^{4}\left(\Theta_i^{final} - \Theta_i^{\text{intial}}\right)^2 \\ &subject\ to: \overline{P}_{hand}(\overline{\Theta}) = \overline{P}_{\text{target}} \end{aligned} \quad (7)$$

Where $\overline{P}_{\text{target}}$ is the position vector of the target point, $\Theta_i^{final}$ is the i[th] DOF angle at the final posture, and $\Theta_i^{intial}$ is the i[th] DOF angle at the initial posture.

Since a 4 DOF arm model is redundant by only one DOF, this optimization problem can be solved by a brute force grid search where solving for the swivel angle that minimizes the cost function under the given constrain, using (7). During the brute force search, the initial posture of the arm remains constant, while the final posture varies with the value of the swivel angle, and the cost function's value changes accordingly.

### *E. The bi-criterion model*

The JRA and MAD criteria are used independently, as previously explained, to calculate the humeral torsion angles $\zeta_{JRA}$ and $\zeta_{MAD}$ respectively. The bi criterion model merges the two results by calculating the weighted average of the humeral torsion angle $\zeta_{optimal}$ as defined by

$$\zeta_{optimal} = k \cdot \zeta_{MAD} + (1-k) \cdot \zeta_{JRA} \quad (8)$$

The weight factor *k* in (8) is optimized to match experimental results with the model prediction.

## III. Results

Testing the model's prediction is based on experimental results that were previously published by Admiraal et al. (see 5). As part of this protocol seven subjects pointed to five different target points in space while sitting with constrained torsos. Four pointing movements were conducted from different starting locations to each target point. Each subject performed 20 pointing tasks. The data was collected with motion capturing equipment.

The final arm postures are reported in terms of torsion angles. Data points used in the current study were extracted in terms of final arm postures for three subjects denoted by O, Δ and □. This subset of the database renders data for 60 pointing movements which enables us to test the prediction performance of our bi-criterion model.

We ran simulations of these 60 pointing movements with the bi-criterion model and compared the predicted arm postures to the ones measured experimentally.

The correlation between the combined prediction model and the experimental results is depicted in Fig. 3. An ideal correlation between the prediction models and the experimental results would be depicted by a linear relationship with a slope and a correlation factor ($r^2$) of one. Comparing the experimental data with each one of the criteria (MAD and JRA) separately is summarized in Table I. Correlating the experimental results with those of MAD model prediction is represented by a slope of 0.76 and a correlation factor ($r^2$) of 0.67 whereas the JRA model prediction is represented by a slope of 1.0, but with a relatively lower correlation factor ($r^2$) of 0.56. Moreover, this model, being a posture based model, does not predict the influence of the initial posture on the final posture.

The best correlation of the experimental results with the synthesized model was achieved with weight factor k=0.7 for which 70% of the output is contributed by the MAD model and 30% of the output is contributed by the JRA model. The linear correlation of the synthesized model results and experimental data is represented a by a slope of 0.97 and a correlation factor ($r^2$) of 0.81 (Fig. 3). The same value of *k* was obtained by analyzing the data of each individual subject (see Table II). This may imply that this specific value of $k$ weighted is an invariant feature for neural motor control system associated with pointing task.

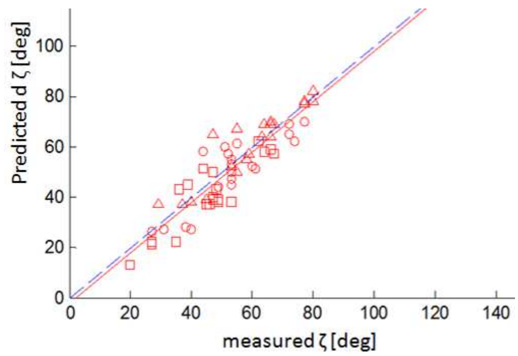


Fig. 3 The humeral torsion angle comparison between the prediction of the combined model and the experimental data. The solid line (red) is the trend line. The dashed line (blue) is the ideal trend line (slope=1).

Table 1 Prediction performance of the three models

| Model | Slope | $r^2$ |
|---|---|---|
| MAD | 0.76 | 0.67 |
| JRA | 1.0 | 0.56 |
| MAD + JRA | 0.97 | 0.81 |

Table 2 Prediction performance for the three subjects by the bi-criterion model

| Subject | Slope | $r^2$ |
|---|---|---|
| O | 0.97 | 0.81 |
| Δ | 1.0 | 0.88 |
| □ | 1.02 | 0.81 |

## IV. Discussion

This research effort is focused on a synthesized model for redundancy resolution of the human arm in pointing tasks. For a 4 DOF model of the human arm, where there is only one redundant DOF, the synthesized model using two criteria provided high correlation with previously reported experimental data by Admiraal et al. 5.

Correlation of alternative criteria such as the minimal work and minimal torque change were associated with a slope of 0.3 and $r^2$=0.56 for both criteria using the same database 5.

Using the minimum peak kinetic energy criterion 4 led to correlation with slope of approximately 1 and $r^2$ in the range of 0.522 to 0.915 using a database of four subjects.

Future work will focus on establishing a large database for pointing tasks along with a comparative research effort of synthesizing various combinations of criteria and their correlation with the tasks under study.

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