

Optimisation of the mean boat velocity in rowing

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(Received 18 November 2010; final version received 7 February 2011)

In rowing, motor learning may be facilitated by augmented feedback that displays the ratio between actual mean boat velocity and maximal achievable mean boat velocity. To provide this ratio, the aim of this work was to develop and evaluate an algorithm calculating an individual maximal mean boat velocity. The algorithm optimised the horizontal oar movement under constraints such as the individual range of the horizontal oar displacement, individual timing of catch and release and an individual power–angle relation. Immersion and turning of the oar were simplified, and the seat movement of a professional rower was implemented. The feasibility of the algorithm, and of the associated ratio between actual boat velocity and optimised boat velocity, was confirmed by a study on four subjects: as expected, advanced rowing skills resulted in higher ratios, and the maximal mean boat velocity depended on the range of the horizontal oar displacement.

Keywords: biomechanical analysis; genetic algorithm; individual constraints; individual feedback; rowing simulator

1. Introduction

In the refining stage of motor learning, augmented feedback in terms of knowledge of results should provide lasting performance benefits. By the knowledge of results, the athlete can confirm his own assessment based on taskintrinsic feedback. Furthermore, knowledge of results will facilitate motor learning if the outcome of the performance cannot accurately be assessed by the athlete himself (Magill 2003).

In rowing, the mean boat velocity, i.e. the averaged boat velocity of at least one stroke, represents the outcome of the rowing performance in general. The mean boat velocity is hardly accurately, task intrinsically available to the athlete, thus, it is predestinated to be presented as an augmented feedback. Knowing the actual mean boat velocity may provoke that the rower elaborates movement characteristics through trial and error to increase the velocity. The motivation to elaborate different movement characteristics should be even higher if the actual mean boat velocity is related to a theoretical, individual maximal mean boat velocity that can be achieved by the rower.

We have developed a sweep rowing simulator, which serves as a high-level indoor training tool. The underlying rowing model takes the displacement of the oar and of the seat into account to calculate both the actual boat velocity and the forces to be displayed at the oar. In our virtual rowing environment, the rower also visually and acoustically perceives his actual rowing performance as during on-water rowing, e.g. the landscape passes by and

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ISSN 1025-5842 print/ISSN 1476-8259 online © 2012 Taylor & Francis http://dx.doi.org/10.1080/10255842.2011.561794 http://www.tandfonline.com the immersion of the oar into the water is sonified (von Zitzewitz et al. 2008; Rauter et al. 2010). Displaying concurrent, augmented feedback in terms of knowledge of performance has already been realised, e.g. foot-stretcher forces can be visualised (Krumm et al. 2010). To provide also knowledge of results in terms of a feedback relating actual mean boat velocity to individual maximal mean boat velocity, the latter one has to be determined.

This paper presents a mathematical optimisation that maximises the mean boat velocity based on important rowing variables, and on the individual physiology and anthropometry of the rower. This is the first approach that relates actual to individual, maximal mean boat velocity to provide a general feedback of the actual rowing performance.

2. Methods

2.1 The rowing simulator and the applied rowing model to provide the actual rowing velocity

The rowing simulator at the Sensory-Motor Systems Lab of the ETH Zurich and University Zurich consists of a shortened skiff mounted on a podium in the middle of a CAVE. The skiff is surrounded by three projection screens that visually display a virtual rowing scenario, and by a closed ring of loudspeakers that displays corresponding sounds. The rowing simulator was set up for sweep rowing, i.e. one user interacted with the virtual rowing scenario through one shortened oar whose blade was virtually displayed. At the outer end of the shortened oar, two ropes of a tendon-based parallel robot were attached. The tendon-based robot applied water resistance forces $F_{\Omega,x}$ at the oar in the direction of boat motion. These forces were rendered in real time by a rowing model. Another outcome of the rowing model was the actual boat velocity $\dot{x}_{\text{B,act}}$. Inputs to the rowing model were the trajectories of the three oar angles, i.e. the horizontal oar angle $\theta(t)$, the vertical oar angle $\delta(t)$ and the angle around the longitudinal axis of the oar $\phi(t)$, as well as the linear displacement of the seat $x_{\rm S}(t)$, representing the movement of the rower's mass over the course of time (Figure 1). Parameters set in the rowing model were the number of rowers, the mass of the boat, the masses of the oars and of all rowers, the length of the user's oar, and the drag coefficients of the boat and the oars. As sweep rowing is only performed by an even number of rowers, the additionally needed rowers were simulated to perform in the same way as the human rower (von Zitzewitz et al. 2008).

2.2 Approach to determine the individual maximal mean boat velocity

To calculate the individual maximal mean boat velocity $\dot{x}_{B,max}$, the actual mean boat velocity $\dot{x}_{B,act}$ was combined with an optimisation algorithm that maximised the boat velocity through optimisation of the input functions, $\theta(t)$, $\delta(t)$, $\phi(t)$ and $x_S(t)$. In order to individualise the maximal mean boat velocity, the personal abilities of force and power generation were considered during the optimisation process (Figure 1).

Prior to the optimisation, the maximal range of the oar angle θ and the duration of the drive phase, as well as the recovery phase, had to be known. In this paper, the beginning of the drive phase, i.e. the catch, was defined

at the minimal displacement of the horizontal oar angle. The end of the drive phase, i.e. the release, was defined at the maximal displacement of the horizontal oar angle. These values were determined at different stroke rates during the warm-up of the rower on the rowing simulator. Thereafter, the optimisation problem (1) of maximising the mean boat velocity $\dot{x}_{B,max}$ depending on the three oar angular displacements and the seat movement could be performed.

$$\max_{k} \dot{x}_{B,\max}(k) \quad \text{with } k = \begin{bmatrix} \theta(t) \\ \delta(t) \\ \phi(t) \\ x_{S}(t) \end{bmatrix}.$$
(1)

2.2.1 Adaptations of the rowing variables to optimise

As all four functions to optimise were time series with hundreds of data points, each time series was considered as a continuous function with a predefined number of parameters in order to simplify the optimisation problem. These simplifications are described in the following paragraphs. Parameters of the rowing model were not changed throughout all further considerations. The number of additional rowers was one, thus, a coxless pair was simulated.

As small changes in the trajectory of the seat had no considerable impact on the boat velocity, seat position data were taken from a professional rower who participated twice in the Olympic Games (Figure 2). As both the duration of the stroke and the relation of the drive phase to the recovery phase depend on the stroke rate, the professional rower



Figure 1. Flowchart of the model displaying the actual mean boat velocity $\dot{x}_{B,act}$ in relation to the individual maximal mean boat velocity $\dot{x}_{B,max}$. The actual mean boat velocity was given by an existing rowing model fed with variables of the actual performance (von Zitzewitz et al. 2008). The maximal mean boat velocity was based on an optimisation of the horizontal oar angle θ . The optimisation also considered an optimal vertical oar angle δ , an optimal angle about the longitudinal axis of the oar ϕ , the seat movement data of a professional rower x_{s} , pro and the power P_{need} needed to perform the oar movement.



Figure 2. Seat position x_s during one rowing cycle exemplified for three stroke rates: 32 strokes/min, 26 strokes/min and 20 strokes/min. In the optimisation, seat position data corresponding to the actual stroke rate of the rower under investigation were considered.

rowed at different stroke rates. This enabled to consider optimal seat position data at the individual stroke rate of that rower whose maximal mean boat velocity should be determined.

To apply maximal forces during the drive phase, the blade has to be oriented vertically with respect to the water surface. During the recovery phase, the orientation of the blade should result in a minimal air drag. As air drag of the blade was not considered in the rowing model (von Zitzewitz et al. 2008), the longitudinal oar angle ϕ was not optimised but held constant at 90° over the whole rowing stroke.

The relative velocity between oar and water determines those points in time, when the blade has to be immersed in the water and when it has to be removed out of the water. When the velocity of the oar in the direction of motion is lower than the velocity of the water, the blade has to be in the air to avoid breaking forces which would slow down the boat. When the oar moves faster than the water, the blade should be fully immersed in the water to generate the full amount of propulsive forces. On the basis of these basic considerations, just two states of the vertical oar angle δ were considered: 'blade in the air' and 'blade fully immersed in water' (Figure 3). This simplification of the oar angle δ ensured that no breaking forces occurred, accepting that an immersion phase was ignored and that a human rower will never be able to generate such stepwise movements.

Out of the four variables considered in the presented optimisation, the oar angle θ has the greatest impact on the boat velocity. The velocity of the horizontal oar movement mainly determines the force $F_{O,x}$ at the blade in the main direction of boat motion. This force is the only propulsive force during the drive phase that can be generated by the



Figure 3. Example of an adapted oar angle δ during one stroke (shown in black): in contrast to the usually stepless immersion and lifting of the blade (shown in grey), the blade was either fully immersed or in the air.

rower and, therefore, it is crucial for the boat velocity. Thus, the oar angle θ was in the focus of the optimisation.

As the oar angle θ was a time-dependent variable, $\theta(t)$ was parameterised for both the drive phase and the recovery phase. On the basis of a common trajectory of $\theta(t)$ (Figure 4), the function f_1 to describe the oar angle θ during drive phase had to start (t = 0) at the minimal displacement θ_{\min} , and had to end (time of oar release t_{release}) at the maximal displacement θ_{\max} . At these points in time, no angular velocity had to be present. Accordingly, the function f_2 to describe the recovery phase had to start at t_{release} at the maximal displacement θ_{\max} , and to end at the



Figure 4. Example of a typical horizontal oar displacement. Constraints of the optimisation are also indicated: Besides the individual range of angular displacement given by θ_{\min} and θ_{\max} , the points in time of the release t_{release} and the next catch t_{catch} were considered.

next catch t_{catch} at the minimal displacement θ_{\min} . The transition between the functions was assumed to be continuous, i.e. oar angle, velocity and acceleration must not change. These constraints resulted in 10 equations (2) to be fulfilled.

$$f_{1}(0) = \theta_{\min},$$

$$f_{1}(t_{\text{release}}) = \theta_{\max},$$

$$\dot{f}_{1}(0) = 0,$$

$$\dot{f}_{1}(t_{\text{release}}) = 0,$$

$$f_{2}(t_{\text{release}}) = \theta_{\max},$$

$$f_{2}(t_{\text{catch}}) = \theta_{\min},$$

$$\dot{f}_{2}(t_{\text{catch}}) = 0,$$

$$\dot{f}_{2}(t_{\text{catch}}) = 0,$$

$$\ddot{f}_{1}(0) = \ddot{f}_{2}(t_{\text{catch}}) \text{ and}$$

$$\ddot{f}_{1}(t_{\text{release}}) = \ddot{f}_{2}(t_{\text{release}}).$$

$$(2)$$

To have at least one parameter for each of the two functions f_1 and f_2 to be optimised, two fifth-order polynomial functions were chosen. The first polynomial function P_1^5 , with the parameters a, b, c, d, e and f, had to reflect the horizontal angular displacement of the oar during the drive phase of one single stroke ($0 \le t < t_{\text{release}}$). The second function P_2^5 , with the parameters g, h, i, k, l and m, described the recovery phase ($t_{\text{release}} \le t < t_{\text{catch}}$):

problem depending only on two parameters, namely f and g:

$$\max_{\theta(f,g)} \dot{x}_{B,\max}(\theta(f,g;t))$$
s.t.
$$P_{1}^{5}(f,g;t) \quad [0 \le t \le t_{\theta_{\max}}]$$

$$P_{2}^{5}(f,g;t) \quad [t_{\theta_{\max}} \le t \le t_{\theta_{\min}}]$$
(5)

As a further constraint, the first polynomial function $P_1^5(f,g;t)$ had to be monotonically increasing and the second function $P_2^5(f,g;t)$ monotonically decreasing as discontinuities are not in line with biomechanical principles and are hardly realisable by a rower:

$$\dot{P}_1^5(f,g;t) \ge 0 \quad [0 \le t \le t_{\theta_{\max}}],$$

$$\dot{P}_2^5(f,g;t) \le 0 \quad [t_{\theta_{\max}} \le t \le t_{\theta_{\min}}].$$
(6)

The minimal and maximal values of f and g, i.e. f_{min} , f_{max} , g_{min} and g_{max} , that fulfil the monotony constraint for the two functions within the corresponding time intervals were determined by solving system (6) with Maple (Maplesoft, Canada).

2.2.3 The genetic algorithm solving the optimisation

The mean boat velocity is calculated by the rowing model which has a nonlinear and hybrid behaviour. Thus,

$$\theta(t) = \begin{cases} P_1^5 = a + bt + ct^2 + dt^3 + et^4 + ft^5 & \text{for } 0 \le t < t_{\text{release}}, \\ P_2^5 = g + ht + jt^2 + kt^3 + lt^4 + mt^5 & \text{for } t_{\text{release}} \le t < t_{\text{catch}}. \end{cases}$$
(3)

It was important to solve the system of equations in (2) and (3) for one odd parameter in t and for one even parameter in t to find a valid solution. It did not matter in which equation the odd or the even parameter was solved because they were linked with each other through the constraints. After solving equation systems (2) and (3), the 10 parameters depended only on 2 parameters; in this work, f and g were chosen. With these two parameters, an entire θ trajectory could be generated which still behaved like two linked fifth-order polynomial functions having the demanded properties of (2):

$$\theta(f,g;t) \begin{cases} P_1^5(f,g;t), & \text{for } 0 \le t < t_{\text{release}}, \\ P_2^5(f,g;t), & \text{for } t_{\text{release}} \le t < t_{\text{catch}}. \end{cases}$$
(4)

2.2.2 Simplification of the optimisation

The initial optimisation problem (1) depending on four time-dependent variables was adapted to an optimisation simple gradient methods to solve the optimisation could not be applied. Instead, a genetic algorithm was chosen to approximate the optimal solution that maximises the mean boat velocity (5). The basic idea of the algorithm is known from biology as the survival of the fittest: first, a random population is generated. This population is tested in a fitness function. The members of the population with the best results are selected and used to generate a new population until a termination criterion is fulfilled.

In the first step of the optimisation algorithm, initial populations of f and g from Equation (4) were randomly generated by a uniform distribution within the intervals $f_{\min} \leq f \leq f_{\max}$ and $g_{\min} \leq g \leq g_{\max}$. The population size was set to 250 based on pilot optimisations showing that larger populations had no significant influence on the optimised mean boat velocity (but smaller had). For every parameter pair of f and g, a resulting trajectory of θ with the length of one rowing cycle was computed. To get a

stationary behaviour of the mean boat velocity, each resulting trajectory of θ and corresponding trajectory of δ as well as the predefined trajectories of ϕ and x_S were replicated eight times, and then used as an input for the rowing model. After eight optimised strokes, the mean boat velocity did not change anymore. Thus, only the last of the eight rowing strokes was considered to calculate the mean boat velocity. The calculated mean boat velocity indicated the fitness of the θ trajectory to achieve a maximal mean boat velocity.

Thereafter, the parameter pairs of *f* and *g* were sorted by their resulting mean boat velocity and the 10% with the highest velocities were selected. From these selected parameters, the covariance matrix and the mean of each parameter were calculated. The parameter pair which produced the highest mean boat velocity was saved together with the value of the maximal mean boat velocity $\dot{x}_{B,max}$.

To create a new population of 250 parameter pairs for the next iteration, a 2D multivariate distribution was used. The two mean values for this multivariate distribution were obtained from the mean values of the selected parameter pairs of the current iteration.

To create the covariance matrix for the multivariate distribution, 80% of the covariance matrix obtained from the selected parameter pairs of the current iteration was added to 20% of the covariance matrix used in the last iteration.

The optimisation algorithm was stopped in case (i) the maximal mean boat velocity changed by less than 0.001 m/s since the last iteration or (ii) 10 iterations (to get results in a reasonable time) were done.

2.2.4 Individual power limitation to individualise the optimisation

To further individualise the optimisation, a power constraint was included: the needed power P_{need} to move the oar had to be always smaller than or equal to the individual maximal power P_{max} of the rower at each horizontal oar angle θ during the drive phase ($0 \le t \le t_{\theta_{\text{release}}}$):

$$\max_{\theta(f,g)} \dot{x}_{B,\max}(\theta(f,g;t))$$
s.t.
$$f_{\min} \leq f \leq f_{\max} \qquad (7)$$

$$g_{\min} \leq g \leq g_{\max}$$

$$P_{need}(\theta(f,g;t)) \leq P_{\max}(\theta) \quad [0 \leq t \leq t_{\theta_{release}}].$$

This individual power was determined by a full body force test which was executed on the rowing simulator in order to copy the movements occurring during rowing as well as possible.



Figure 5. Example of an angular velocity-force-angle relation. A fourth-order polynomial function was fitted to the data points (grey diamonds) gained in the force tests.

The force test was performed at different constant resistances, starting at 500 N and decreasing by steps of 50 N until a minimal resistance of 250 N was reached. In every test, the rower held the oar in his hands and performed one drive phase as fast as possible. Four to six repetitions were performed at each resistance with breaks of about 1 min.

The different resistance forces were applied at the oar via the rope robot. Ideally, resistance forces should have been applied independently from the angular velocity and from the beginning of the oar movement onwards. Due to safety issues, this could not be realised. Instead, resistance forces were provided by a model whose only input was the angular velocity of the horizontal oar angle ($\dot{\theta}$). After passing a low-pass filter, the square of the angular velocity was taken to get similar behaviour of the resistance force as in the rowing model. Then, the squared angular velocity was multiplied by a constant C (80 kg m) to generate a steep force increase to quickly reach the desired resistance force F_{res} . Thereafter, the produced force went through a saturation block, setting the desired resistance force $F_{\rm res}$. Thus, the predefined constant resistance force $F_{\rm res}$ was only applied when the angular velocity $\dot{\theta}$ raised above a threshold $\dot{\theta}_{\text{lim}}$ given by

$$\dot{\theta}_{\rm lim} = \sqrt{\frac{F_{\rm res}}{C}}.$$
 (8)

The minimal angle of θ , when the threshold velocity could be achieved for the first time, and the maximal angle of θ , before the velocity fell below the threshold again, were used as data points to fit a fourth-order polynomial function providing the force–angle relation over the tested range of θ (Figure 5).

Considering the outboard length of the oar to the attachment of the ropes l_{out} , the maximal power at the blade

depending on θ could be calculated:

$$P_{\max}(\theta) = \dot{\theta}(\theta) F_{\text{res}}(\theta) l_{\text{out}}.$$
 (9)

The gained power curve $P_{\text{max}}(\theta)$ was then used to limit the optimisation (7): if at any angle of the optimised trajectory the needed power was higher than the maximal power that could be produced, the mean boat velocity of the corresponding parameter pair of *f* and *g* was set to -1 m/s to ensure that this parameter pair was not considered in the next iteration.

2.3 Application of the optimisation

The general feedback in terms of the ratio between achieved mean boat velocity and optimised, maximal mean boat velocity was determined for four healthy men aged between 17 and 33 years, rowing at different levels: rower 1 was a beginner, rower 2 an intermediate and rowers 3 and 4 were members of the national youth squad in sweep rowing. Informed written consent was obtained from all rowers in accordance with the research ethics committee of ETH Zurich, Switzerland.

The force tests were performed by the rowers at first. Therefore, the rowing simulator described in Sections 1 and 2.1 was used to apply resistance forces as described in Section 2.2.4. Thereafter, rower 1 performed two runs and rower 2 one run, at a self-selected stroke rate. Rowers 3 and 4 were asked to row runs at three different stroke rates (20, 26 and 32 strokes/min) which were provided by a metronome. Those two rowers were also asked to row erroneously: in error state 1, the inner arm was to be not completely stretched; in error state 2, the shoulder axis was to be not parallel to the inboard of the oar; in error state 3, the wrists were to be lifted too high; in error state 4, the shoulders were to be lifted up in the middle of the drive phase. All runs included between 25 and 30 strokes.

The optimisation of the mean boat velocity was performed for a representative stroke in the middle of each run. As rowers 3 and 4 could not perform the error states at 20 strokes/min or faster, only that stroke with the highest stroke rate was considered for the optimisation. (Optimal seat movement data were only available from 20 strokes/min onwards, and using optimal seat movement data of a higher stroke rate than rowed would increase the optimal mean boat velocity.)

To test the robustness of the optimisation, 10 different initial populations were used for the optimisation of representative strokes of rowers 1 and 2.

3. Results

In more than 90% of all optimisations, the optimisation was terminated before the set maximum of 10 iterations

was reached, i.e. the optimal velocity changed by less than 0.001 m/s within less than 10 iterations. Repeated optimisations with different initial populations of parameter pairs *f* and *g*, performed for one representative stroke of each run, resulted in optimised mean velocities with a standard deviation of 0.02 and 0.01 m/s for rower 1, and of 0.07 m/s for rower 2 (Table 1).

The two professional rowers (rowers 3 and 4) showed larger angular displacement ranges of the horizontal oar angle θ than the other two rowers, e.g. rower 3 from -52° to $+41^{\circ}$ at 26 strokes/min in contrast to rower 1, -37° to $+44^{\circ}$ at 28 strokes/min. At the highest stroke rate, rower 3 reduced his angular displacement range by 5° compared to the other two stroke rates.

Both professional rowers achieved higher mean boat velocities than the other two rowers. Their ratios of achieved mean boat velocity to optimised mean boat velocity were also higher. In different error states, the achieved mean boat velocity, the optimised mean boat velocity and the corresponding ratio were reduced compared to correct rowing at similar stroke rates (Table 1).

Optimised trajectories of θ are exemplified in Figure 6, as well as the required power to provide the trajectory. Due to the individual power limitation given in Equation (7), the required power was always lower than the maximal power determined by the force tests. Rower 1 also produced considerable breaking forces, as the power–angle relation showed negative values at the end of the drive phase (Figure 6).

4. Discussion

In this paper, a model to calculate the individual maximal mean boat velocity during rowing was presented. It is intended to use this individual maximal mean boat velocity as a general feedback, when set in relation to the actual mean boat velocity. Such a general feedback will motivate the rower to elaborate by himself different movement characteristics by trial and error to increase his actual mean boat velocity.

The applied optimisation was in about 10% terminated by the criterion '10 iterations performed'. This rather small number of maximally done iterations was chosen to terminate the optimisation in a reasonable time, i.e. in less than half an hour (2 Quad CPU at 3 GHz). Together with the extraction of the data from force tests and the calculation of the borders for the two parameters f and g, about an hour was needed to get the individual maximal mean boat velocity. Time is a crucial factor for our future experiments on rowers, because the optimisation is intended to be performed after the force tests of the rower, and should be completed before the rower starts to train on the simulator. Therefore, it is not desirable to spend hours on the optimisation because the rowers want to train on the simulator as soon as possible.

Table 1.	Achieved m	ean boat velocity of selected strokes	at different stroke rates as w	ell as corresponding optimised mean boat veloc	ity.	
Rower	Run/error	Stroke rate (mean, SD) ^a [1/min]	Range of oar angle θ [°]	Achieved mean velocity (mean, SD) ^a [m/s]	Optimised mean velocity [m/s]	Ratio
1 PW	1	28 (28, 1)	- 26.4 to 42.3	3.40 (3.22, 0.17)	$4.55(0.02)^{b}$	0.75
	2	22 (22, 1)	-36.7 to 43.3	3.34(3.42, 0.22)	$4.76(0.01)^{\rm b}$	0.70
2 GR	1	30(30,1)	- 46.9 to 38.63	3.95(3.89, 0.10)	$5.22(0.07)^{b}$	0.76
3 JO	1	20(20, <1)	-52.1 to 40.0	3.91(3.90, 0.03)	4.45	0.88
	2	26(26, <1)	- 52.5 to 40.3	4.37 (4.38, 0.02)	5.04	0.87
	3	34 (32, 1)	-50.4 to 38.5	4.61(4.62, 0.04)	5.67	0.81
	Error 1	19	- 34.8 to 40.1	3.16	4.19	0.75
	Error 2	20	-35.4 to 41.0	3.42	4.36	0.80
4 RR	1	20(20, <1)	- 53.4 to 45.1	4.42(4.47, 0.14)	4.92	0.90
	2	26(26, <1)	- 53.4 to 44.5	5.10(5.02, 0.07)	5.59	0.91
	3	35(35,1)	- 52.0 to 44.7	5.50(5.43, 0.17)	5.90	0.93
	Error 3	18	-51.7 to 44.3	3.79	4.68	0.81
	Error 4	17	-51.7 to 43.5	3.35	4.66	0.72
^a Mean va	lue and related s	standard deviation (SD) are reported for the	the whole run.			

^bThe optimisation was 10 times repeated, thus, mean value and related SD are reported here. The mean value is used for calculating the ratio.

Higher variability in the parameters f and g can be obtained by considering more than the used 20% of the covariance matrix from the last iteration or more than the used 10% of the best of the actual iteration results to generate the next population of parameter pairs. However, a higher variability in f and g may result in more iterations needed to completed the optimisation. We exemplarily tested higher variabilities without getting maximal mean boat velocities deviating from the prior calculated mean out of 10 times repeated optimisations by more than 2 times the corresponding standard deviation. The highest standard deviation of the repeated optimisations was 0.07 m/s (see Table 1). Differences in mean boat velocity in this magnitude are relevant in rowing; however, it is our intention to provide general feedback and not to precisely predict a theoretically possible mean boat velocity. Furthermore, the mean boat velocity was much more affected by other constraints such as stroke rate and the range of the oar angle θ (see Table 1). Nevertheless, the presented optimisation enables also a precise prediction of the theoretically maximal possible, individual mean boat velocity when computational effort has not to be constrained.

4.1 Individualisation of the optimisation

The minimal and maximal displacement of the horizontal oar angle, θ_{\min} and θ_{\max} , were individually measured to get an optimised angular trajectory of θ that could be produced by the rower. As θ_{\min} and θ_{\max} have usually to be determined during warm-up, the angles will not exactly match those in the actual stroke that is analysed. Depending on the skill level of the rower, the fluctuations in these angles will be remarkable (novice rower) or small (professional rower). This will have an impact on the ratio between actual mean boat velocity and optimised mean boat velocity, i.e. on the general feedback during practice. In this work, θ_{\max} and θ_{\min} of a measured stroke during practice were used, because the optimisation was done after all measurements.

The force tests were performed on the simulator in order to test individual physical abilities during the rowing movement itself. If force tests had been executed on conventional force machines, only isolated muscles or muscle groups could have been tested. Using rowing ergometers would have been limited by the impossibility to set a constant resistance force.

The force tests on the simulator provided information about the angular displacement velocity $\dot{\theta}$ that a rower could achieve at a constant resistance force. This was a rather general statement on his physical abilities without knowing the contribution of muscle groups acting in the subphases of the stroke, namely the legs, the back and the arms. If the power curves of these muscle groups were placed within the displacement range of θ , the presented



Figure 6. Application of the optimisation. On the left side, measured and optimised/adapted oar angles are shown for rower 1 (beginner) and for rower 4 (expert) rowing correctly and erroneously. On the right side, power-angle relations are shown as determined in the force tests, and as required during the optimised and measured stroke.

model could also consider when leg drive, back rotation and arm pull should take place. Therewith, the coordination pattern of these muscle groups can be optimised, under the assumption that the rower is able to fully activate the muscles as requested in the subphases. Such an improved model may be quite interesting for rowing experts. However, the additional force tests and the increased complexity of the model will increase the time needed for the optimisation which is not desired for our intention of providing general feedback in a training session.

As the individual maximal mean boat velocity is limited with results from maximal force tests, the optimised velocity can surely not be achieved during rowing longer distances. Therefore, energy efficiency might additionally be considered. Pulman (2004) stated that symmetrical shapes of the force curves are the most efficient. This constraint might further improve the validity of the optimisation. Furthermore, only a percentage of the optimised mean boat velocity might be used to provide general feedback. To determine the magnitude of this percentage was not in the focus of this work. The applicability of the general feedback considering the optimised mean boat velocity as presented is not limited because rowing experts showed a quite high ratio between actual velocity and optimised velocity of about 0.9 which leaves a small, but always present, potential to improve (Table 1).

4.2 Application of the optimisation

The runs at different stroke rates of the professional rowers and the different ranges of the horizontal oar angle θ seen for professional and non-professional rowers clearly showed the impact of these two factors on the optimised mean boat velocity: higher stroke rates and greater ranges of θ resulted in higher optimised velocities. An increased range of θ could thereby compensate for a reduced stroke rate as observed for the second run of rower 1 compared to his first run (Table 1).

As the immersion of the blade was simplified to a steplike instantaneous movement, the blade was not immersed for the entire drive phase but shorter. Due to this simplification and the made constraint that the optimisation considered the whole displacement of the horizontal oar angle (3), the power–angle relation of the optimised stroke showed a range without power at the beginning and at the end of the drive phase (Figure 6). This range without power will not occur if in a future optimisation algorithm the immersion phase of the blade is also considered.

The determined power-angle relations reflected the skill level of the different rowers quite well: as determined in the force tests, the professionals were able to generate more power over the whole range of the horizontal oar displacement than the non-professionals. In contrast to the non-professionals, this maximally available power of the professionals clearly exceeded the power needed for their optimised strokes. The power needed for an optimised stroke was exceeded by the professionals in the rowing runs (power in optimised run vs. power in measured run, Figure 6) but not by the non-professionals. This indicates that the professionals will be able to adapt their oar displacement but the non-professionals probably not.

For the investigated rowers, the skill level in sweep rowing was also quite well reflected in the determined general feedback, i.e. the ratio of achieved to optimised mean boat velocity: both professional rowers rowed more than 10% better than the other two rowers. The differences between the two professional rowers can be explained by their usual position in sweep rowing: rower 3 was not used to row on the side available on the simulator, thus, especially at the fastest stroke rate, he showed a considerably reduced performance: compared to his runs at slower stroke rates his ratio decreased by more than 5%, and compared to rower 4 at his fastest stroke rate, his ratio was more than 10% smaller.

Rowing error states were somehow challenging for the professionals as the requested stroke rate of 20 strokes/min could hardly be realised and the generated forces were about 500 N lower than during normal rowing (Table 1 and Figure 6). Consequently, achieved mean boat velocities were lower than during normal runs, but also the optimised mean boat velocities were lower and the calculated ratios smaller. The smaller ratios revealed that the movement was erroneously performed. On the basis of these systematic changes which were also observed for lower skill levels, it can be concluded that the proposed ratio is sensitive to the rowing performance and thus, can be used as a general feedback in rowing.

The presented optimisation can also be elaborated to systematically investigate the impact of different horizontal oar angle ranges on the mean boat velocity while considering an individual power-angle relation; for instance, at the beginning of the drive phase of rower 4 in run 1, the generated power was quite close to his maximal available power (see Figure 6). To let the athlete row within his optimal range, such investigations may justify adaptations of the boat set-up. Note that adaptations of the boat set-up get more and more popular in research on rowing (Barrett and Manning 2004; Baudouin and Hawkins 2002; Cornett et al. 2008). If only millimetres can be gained within one stroke, that might be worth the effort, because these millimetres can decide a 2000-m race.

5. Conclusion

The presented algorithm provides an individual maximal mean boat velocity that can be related to the actual mean boat velocity. The presented algorithm is capable of showing differences on individual maximal mean boat velocity, on optimal angular trajectory and on power needed to produce the optimal trajectory for different ranges of motion of the horizontal oar movement θ . The ratio between achieved mean boat velocity and individual maximal mean boat velocity will now be used as a general feedback in our rowing simulator because different skill levels were well reflected by this ratio.

With the presented algorithm, different factors that have an impact on the individual maximal mean boat velocity such as the range of motion of θ , shifted power curves and durations of drive time and recovery time can now systematically be investigated.

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