Modelling the Rowing Stroke in Racing Shells
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Modelling the rowing stroke in racing shells

MAURICE N. BREARLEY, NEVILLE J. de MESTRE and DONALD R. WATSON

1. *Introduction*

In this article we set up a mathematical model to represent the effects of the forces which operate during the rowing of racing shells. The analysis is conducted in terms of eights, but could apply equally well to fours, pairs and double or quad sculls, and even (with obvious verbal changes) to single sculls. McMahon [1] as well as McMahon and Bonner [2] have previously considered various numbers of rowers in racing shells, and reached conclusions suggesting that consideration of an eight is representative of all possible combinations of rowers.

The rowing stroke is divided into two parts: the power stroke, during which the blades of the oars are in the water and the rowers pull on the oar handles and straighten their legs, thus moving their bodies towards the bow on their sliding seats; and the recovery phase, during which the blades are clear of the water and the rowers move stern-wards by bending their legs and leaning forward.

The main resistance to the forward motion of a boat is provided by the drag of the water. Air resistance plays a much smaller part, in general, and is neglected in the analysis. A formula for the drag on the hull of a typical racing eight is obtained from experimental data in a Report of the UK National Physical Laboratory (Wellicome [3]), and this enables numerical results to be obtained for a typical eight. Boat-flexing, pitching and 'fish-tailing' are all neglected, their influence being negligible compared with that of the forces considered in the analysis.

Assumptions are made in later Sections about the variation of the forces on the oar handles and of the rowers' displacements on their slides. To our knowledge only one other author (Millward [4]) has endeavoured to construct a mathematical model of rowing. His model ignored any movement of the rowers within the boat. The test of any model is how well its predictions correspond to observed results in a practical situation; in Section 5 the variation of boat speed during a stroke and race duration are predicted by our model for an eight in a 2000 m race.

Graphs are obtained of the variation in boat velocity while accelerating from a stationary start and during a full stroke after top speed has been reached. The idea of two-phase rowing is introduced, and an analysis of its effectiveness is made.

2. *Notation and conventions*

The water on which the boat travels may, of course, be regarded as a fixed reference frame.

A modern racing oar has a mass of only about 2 kg, so the masses of the oars may reasonably be neglected in the analysis. Let

m = mass of boat (including the cox, if present),
M = combined mass of rowers,
t = time from start of power stroke,
v = velocity of boat at time t,
f = dv/dt = acceleration of boat at time t,
τ_1 = duration of power stroke,
τ_2 = duration of recovery phase,
t' = $t - \tau_1$ = time from start of recovery phase,
D = drag of the water on the hull.

In the Appendix it will be shown that for a typical racing-eight hull,

$$D = a + bv + cv^2, \tag{1}$$

where a, b, c, are constants which are calculable from data in Wellicome [3]. It will be assumed that this formula remains valid over the whole range of boat velocities considered in this analysis of rowing.

It is important to distinguish the directions of the forces which operate. The word 'forward' will be taken to mean the direction in which the boat is moving, and 'backward' to mean the opposite direction.

The word 'rowers' will be used instead of the traditional (and sex-discriminatory!) word 'oarsmen'.

3. *The power stroke*

This occurs during the time interval $0 \leqslant t \leqslant \tau_1$.

The feet of the rowers are strapped to footrests fixed to the hull. During the power stroke the rowers straighten their legs and exert a combined backward force, Q say, on the footrests and hence on the boat. By Newton's Third Law, an equal and opposite force is exerted on the rowers by the footrests. These forces are depicted in Figure 1 for a single oar, but the forces shown are the combined values for all rowers in the boat.

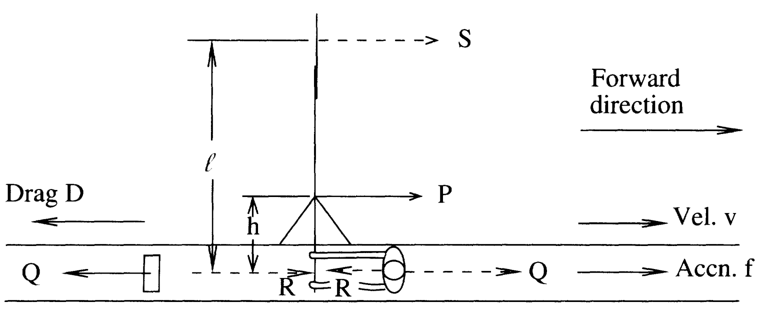

FIGURE 1 Plan view of the power-stroke situation

In Figure 1, forces acting on the boat itself are drawn with full arrows; others are shown as dashed lines. Since force components perpendicular to the direction of travel will cancel for each pair of oars on opposite sides of the boat, it is sufficient to consider only components parallel to the direction of travel.

The rowers exert a combined forward force R on the oars, and experience themselves an equal and opposite backward force. Not being forces on the boat, both of these forces are shown dashed in Figure 1.

The water exerts forces on the blades of the oars, the combined component in the forward direction being denoted by S. It is shown dashed in the figure since it does not act on the boat. It may be considered as acting at the centre of each blade. It has been observed by us that the blades move very little through the water during the power stroke; in this model they are regarded as fixed fulcrums of the levers formed by the oars. A case can be made for allowing for some small motion of the blades through the water by taking the fulcrum of each oar to be inboard of the blade itself. Such a change would affect some of the numerical work in Section 5 but would not alter the basic form of the mathematical model.

The oars exert forces on the boat at the swivels, their combined components in the forward direction being denoted by P in Figure 1. The mechanical advantage of the oar-lever system ensures that $P > Q$. It is the combined difference $P - Q$ for all oars that drives the boat forward against the drag D of the water during each power stroke.

As depicted in Figure 1, let

ℓ = oar length from centre of grip to centre of blade,

h = distance from centre of grip to swivel.

By considering moments about the centre of the blade it is seen that

$$R\ell = P(\ell - h).$$

It follows that

$$P - R = (h/\ell)P, \tag{2}$$

and this relationship evidently holds throughout the power stroke, irrespective of the angle which the oars make with the boat.

Oar flexing is ignored in this approach, even though it may modify slightly the values of P and ℓ. Its effect on the conclusions will be small.

Relative to the boat, the rowers begin and end their forward motion with zero velocity, and attain smoothly a maximum velocity at about the centre of their travel. This relative motion is such that it may reasonably be taken as half of a cycle of simple harmonic motion (SHM). This suggests writing the relative forward displacement of the rowers from the central point of their travel during $0 \leqslant t \leqslant \tau_1$ as

$$x_1 = -a_1 \cos n_1 t, \tag{3}$$

where the forward direction of the boat is taken as positive and

a_1 = the amplitude averaged over all rowers of the SHM of the centres of mass of the rowers' bodies,

n_1 = π / τ_1 = the circular frequency of the SHM.

The forward acceleration of the rowers relative to the boat is $\ddot{x}_1$, and relative to the water is $\ddot{x}_1 + f$. The equation of motion of the rowers in the forward direction is therefore

$$Q - R = M(\ddot{x}_1 + f) = M\left(n_1{}^2 a_1 \cos n_1 t + \frac{dv}{dt}\right).$$

The equation of motion of the boat is

$$P - Q - D = m\frac{dv}{dt}.$$

Adding these two equations and using (2) produces

$$(m + M)\frac{dv}{dt} = \frac{h}{\ell'}P - Mn_1{}^2 a_1 \cos n_1 t - D.$$

The force P begins and ends with small magnitudes in $0 \leqslant t \leqslant \tau_1$, and attains smoothly a maximum near the centre of this interval (Mason et al [5]). The salient features of P will be adequately represented in a mathematically tractable way by taking

$$\frac{h}{\ell'}P = P_m \sin n_1 t, \tag{4}$$

where $n_1 = \pi / \tau_1$ as before, and P_m is the maximum value of $(h / \ell')P$.

The previous differential equation then becomes, by virtue of (1),

$$(m + M)\frac{dv}{dt} = P_m \sin n_1 t - Mn_1{}^2 a_1 \cos n_1 t - a - bv - cv^2.$$

This power-stroke equation may also be written as

$$\frac{dv}{dt} = K_1 \sin n_1 t + K_2 \cos n_1 t + A + Bv + Cv^2, \tag{5}$$

where $0 \leqslant t \leqslant \tau_1$, $n_1 = \pi / \tau_1$, and

$$K_1 = \frac{P_m}{m + M}, \qquad K_2 = \frac{-Mn_1{}^2 a_1}{m + M}. \tag{6a,b}$$

$$A = \frac{-a}{m + M}, \qquad B = \frac{-b}{m + M}, \qquad C = \frac{-c}{m + M}. \tag{7a,b,c}$$

4. *The recovery phase*

This occurs during the time interval $\tau_1 \leqslant t \leqslant \tau_1 + \tau_2$, or $0 \leqslant t' \leqslant \tau_2$ where $t' = t - \tau_1$.

During the recovery the rowers bend their legs to draw themselves on their sliding seats towards the stern of the boat. The combined force F on the rowers which produces their motion is shown dashed in Figure 2, and the equal and opposite force F on the footrests is shown as a full arrow because it is a force on the boat. Just as for Figure 1, it is enough to illustrate the situation for a single oar.

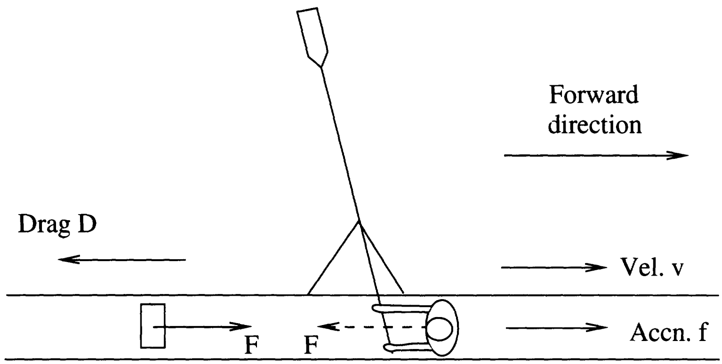

FIGURE 2 Plan view of the recovery-phase situation.

During the first half of the recovery phase the directions of the forces F are as shown in Figure 2, and the boat accelerates in the forward direction. For the second half of the recovery the forces F and the boat's acceleration reverse their directions, and this will shortly be seen analytically.

During the recovery phase the positions occupied by the rowers' bodies are very similar to those during the power stroke, but assumed of course in the opposite direction and over a different time span τ_2. The relative forward displacement of the rowers from the central point of their travel during $0 \leqslant t' \leqslant \tau_2$ may thus be taken as

$$x_2 = a_1 \cos n_2 t', \tag{8}$$

where the amplitude a_1 is the same as in equation (3), and $n_2 = \pi / \tau_2 =$ the circular frequency of the SHM.

The forward acceleration of the rowers relative to the boat is $\ddot{x}_2$, and relative to the water is $\ddot{x}_2 + f$. The forward equation of motion of the rowers is therefore

$$-F = M(\ddot{x}_2 + f) = M\left(-n_2^2 a_1 \cos n_2 t' + \frac{dv}{dt'}\right).$$

The equation of motion of the boat is

$$F - D = m\frac{dv}{dt'}.$$

On adding the last two equations and using (1) it is seen that

$$(m + M)\frac{dv}{dt'} = M n_2^2 a_1 \cos n_2 t' - a - bv - cv^2.$$

Dropping the dash from t' (for convenience only), this recovery phase equation may also be written as

$$\frac{dv}{dt} = K_3 \cos n_2 t + A + Bv + Cv^2, \tag{9}$$

where $0 \leqslant t \leqslant \tau_2, n_2 = \pi / \tau_2$, and

$$K_3 = \frac{M n_2^2 a_1}{m + M}, \tag{10}$$

with A, B, C given by (7 a,b,c).

5. A particular numerical example

To apply the foregoing theory to a particular case, a racing eight will be considered. The constants a, b, c in equation (1) are known for such a boat from work done in the Appendix. It is shown there that the drag on the hull (in newtons) is given by

$$D = 24.93 - 11.22v + 13.05v^2,$$

where the boat speed v is in m/s. The values of the constants in (1) in SI units are therefore

$$a = 24.93, \qquad b = -11.22, \qquad c = 13.05.$$

A video of an Australian Olympic eight in action over a 2000-metre course enabled estimates to be made of the power-stroke and recovery durations, and it was decided to take $\tau_1 = 0.7$ s, $\tau_2 = 0.9$ s. Of course in a race the durations would vary, but for purposes of calculation they are taken as constant. The time for a complete stroke is 1.6 s, which corresponds to a stroke rate of 37.5 per minute.

The amplitude a_1 of the motion of the centres of mass of the rowers' bodies is estimated to be 0.36 m.

The mass of the boat plus cox, and the combined masses of the eight rowers are taken to be respectively $m = 146$ kg, $M = 680$ kg.

Measurements of the force exerted by rowers while using a rowing ergometer yielded an average maximum value of 447.4 N. This is not meant to imply that this force is known to within 0.1 N, but that this number of significant figures is needed to ensure that the results of subsequent calculations are sufficiently accurate. This force estimate lies within the range obtained by Mason et al [5]. For all eight rowers combined, the value of the force R in Figure 1 is then 3579 N. Equations (2) and (4) show that

$$P_m = [(\ell / h) - 1]^{-1} \times \max R,$$

where ℓ and h are the lengths depicted in Figure 1. If the estimates $\ell = 3.40$ m and $h = 1.02$ m are used, then

$$P_m = 0.4286 \times 3579 = 1534 \text{ N},$$

and this is the value that is used in (6a) to calculate the value of the constant K_1.

In the power-stroke differential equation

$$\frac{dv}{dt} = K_1 \sin n_1 t + K_2 \cos n_1 t + A + Bv + Cv^2, \tag{5}$$

the relevant domain is $0 \leqslant t \leqslant 0.7$, and $n_1 = \pi / 0.7$ rad/s. The values of the other constants in (6 a,b) and (7 a,b,c,) are found to be (in S.I. units)

$$K_1 = 1.8577, \qquad K_2 = -5.9695,$$

$$A = -0.030182, \qquad B = 0.013584, \qquad C = -0.015799.$$

In the recovery-phase equation

$$\frac{dv}{dt} = K_3 \cos n_2 t + A + Bv + Cv^2, \tag{9}$$

the domain is $0 \leqslant t \leqslant 0.9$, and $n_2 = \pi / 0.9$ rad/s. The values of A, B, C are as listed above, and $K_3 = 3.6112$.

Equations (5) and (9) are not very amenable to analytical solution. A computer was therefore used to solve them numerically, using the Runge-Kutta method. To investigate the acceleration of the boat from a stationary start it was assumed that (5) and (9) applied from the outset, and the following iterative procedure was used.

An initial velocity of $v = v_{01} = 0$ was used, and (5) was solved to find the velocity $v = v_{11}$ at $t = 0.7$, the end of the first power-stroke. This value v_{11} was used as a starting value with the recovery equation (9), which was then solved numerically to yield the velocity $v = v_{21}$ at $t = 0.9$, the end of the first complete stroke. The whole procedure was then repeated, using in (5) the new initial velocity $v_{02} = v_{21}$ and arriving at a new value for v_{22} at the end of the second complete stroke. The iteration was continued until the value v_{2n} was repeated to sufficient accuracy after successive strokes, showing that the boat had reached a 'steady state'.

The distance travelled by the boat during each stroke was calculated by numerical integration of the velocity, from which the mean boat velocity $\tilde{v}$ during each stroke was found by dividing the distance by the stroke duration of 1.6 seconds. Because the velocity varies greatly during a stroke, $\tilde{v}$ is a more suitable quantity to plot as a function of time than the instantaneous

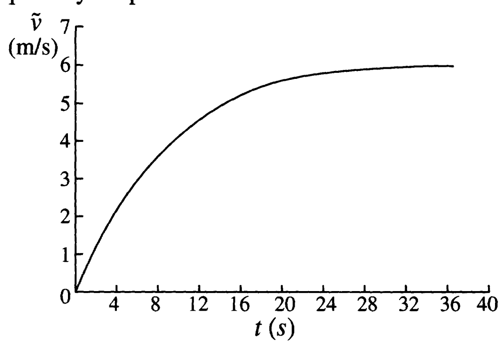

FIGURE 3 Mean boat velocity $\tilde{v}$ versus time t during acceleration

velocity. Figure 3 was formed by drawing a smooth curve through the points $(t, \tilde{v})$, where t is the time from the start to the middle of the stroke to which the $\tilde{v}$ value refers.Figure 3 shows that the boat reaches a constant mean speed after about 40 seconds, which corresponds to 25 complete strokes.

The computer solution was also used to find the boat velocity at 0.1 second intervals throughout a complete stroke after the 'steady state' had been achieved. Figure 4 was formed by drawing a smooth curve through the resulting points (t, v).

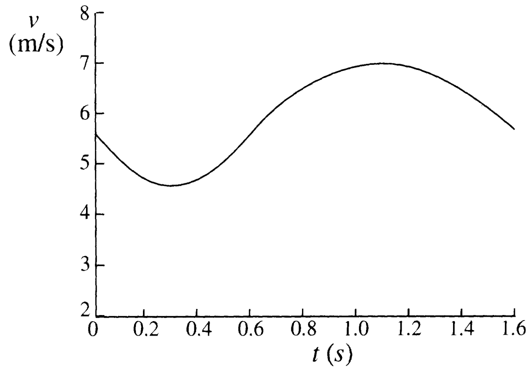

FIGURE 4 Boat velocity v during one stroke in the 'steady state' situation

Figure 4 shows that the boat speed drops to below 4.6 m/s near the middle of the power stroke, and that it reaches nearly 7 m/s near the middle of the recovery phase. This speed variation is mainly the result of the forces exerted by the rowers on their footrests; during the power stroke they are driving the boat back against its predominantly forward motion, and during the recovery phase they are dragging the boat forward and augmenting its velocity. From a dynamical viewpoint one would say that there is an exchange of momentum between the rowers and the boat, with the motion of their combined centre of mass being much more uniform than the motion of either component.

The distance travelled by the boat during one 'steady state' stroke was also calculated and found to be 9.488 m, which corresponds to a mean velocity $\tilde{v}$ of 5.93 m/s.

The time taken for the eight to row the 2000 m course can now be calculated. During the acceleration phase of 40 seconds duration, the distance travelled by the boat was found from the computer solution to be 200.7 m. The time taken to cover the remaining 1799.3 m at the mean 'steady' speed of 5.93 m/s is 303 s, making the total race time 5 m 43 s. This would be a reasonable time for an Olympic eight, and a very good time for a club crew. It suggests that the value taken for the maximum force exerted by each rower was a reasonable one.

The principles used in the foregoing example of a racing eight would apply equally well to fours and pairs, and even to double, quad and single sculls, provided the obvious modifications were made for the numbers of rowers and oars involved, and for the mass and drag of the hull.

6. *Two-phase rowing*

The variation in boat speed during a complete stroke causes the water resistance to be greater than it would be if the speed were constant. One way of achieving a more uniform speed is to have the rowers operating out of phase with each other. It is not feasible to have all rowers with different phases for several reasons, including the extra boat length that would be needed to permit such an arrangement. It is, however, quite practicable to have two groups of rowers which are exactly out of phase with each other. In the case of an eight, a group of four at the bow end could row in unison and exactly out of phase with a group of four at the stern end. Calculations show that an extra space of 2m between numbers 4 and 5 is sufficient to avoid a clash between these rowers, and between the oars of 3 and 5, and of 4 and 6. It will be assumed that the drag D of the boat is unchanged by this modification.

An analysis will be made of the effect of such a two-phase arrangement for an eight, on the assumption that the two groups are equally matched as regards their mass, strength and rowing efficiency, and using the same durations for the recovery phase and power stroke as for the single-phase case considered earlier.

Figure 5 shows schematically the relationships between the recovery and power strokes for the Bow Four and Stern Four over a complete cycle. The terms Up and Down refer to the blades of the oars being out of the water during the recovery and in the water during the power stroke. Clearly it will be sufficient to analyse the situation for half a cycle, such as that corresponding to the period $0 \leqslant t \leqslant 0.8$ depicted in the figure.

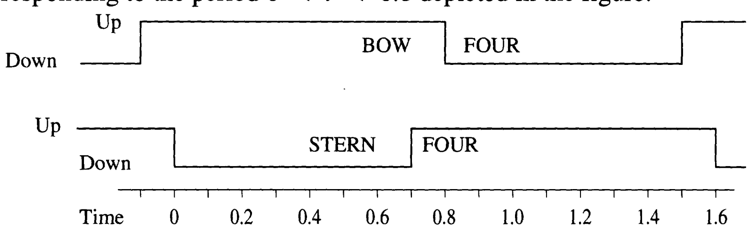

FIGURE 5 Two-phase rowing arrangement for an eight

The notation employed in the single-phase case will again be used. In the two-phase case the combined mass of each group of four rowers will be taken as $\frac{1}{2}M$, and in Figures 1 and 2 the forces shown will all be halved when applied to each group.

From Figure 5 it is clear that the intervals $0 \leqslant t < 0.7$ and $0.7 < t \leqslant 0.8$ must be considered separately.

(i) The period $0 \leqslant t < 0.7$

For the Bow Four, this period begins 0.1 seconds after the start of their recovery phase. The equation corresponding to (8) which gives the relative forward displacements of the Bow Four from their central position is

$$x_2 = a_1 \cos n_2 (t + 0.1),$$

where t is used instead of t'.

For the Bow Four, instead of F in Figure 2, let F_1 denote the corresponding combined force of these four rowers. Their forward equation of motion is therefore

$$-F_1 = \tfrac{1}{2}M(\ddot{x}_2 + f) = \tfrac{1}{2}M\left[n_2^2 a_1 \cos n_2 (t + 0.1) + \frac{dv}{dt}\right]. \tag{11}$$

For the Stern Four, Figure 5 shows that the interval $0 \leqslant t < 0.7$ corresponds to a complete power stroke. The relative forward displacements of these four rowers from their central position is again given by equation (3).

For the Stern Four, instead of P, Q, R in Figure 1, let P_1, Q_1, R_1 denote the corresponding combined forces for these four rowers. Their forward equation of motion is then

$$Q_1 - R_1 = \tfrac{1}{2}M(\ddot{x}_1 + f) = \tfrac{1}{2}M\left[-n_1^2 a_1 \cos n_1 t + \frac{dv}{dt}\right]. \tag{12}$$

The equation of motion of the boat is

$$F_1 + P_1 - Q_1 - D = m\frac{dv}{dt}. \tag{13}$$

Adding equations (11), (12), (13) leads to

$$(m + M)\frac{dv}{dt} = P_1 - R_1 + \tfrac{1}{2}Ma_1\left[n_2^2 \cos n_2(t + 0.1) - n_1^2 \cos n_1 t\right] - D.$$

Exactly as in equation (2) it is seen that

$$P_1 - R_1 = \frac{h}{\ell}P_1 \tag{14}$$

and, as in equation (4),

$$\frac{h}{\ell}P_1 = P_{1m} \sin n_1 t \tag{15}$$

by the same logic as before. Since only four rowers are in power-stroke action when it occurs, it is clear that

$$P_{1m} = \tfrac{1}{2}P_m. \tag{16}$$

It is assumed that the drag D on the boat is still given by (1) with the values of a, b, c unchanged.

On using (1), (14), (15) and (16) the power-stroke differential equation above becomes

$$(m + M)\frac{dv}{dt} = \tfrac{1}{2}P_m \sin n_1 t + \tfrac{1}{2}Ma_1\left[n_2^2 \cos n_2(t + 0.1) - n_1^2 \cos n_1 t\right]$$
$$-a - bv - cv^2.$$

This may also be written as

$$\frac{dv}{dt} = \tfrac{1}{2}K_1 \sin n_1 t + \tfrac{1}{2}K_2 \cos n_1 t + \tfrac{1}{2}K_3 \cos n_2 (t + 0.1) + A + Bv + Cv^2, \quad (17)$$

where $0 \leqslant t < 0.7$, $n_1 = \pi / 0.7$, $n_2 = \pi / 0.9$, K_1, K_2 are given by (6 a,b), K_3 is given by (10), and A, B, C are given by (7 a,b,c).

(ii) The period $0.7 < t \leqslant 0.8$.

Figure 5 shows that during this brief period both groups of rowers are in recovery mode. For the Bow Four, equation (11) shows that the force corresponding to F in Figure 2 is

$$F_1 = \tfrac{1}{2}M\left[n_2^2 a_1 \cos n_2 (t + 0.1) - \frac{dv}{dt}\right]. \quad (18)$$

This equation is valid for $-0.1 \leqslant t \leqslant 0.8$, which includes the period now being considered. From Figure 5 it can be seen that the recovery period $0.7 < t \leqslant 0.8$ for the Stern Four is precisely the same as the recovery period $-0.1 \leqslant t \leqslant 0$ for the Bow Four. If F_2 denotes the combined force for the Stern Four that corresponds to F in Figure 2, it follows from (18) that

$$F_2 = \tfrac{1}{2}M\left[n_2^2 a_1 \cos n_2 (t - 0.7) - \frac{dv}{dt}\right].$$

The sum of these forces on the boat is

$$\begin{aligned} F_1 + F_2 &= \tfrac{1}{2}Mn_2^2 a_1 \left[\cos n_2 (t + 0.1) + \cos n_2 (t - 0.7)\right] - M\frac{dv}{dt}, \\ &= Mn_2^2 a_1 \cos n_2 (t - 0.3) \cos (0.4 n_2) - M\frac{dv}{dt}. \quad (19) \end{aligned}$$

The forward equation of the motion of the boat is

$$F_1 + F_2 - D = m\frac{dv}{dt},$$

which, by virtue of (1) and (19), becomes

$$(m + M)\frac{dv}{dt} = Mn_2^2 a_1 \cos (0.4 n_2) \cos n_2 (t - 0.3) - a - bv - cv^2.$$

This may also be written as

$$\frac{dv}{dt} = K_4 \cos n_2 (t - 0.3) + A + Bv + Cv^2, \quad (20)$$

where $0.7 < t \leqslant 0.8$, A, B, C are given by (7 a,b,c) and

$$K_4 = \frac{Mn_2^2 a_1 \cos (0.4 n_2)}{m + M} = K_3 \cos (0.4 n_2). \quad (21)$$

For computational purposes it is more convenient to introduce $t' = t - 0.7$, so that the interval to be considered is $0 < t' \leqslant 0.1$. Then (21) becomes

$$\frac{dv}{dt'} = K_4 \cos n_2 (t' + 0.4) + A + Bv + Cv^2. \quad (22)$$

In practice the dash may be dropped from t' in (22), it being remembered that then $0 < t \leqslant 0.1$.

7. Comparison of conventional and two-phase rowing

To compare the effectiveness of the two modes of rowing in an eight, the particular example discussed in Section 5 was used. The relevant two-phase equations of Section 6 are:

$$\frac{dv}{dt} = \tfrac{1}{2}K_1 \sin n_1 t + \tfrac{1}{2}K_2 \cos n_1 t + \tfrac{1}{2}K_3 \cos n_2 (t + 0.1) + A + Bv + Cv^2, \quad (17)$$

where $0 \leqslant t < 0.7$, $n_1 = \pi / 0.7$, $n_2 = \pi / 0.9$, and

$$\tfrac{1}{2}K_1 = 0.92887, \qquad \tfrac{1}{2}K_2 = -2.9847, \qquad \tfrac{1}{2}K_3 = 1.8056,$$

$$A = -0.030182, \qquad B = 0.013584, \qquad C = -0.015799;$$

and

$$\frac{dv}{dt} = K_4 \cos n_2 (t + 0.4) + A + Bv + Cv^2, \quad (22)$$

where $0 < t \leqslant 0.1$, $n_2 = \pi / 0.9$, $K_4 = K_3 \cos(0.4 n_2) = 0.62707$, and A, B, C have the values listed above.

These equations were solved with the aid of a computer in the same way as that described in Section 5 for the single-phase case. To investigate the acceleration of the boat from a stationary start, the same iterative procedure was used with (17) and (22) as was done with (5) and (9). The distance travelled by the boat during each time interval of 1.6 seconds was found as before by numerical integration of the velocity, and the mean velocity $\tilde{v}$ during each of these intervals was found by dividing the distance by 1.6. The acceleration of the boat from rest was found just as for the single-phase case, and is depicted by the dashed curve in Figure 6. To enable the

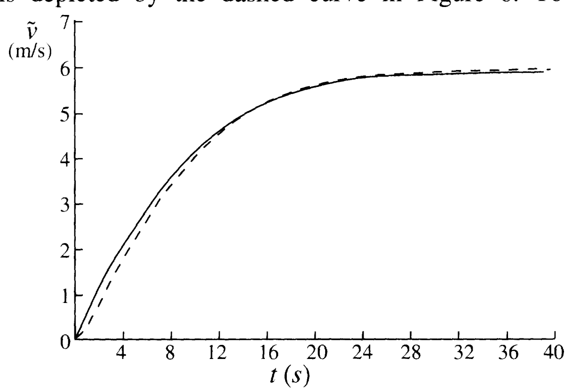

FIGURE 6 Comparison of boat velocity $\tilde{v}$ for single-phase and two-phase rowing during acceleration

——————— single-phase; - - - - - - - two-phase accelerations in the two modes to be compared, the single-phase curve of Figure 3 is repeated in Figure 6 as a full line.

Figure 6 shows that if two-phase rowing is used from the start it will take about 19 seconds (or 12 complete strokes) to overtake the single-phase performance, and that thereafter a two-phase boat would be travelling faster. The figure also shows that the acceleration for single-phase rowing is greater than that for two-phase only during the first complete stroke, thereafter it is smaller. Clearly it would theoretically be better to switch from single to two-phase after the first complete stroke; but in practice it would probably be better to continue with single-phase for a few more strokes to get the boat well under way before switching to two-phase rowing.

The velocity of the boat throughout a complete stroke in the 'steady state' was found just as for the single-phase case, and is shown as a dashed curve in Figure 7. For comparison purposes the single-phase velocity curve of Figure 4 is reproduced as a full line in Figure 7.

Figure 7 shows that the variation in boat velocity throughout the stroke is much less for two-phase rowing than for single-phase, varying between about 5.8 m/s and 6.2 m/s. It is this property which is responsible for the improved performance obtainable from two-phase rowing, for the drag D of the water is least when the variation in boat velocity is smallest.

The distance travelled by the boat during one complete two-phase 'steady-state' stroke was calculated and found to be 9.588 m, which corresponds to a mean velocity $\tilde{v}$ of 5.99 m/s.

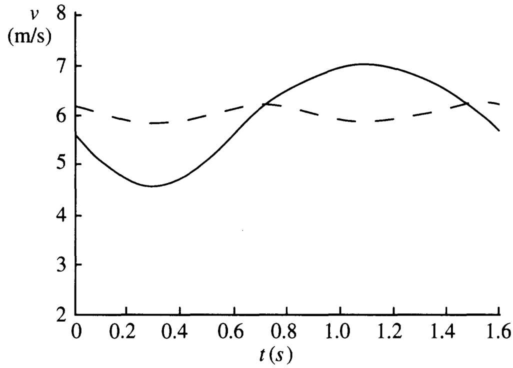

FIGURE 7 Comparison of boat velocity v for single-phase and two-phase rowing in the 'steady state'

——— single-phase; - - - - - - - two-phase

One can now calculate the improvement in terms of boat lengths in a 2000-metre race for eights which would come from using two-phase instead of single-phase rowing. For simplicity, the small gain achievable during the acceleration in the early stages of the race will be ignored. The 'steady state' was found to be reached by single-phase rowing after 200.7 m, leaving

1799.3 m still to be travelled. The distance travelled per stroke at that stage was 9.49 m for single-phase and 9.59 m for two-phase. The two-phase boat would cover the 1799.3 m in $1799.3 / 9.59 = 187.6$ strokes, during which time the single-phase boat would have travelled $187.6 \times 9.49 = 1781$ m, leaving it about 18 m behind the two-phase boat. Taking the boat length to be 17 m, the gain resulting from using two-phase rowing would be just over one boat length in a 2000-metre race.

8. *Conclusions*

A mathematical model was set up to represent the rowing stroke in a racing shell. An eight was used for the numerical work, but the principles involved apply also to fours and pairs and to single, double and quad sculls. The validity of the model is verified by its success in predicting quantitatively the familiar variation in boat speed during a stroke. The reason for this speed variation is revealed precisely by the model.

The concept of two-phase rowing was introduced, and the mathematical analysis of it showed that its use could lead to a gain of just over one length for an eight in a 2000 m race assuming that it caused no change in the drag on the boat. Although attention was confined to its effect in an eight, two-phase rowing could also be used in fours and in double and quad sculls, the changes required in these other cases being obvious.

9. *Acknowledgements*

The authors are grateful to racing shell builder Mr Jeff Sykes, of Jeff Sykes and Associates, Geelong, for information and advice; to Professor B.C. Rennie for advice and for drawing their attention to the NPL Technical Memorandum mentioned in Reference [3], and for providing a copy of it; and to British Maritime Technology Limited for permission to reproduce in the Appendix a graph from that Technical Memorandum.

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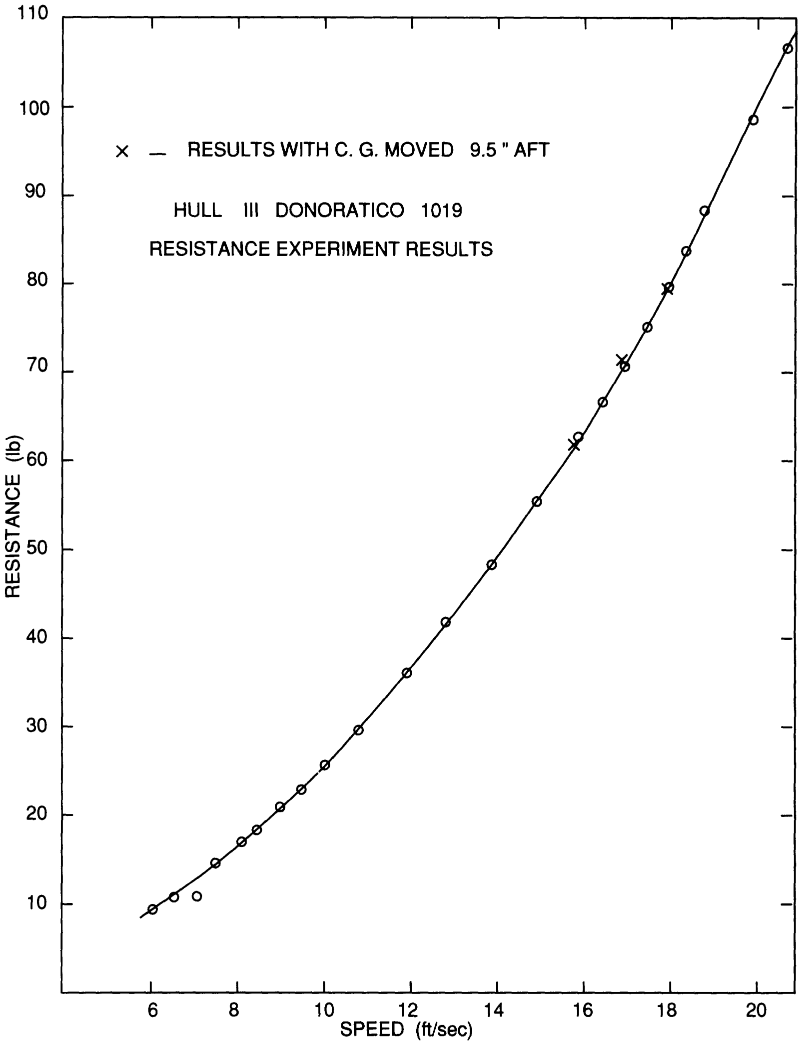

FIGURE 8 Hull resistance in lb wt versus speed in ft/sec for a typical racing eight hull (reproduced from Wellicome (1967) by permission of British Maritime Technology Limited).

Appendix

The formula for the resistance of a racing shell

Wellicome [3] describes resistance measurements made on racing eight hulls in a water tank at the Ship Division of the National Physical Laboratory. The results of one set of experiments are shown in Figure 8, reproduced unchanged from the above Reference.

The method of least squares was used to fit a quadratic velocity equation to the drag curve of Figure 8. In terms of the units used in that Figure the fitted equation is

$$D = 5.6027 - 0.7685V + 0.2725V^2 \text{ lb wt},$$

where V is the boat speed in ft/s. In S.I. units this is

$$D = 24.93 - 11.22v + 13.05v^2 \text{ N},$$

where v is in m/s. This is the origin of the equation stated near the beginning of Section 5.

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top quality	Get 13 months'
Home	home insurance
Contents	cover for the
Insurance	price of 12

From two sides of an advertising leaflet from Royal & Sun Alliance, sent in by Hamish Sloan who clearly does not think they are equivalent.

Does it have $\sqrt{80}$ lives?

Snapped by Peter Ransom at Foxton Locks in May 1998.