Synthesizing Two Criteria for Redundancy Resolution of Human Arm in Point Tasks

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Abstract**—The human arm may be considered to be a redundant mechanism given a pointing task. As a result, multiple arm configurations can be used to complete a pointing task in which the tip of the index finger is brought to a preselected point in space. A kinematic model of the human arm with four degrees of freedom (DOF) and the synthesis of two criteria were developed as an analytical tool for studying position tasks. The two criteria were: (1) minimizing angular joint displacement (Minimal Angular Displacement (MAD)) and (2) averaging limits of the shoulder joint range (Joint Range Availability (JRA)). Joint angles predicted by a weighted model synthesizing the MAD and JRA models was linearly correlated (slope=0.97; r^2=0.81) with experimental data compared to individual criteria (MAD slope=0.76; r^2=0.67 or JRA slope=1; r^2=0.56). The partial contributions to the synthesized criterion were 70% MAD and 30% JRA. Solving the inverse kinematics problem of articulated redundant serials mechanism such as the human or robotic arm has applications in fields of human-robot interaction and wearable robotics, ergonomics, and computer graphics animation**.

Keywords-human arm; redundancy; pointing task; kinematics; optimization

I. INTRODUCTION

Pointing with the fingertip to a preselected point in space is a task that involves three degrees of freedom (DOF), which are the X,Y and Z coordinates. Whereas the human arm includes seven DOF excluding scapular motion. When the wrist joint is fixed, four DOF (θ,η,ζ,ϕ - Fig. 1a) remain active. Because the number of DOF of the arm is greater than the number of DOF required for the task, the arm is considered a redundant manipulator. As such, a specific pointing task can be accomplished by infinite arm configurations. As a result, there is not a unique solution for the inverse kinematics (IK) problem involved in defining the joint angles of the human arm given a pointing task.

Despite the human arm redundancy, it has been shown experimentally that a small range of unique solutions for the joint angles are selected by human subjects in pointing tasks, a result consistent within and across multiple participants [1-4]. It has also been shown experimentally that the final arm configuration depends on its initial posture [2-5].

One approach for solving the under-determined IK problem of the redundant human arm is by adding additional kinematics, dynamics, or energy-based criteria, formulated as a cost function. As part of the solution the cost function is either minimized or maximized to provide a unique solution to the IK problem when applied to points along the trajectory of the human arm end effector (i.e. the finger tip for a point task)[9] [3-7, 9-11].

The optimization criteria may be divided into two classes: (1) biomechanical (kinematics and dynamics of the human body) criteria and (2) anatomical based criteria. The first class includes the minimal angular displacement (MAD) model [12], the minimal work model [3, 5], the minimal peak kinetic energy model [4], the minimal torque change model [5, 6] and the minimal potential energy change model [9]. Physical quantities such as energy, torque or displacement form the cost function which is further minimized or maximized as part of the solution. The second criterion class is based on anatomical models such as the joint range availability (JRA) criterion (also called Dexterity) [7] and the minimum discomfort criterion [10]. The cost functions in this class are based on anthropometric data of joint motion ranges, the intension of which is to quantify psychophysical discomfort related to the proximity to joint limits or nominal arm configuration.

The majority of these models, when studied individually, and validated experimentally, have

978-1-4577-1124-4/11/$26.00

demonstrated limited capabilities for solving the IK problem of a redundant human arm and predicting arm configuration. In order to overcome the limited capabilities of individual criteria, it was suggested [3-5, 13-15] that two or more criteria should be synthesized with weighted factors.

The objective of this research effort is to develop a model that synthesis two criteria for solving the IK of a redundant human arm given a pointing task. The leading hypothesis is that the human arm adopts a configuration that takes into account energy expenditure (MAD model, as an implicit expression of work) and comfortable posture given limited joint range (JRA model).

The contribution of the reported research is an improved version of the JRA model including realistic description of the shoulder joint, which in conjunction with the MAD model forms the cost function.

Solving the IK problem of an articulated redundant serials mechanism such as the human or robotic arms has applications in fields of robotics, ergonomics and computer graphics animation.

II. METHODS

A. Kinematic arm model

The human arm may be modeled as a serial kinematic chain. For the purposes of this study it is modeled as a four DOF kinematic linkage, consisting of two links (upper arm and forearm along with the hand) and two joints (shoulder joint and elbow joint, with a fixed wrist joint). The shoulder joint is simplified as a ball and socket joint with 3 DOFs and the elbow is simplified as a revolute joint with 1 DOF. This model implies that the forearm and hand are aligned with a fixed wrist angle joint during a reach and point movement.

While the position of the hand in Cartesian space is defined by three coordinates (X,Y,Z), the posture of the arm requires four angles to fully and uniquely specify its configuration. As a result the human arm model may be treated as a kinematically redundant mechanism with respect to a pointing task. The forward kinematic equations of a 4 DOF human arm model depicted in Fig. 1a are defined by (1), (2) and (3) for which θ is the pitch angle, η is the yaw angle, ζ is the torsion angle of the upper arm (shoulder joint) and φ is the flexion/exertion angle of the elbow joint.

$$\begin{bmatrix} {}^{0}\overline{P} \\ 1 \end{bmatrix} = \begin{bmatrix} {}^{0}H_{sh} \end{bmatrix} \cdot \begin{bmatrix} {}^{sh}H_{el} \end{bmatrix} \cdot \begin{bmatrix} {}^{el}\overline{P} \\ 1 \end{bmatrix} \tag{1}$$

Where ${}^{0}\overline{P}$ is the hand's position vector in a shoulder fixed frame, ${}^{el}\overline{P} = (0,0,-L_2)^T$ is the hand's position vector in an elbow frame, L_2 is the distance from the elbow to the hand, ${}^{0}H_{sh}$ and ${}^{sh}H_{el}$ are the 4x4 homogenous coordinate transformations for the shoulder and elbow joints respectively as defined by (2) and (3).

$$\begin{gathered} {}^{0}H_{sh} = R_z(\zeta) \cdot R_x(\theta) \cdot R_z(\eta) = \\ = \begin{bmatrix} C\zeta & S\zeta & 0 & 0 \\ -S\zeta & C\zeta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C\theta & -S\theta & 0 \\ 0 & S\theta & C\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} C\eta & S\eta & 0 & 0 \\ -S\eta & C\eta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{gathered} \tag{2}$$

$$\begin{gathered} {}^{sh}H_{el} = R_x(\phi) \cdot T(0,0,-L_1) = \\ = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C\phi & -S\phi & 0 \\ 0 & S\phi & C\phi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{gathered} \tag{3}$$

Where R_x and R_z are 4x4 coordinate rotations, T is a 4x4 coordinate translation, and L_1 is the distance from the shoulder to the hand.

Once the position of the hand is fixed at a specific target in space, the elbow joint may swivel around a virtual line connecting the shoulder joint and the location of the hand with an angle defined as the swivel angle α which constitutes the redundancy of the human arm (see Fig. 1b). Given that there exist physiological joint limits, the elbow joint may only follow a limited arc out of the full circle depicted in Fig. 1b.

(a)

(b)

Figure 1. A 4 DOF model of the human arm (a) Definition of arm parameters – Shoulder joint pitch angle θ, yaw angle η, torsion angle ζ and elbow joint flexion/extension angle φ. (b) The swivel angle α.

The IK of the human arm model can be derived by solving (2) for ${}^{0}H_{sh}$ and specifying one of the shoulder angles. For the purposes of this study, we used an IK procedure formulated in [18], which finds the configuration of the arm for a given point is space and a swivel angle (see (4)).

$$\overline{\Theta} = f(\overline{P}_{hand}, \alpha) \tag{4}$$

Where $\overline{\Theta}$ is the vector containing the 4 DOF angles, α is the swivel angle, and $\overline{P}_{hand}$ is the hand's position vector.

Note that this algorithm does not provide a solution to the IK problem of kinematically redundant mechanisms, because the swivel angle has to be specified as input to this algorithm.

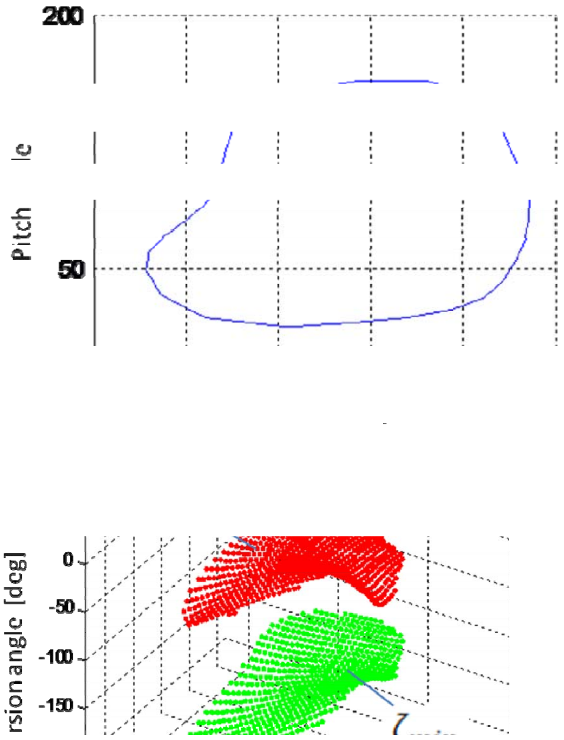
B. . Shoulder and elbow joints motion range

In a previous research effort [16], investigators conducted a series of experiments rendering a mathematical model for the range of motion of the shoulder complex. This model defines the shoulder sinus cone which restricts the angular motion of the shoulder joint's pitch and yaw angles depicted in Fig. 2a. The humeral torsion motion range was found to be dependent on the former two angles as depicted in Fig. 2b using a model based on experimental data [17] . The maximal internal and external upper arm torsion surfaces were fitted into a polynomial function. Fig. 2b shows the upper and lower limits of the torsion range of motion.

The motion range of the elbow joint (flexion/extension angle) is bounded by a minimal and a maximal value and defined by $\phi_{\min} < \phi < \phi_{\max}$. Since the elbow flexion/extension angle is uniquely defined by the distance between the center of the shoulder joint and the hand, it can be calculated directly according to the hand's position and the segments' lengths.

C. Joint Range Availability (JRA) Criterion

The JRA criterion is based on the idea that the human arm tends to adopt postures with joint angles that are as close as possible to their mid-range values and as far as possible from their joint limits. As the elbow joint swivels, and (theoretically) provides an infinite number of possible arm postures, the pitch, yaw and torsion angles of the shoulder joint are adjusted appropriately to maintain the hand position. A valid anatomical posture is achieved if all three angles of the shoulder joint are within their anatomical ranges of motion as in (5).



(a)

(b)

Figure 2. Shoulder joints' dependencies (a) pitch and yaw angles relationships (b) Upper (red) and lower (green) limits of humeral torsion. The distance between the two surfaces defines the torsion motion range.

$$\begin{cases} \theta_{\min}(\eta) < \theta < \theta_{\max}(\eta) \\ \eta_{\min}(\theta) < \eta < \eta_{\max}(\theta) \\ \zeta_{\min}(\theta,\eta) < \zeta < \zeta_{\max}(\theta,\eta) \end{cases} . \tag{5}$$

There is a continuous subset of valid arm postures with a swivel angle in the range of $\alpha_{\min} < \alpha < \alpha_{\max}$. Based on the JRA criterion, the optimal posture is achieved by a mean value of the swivel angle limits defined by

$$\alpha_{mean} = \frac{\alpha_{\min} + \alpha_{\max}}{2} . \tag{6}$$

Fig. 3 illustrates the swivel angle limits and the optimal posture at some hand position in space.

Figure 3. Swivel angle limits and optimal posture. Green dots mark valid postures. Red dots mark invalid postures .

Once the mean swivel angle is defined, the corresponding torsion angle ζ is determined by the IK algorithm. The input for the IK algorithm is the target point in space (hand position) along with the swivel angle, and its output is the four angles of the DOF θ,η,ζ and φ as defined by (2).

D. Min. Angular Displacement Criterion (MAD)

The minimal angular displacement (MAD) criterion minimizes the sum of the difference of the various joint angles, between their initial and final values.

In other words, the final arm posture defined by this criterion yields the shortest distance between the initial and the final value of the joint angles in joint space. This criterion can be formulated as an optimization problem using the following cost function

$$\begin{aligned} &\text{minimize}: f(\overline{\Theta}) = \sum_{i=1}^{4}\left(\Theta_i^{final} - \Theta_i^{\text{intial}}\right)^2 \\ &subject\ to: \overline{P}_{hand}(\overline{\Theta}) = \overline{P}_{\text{target}} \end{aligned} \quad . \qquad (7)$$

Where $\overline{P}_{\text{target}}$ is the position vector of the target point, Θ_i^{final} is the ith DOF angle at the final posture, and $\Theta_i^{\text{int}ial}$ is the ith DOF angle at the initial posture.

Since a 4 DOF arm model is redundant by only one DOF, this optimization problem can be solved by a brute force grid search where solving for the swivel angle that minimizes the cost function under the given constrain, using (7).

During the brute force search, the initial posture of the arm remains constant, while the final posture varies with the value of the swivel angle, and the cost function's value changes accordingly.

E. The Bi-Criterion Model

The JRA and MAD criteria are used independently, as previously explained, to calculate the humeral torsion angles ζ_{JRA} and ζ_{MAD} respectively. The bi criterion model merges the two results by calculating the weighted average of the humeral torsion angle $\zeta_{optimal}$ as defined by

$$\zeta_{optimal} = k \cdot \zeta_{MAD} + (1-k) \cdot \zeta_{JRA} \, . \qquad (8)$$

The weight factor k in (8) is optimized to match experimental results with the model prediction.

III. MODEL VALIDATION

Testing the model's prediction is based on experimental results that were previously published by Admiraal et al. (see [5]). As part of this protocol seven subjects pointed to five different target points in space as depicted in Fig. 4 while sitting with constrained torsos. Four pointing movements were conducted from different starting locations to each target point. Each subject performed 20 pointing tasks. The data was collected with motion capturing equipment.

The final arm postures are reported in terms of torsion angles. A sample of the results in [5] pointing to target 5, are depicted in Fig. 5. Data points used in the current study were extracted in terms of final arm postures for three subjects denoted by O, Δ and □. This subset of the database renders data for 60 pointing movements which enables us to test the prediction performance of our bi-criterion model.

We ran simulations of these 60 pointing movements with the bi-criterion model and compared the predicted arm postures to the ones measured experimentally.

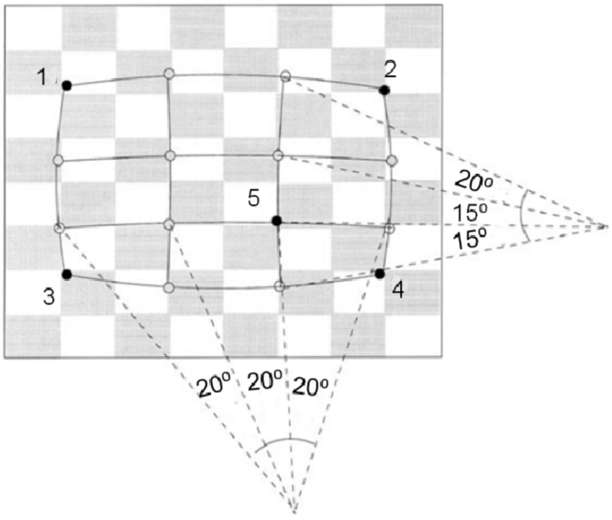


Figure 4. Distribution of five target points is space. Target number 5 is aligned with the shoulder, following [5].

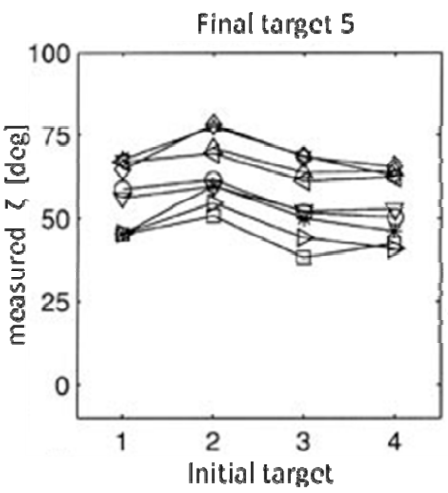


Figure 5. Humeral torsion angles for pointing to target number 5 from starting points 1 to 4. The data was presented for seven subjects – adopted from [5].

IV. RESULTS

The correlation between the combined prediction model and the experimental results is depicted in Fig. 6. An ideal correlation between the prediction models and the experimental results would be depicted by a linear relationship with a slope and a correlation factor (r^2) of one. Comparing the experimental data with each one of the criteria (MAD and JRA) separately is summarized in Table I. Correlating the experimental results with those of MAD model prediction is represented by a slope of 0.76 and a correlation factor (r^2) of 0.67 whereas the JRA model prediction is represented by a slope of 1.0, but with a relatively lower correlation factor (r^2) of 0.56. Moreover, this model, being a posture based model, does not predict the influence of the initial posture on the final posture.

The best correlation of the experimental results with the synthesized model was achieved with weight factor k=0.7 for which 70% of the output is contributed by the MAD model and 30% of the output is contributed by the JRA model. The linear correlation of the synthesized model results and experimental data is represented a by a slope of 0.97 and a correlation factor (r^2) of 0.81 (Fig. 6). The same value of *k* was obtained by analyzing the data of each individual subject (see Table II). This may imply that this specific value of *k* weighted is an invariant feature for neural motor control system associated with pointing task.

V. DISCUSSION

This research effort is focused on a synthesized model for redundancy resolution of the human arm in pointing tasks. For a 4 DOF model of the human arm, where there is only one redundant DOF, the synthesized model using two criteria provided high correlation with previously reported experimental data by Admiraal et al. [5].

TABLE I. PREDICTION PERFORMANCE OF THE THREE MODELS

Model	Slope	r^2
MAD	0.76	0.67
JRA	1.0	0.56
MAD + JRA	0.97	0.81

TABLE II. PREDICTION PERFORMANCE FOR THE THREE SUBJECTS BY THE BI-CRITERION MODEL

Subject	Slope	r^2
O	0.97	0.81
Δ	1.0	0.88
□	1.02	0.81

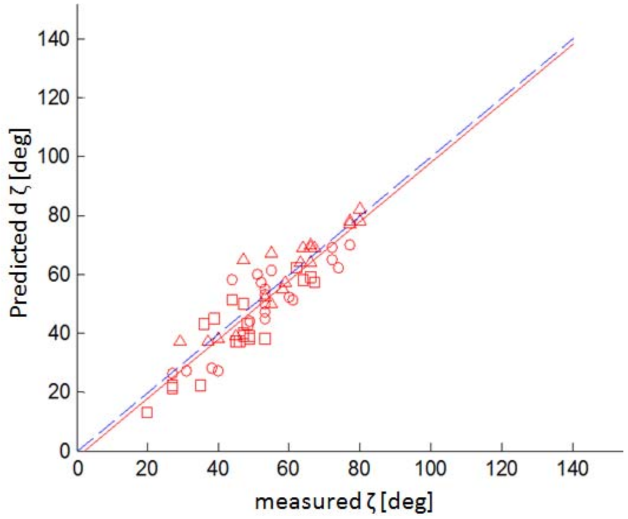


Figure 6. The humeral torsion angle comparison between the prediction of the combined model and the experimental data. The solid line (red) is the trend line. The dashed line (blue) is the ideal trend line (slope=1).

Correlation of alternative criteria such as the minimal work and minimal torque change were associated with a slope of 0.3 and r^2=0.56 for both criteria using the same database [5].

Using the minimum peak kinetic energy criterion [4] led to correlation with slope of approximately 1 and r^2 in the range of 0.522 to 0.915 using a database of four subjects.

Future work will focus on establishing a large database for pointing tasks along with a comparative research effort of synthesizing various combinations of criteria and their correlation with the tasks under study.

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