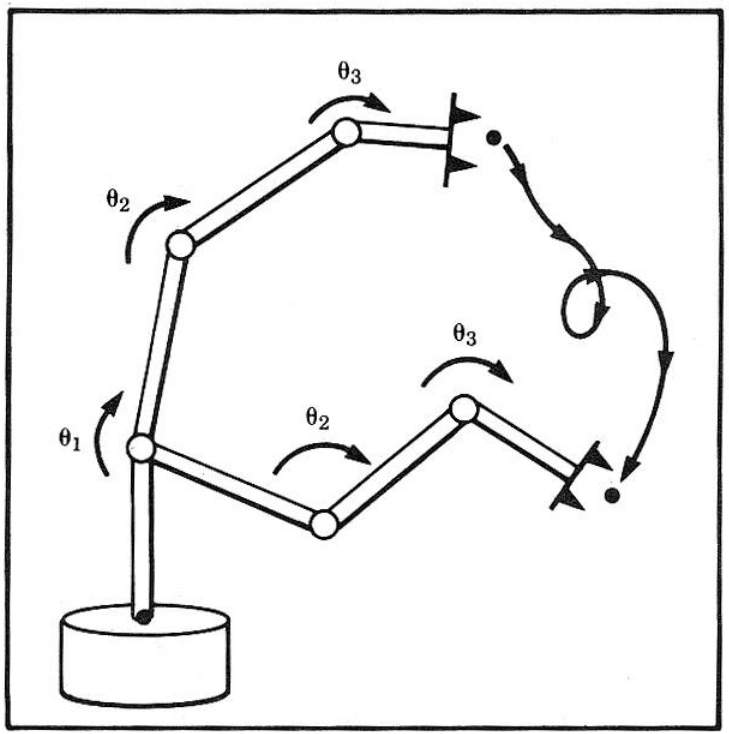
Control of Manipulators

Introduction – Problem Definition

Problem

Given: Joint angles (*sensor readings) links geometry, mass, inertia, friction, Direct /inverse kinematics & dynamics*

Compute: Joint torques to achieve an end effector position / trajectory

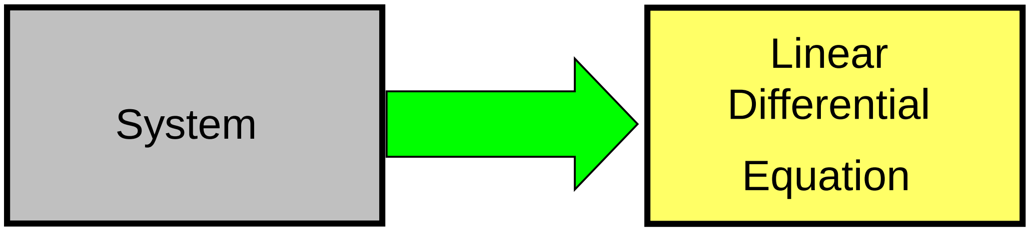


Solution

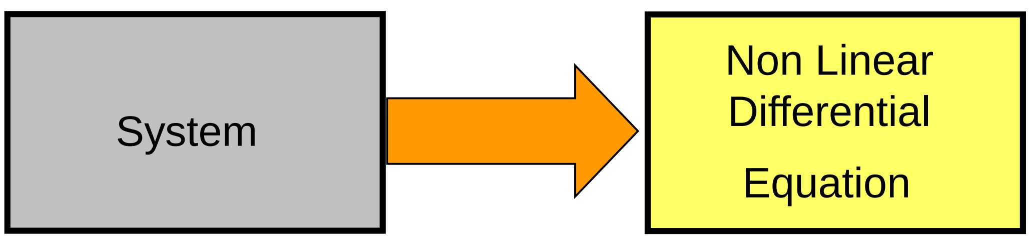
Control Algorithm (PID - Feedback loop, Feed forward dynamic control)

Introduction – Linear Control

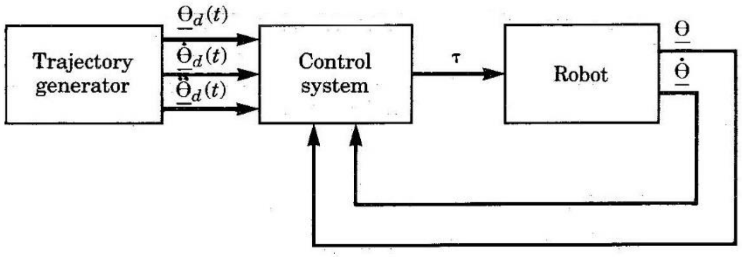
- LDF - Linear Control – Valid Method (strictly speaking)



- NLDF - Linear Control – Approximation (practically speaking)
  - Non Linear Elements (Stiffness, damping, gravity, friction)
  - Frequently used in industrial practice

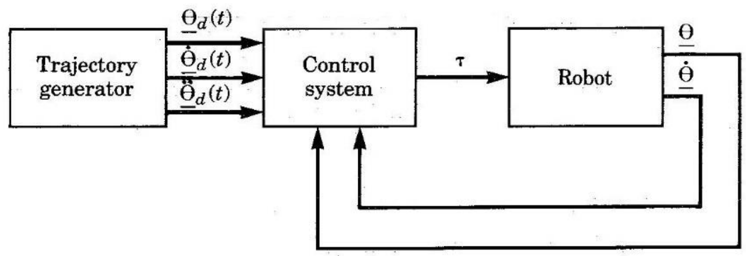


Feedback & Close Loop Control



- Robot (Manipulator) Modeling
  - Mechanism
  - Actuator
  - Sensors (Position / Velocity, Force/toque)
- Task (input command)
  - Position regulation
  - Trajectory Following
  - Contact Force control
  - Hybrid (position & Force)
- Control System – compute torque commands based on
  - Input
  - Feedback

Feedback & Close Loop Control

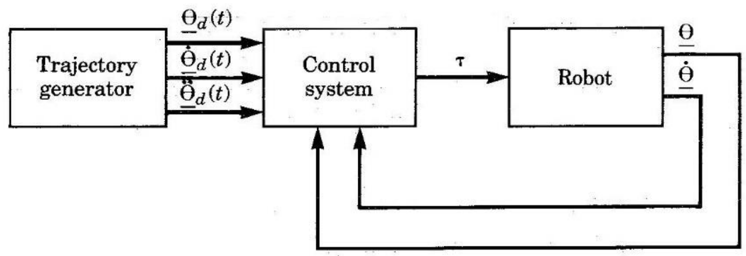


- Open Loop Control System – No feedback from the joint sensor

$$\tau = M(\Theta_d)\ddot{\Theta}_d + V(\Theta_d, \dot{\Theta}_d) + G(\Theta_d)$$

- Impractical - problems
  - Imperfection of the dynamics model
  - Inevitable disturbance

Feedback & Close Loop Control



- Close Loop Control System – Use feedback form joint sensors
- Servo Error – Difference between the desire joint angle and velocity and the actual joint angle and velocity

$$E = \Theta_d - \Theta$$

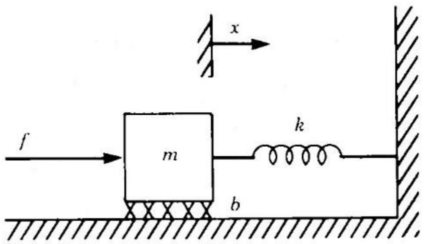
$$\dot{E} = \dot{\Theta}_d - \dot{\Theta}$$

Feedback & Close Loop Control

- Control Design
  - Stability (Servo Errors remain small when executing trajectories)
  - Close loop performance

- Input / Output System
  - MIMO – Multi-Input Multi-Output
  - SISO - Single Input Single Output
  - Current discussion – SISO approach
  - Industrial Robot – Independent joint control (SISO approach)

Position Control – Second Order System
Position Regulation



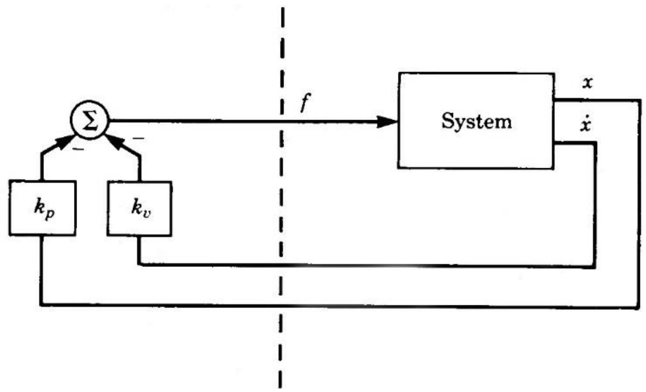
- Problem
  - Option 1: The natural response of the mechanical system is under damped and oscillatory
  - Option 2: The spring is missing and the system never returns to its initial position if disturbed.
- Position regulation – maintain the block in a fixed place regardless of the disturbance forces applied on the block
- Performance (system response) - critically damped
- Equation of motion (free body diagram)

$$m\ddot{x} + b\dot{x} + kx = f$$

Position Control – Second Order System
Position Regulation

- Proposed control law

$$f = -k_p x - k_v \dot{x}$$



- Close loop dynamics

$$m\ddot{x} + b\dot{x} + kx = -k_p x - k_v \dot{x}$$

Position Control – Second Order System
Position Regulation

$$m\ddot{x} + (b + k_v)\dot{x} + (k + k_x)x = 0$$

$$m\ddot{x} + b'\dot{x} + k'x = 0$$

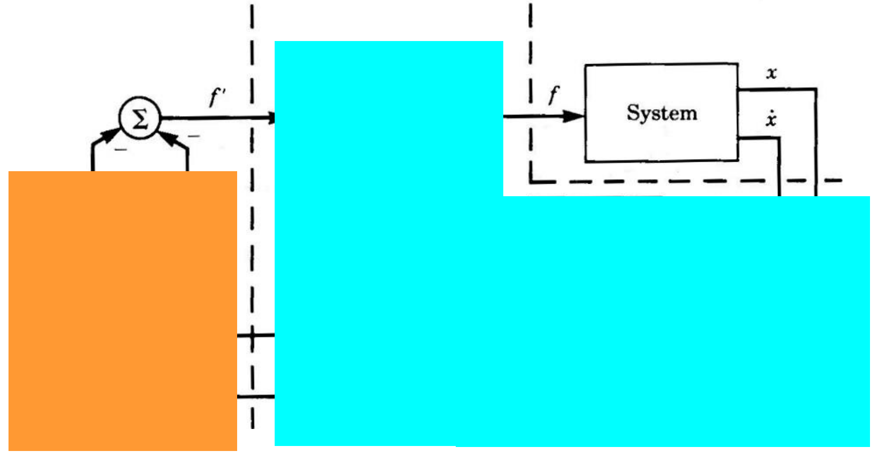
$$b' = b + k_v$$

$$k' = k + k_p$$

- By setting the control gains (k_v, k_p) we cause the close loop system to appear to have ANY second order system behaviors that we wish.
- For example: Close loop stiffness k_p and critical damping $b' = 2\sqrt{mk'}$

Control Law Partitioning

Servo- Based



Model- Based portion

- Partition the controller into
  - **Model- Based portion** – Make use of the supposed knowledge of m, b, k . *It reduce the system so that it appears to be a unite mass*
  - **Servo based portion**
- **Advantages** – Simplifying the servo control design – gains are chosen to control a unite mass (i.e. no friction no mass)

Control Law Partitioning

- Equation of motion

$$m\ddot{x} + b\dot{x} + kx = f$$

- Define the model based portion of the control

$$f = \alpha \ f' + \beta$$

- Combine

$$m\ddot{x} + b\dot{x} + kx = \alpha \ f' + \beta$$

- Define

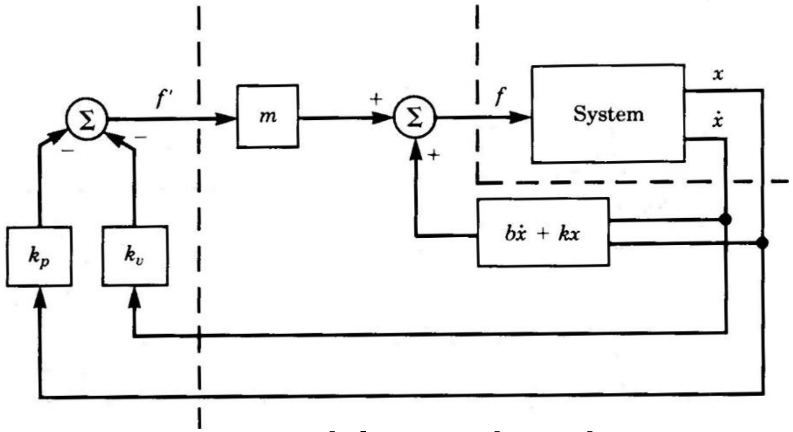
$$m = \alpha$$

$$\beta = b\dot{x} + kx$$

- Resulting

$$\ddot{x} = f'$$

Control Law Partitioning



Servo- Based

Model- Based portion

$$f = \alpha\ f' + \beta$$

$$m = \alpha$$

$$\beta = b\dot{x} + kx$$

- Control law

$$f' = -k_p x - k_v \dot{x}$$

- Combing the control law with the unit mass ($\ddot{x} = f'$) the close loop eq. of motion becomes

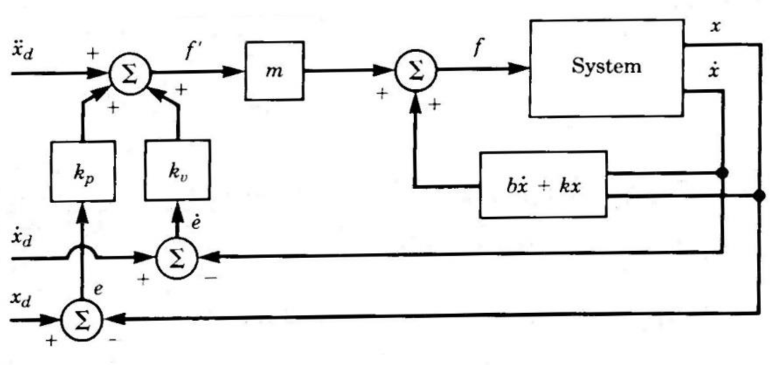
$$\ddot{x} + k_v \dot{x} + k_p x = 0$$

Control Law Partitioning

- Setting the control gains is independent of the system parameters (e.g. for critical damping with a unit mass $m=1$)

$$k_v = 2\sqrt{k_p}$$

Trajectory Following Control



- Trajectory Following – Specifying the position of the block as a function of time $x_d(t)$
- Assumption – smooth trajectory i.e. the fist two derivatives exist
- The trajectory generation $x_d, \dot{x}_d, \ddot{x}_d$
- Define the error between the desired and actual trajectory

$$e = x_d - x$$

Trajectory Following Control

- Servo control

$$f' = \ddot{x}_d + k_v \dot{e} + k_p e$$

- Combined with the eq. of motion of a unite mass leads to

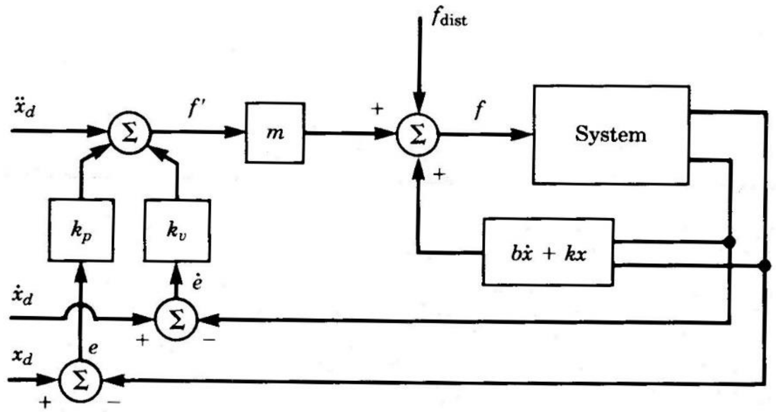
$$\ddot{x} = \ddot{x}_d + k_v \dot{e} + k_p e$$

$$\ddot{e} + k_v \dot{e} + k_p e = 0$$

- Select k_v, k_p to achieve specific performance (i.e. critical damping)
- IF
  - Our model of the system is perfect (knowledge of m, b, k)
  - No noise
- Then the block will follow the trajectory exactly (suppress initial error)

Disturbance Rejection

- Control system provides disturbance rejection
- Provide good performance in the present of
  - External disturbance
  - Noise in the sensors



- Close loop analysis - the error equation

$$\ddot{e} + k_v \dot{e} + k_p e = f_{dist} / m$$

Disturbance Rejection

$$\ddot{e} + k_v \dot{e} + k_p e = f_{dist} / m$$

- If f_{dist} is bounded such that

$$\max_t f_{dist}(t) < a$$

- The the solution of the differential equation $e(t)$ is also bounded

- This result is due to a property of a stable linear system known as bounded-input bounded-output (BIBO) stability

Disturbance Rejection

- Steady state error

$$\cancel{\ddot{e}} + k_v \cancel{\dot{e}} + k_p e = f_{dist} / m$$

$$e = f_{dist} / m k_p$$

- The higher the position gain k_p the small will be the steady state error.
- In order to eliminate the steady state error a modified control low is used

$$f' = \ddot{x}_d + k_v \dot{e} + k_p e + k_i \int e\,dt$$

Disturbance Rejection

- Which results in the error equation

$$\ddot{e} + k_v \dot{e} + k_p e + k_i \int e\,dt = f_{dist} / m$$

- If $e(t) = 0$ for $t < 0$ then for $t > 0$

$$\dddot{e} + k_v \ddot{e} + k_p \dot{e} + k_i e = \dot{f}_{dist} / m$$

- Which in a steady state (for constant disturbance) becomes

$$k_i e = 0$$

$$e = 0$$

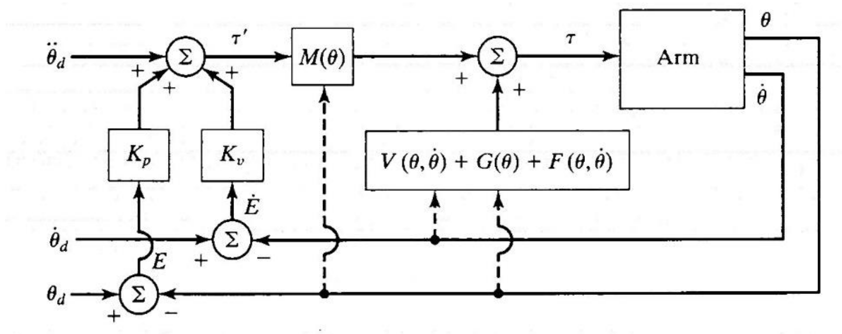
Control Problem of Manipulator – Generalized Approach

- Equation of Motion (rigid body dynamics)

$$\tau = M(\Theta)\ddot{\Theta} + V(\Theta, \dot{\Theta}) + G(\Theta) + F(\Theta, \dot{\Theta})$$

- $M(\Theta)$ Inertia matrix n x n
- $V(\Theta, \dot{\Theta})$ Centrifugal and Coriolis terms n x 1
- $G(\Theta)$ Gravity terms n x 1
- $F(\Theta, \dot{\Theta})$ Friction Term n x 1

Control Problem of Manipulator – Generalized Approach



- Partitioning control scheme

$$\tau = \alpha \tau' + \beta$$

$$\alpha = M(\Theta)$$

$$\beta = V(\Theta,\dot{\Theta}) + G(\Theta) + F(\Theta,\dot{\Theta})$$

- Servo control law

$$\tau' = \ddot{\Theta}_d + K_v \dot{E} + K_p E$$

$$E = \Theta_d - \Theta$$

Control Problem of Manipulator – Generalized Approach

- The close loop system characterized by the error equation

$$\ddot{E} + K_v \dot{E} + K_p E = 0$$

- Note The vector equation is decoupled: the matrix K_p , K_v are diagonal . The equation can be written on a joint by joint basis

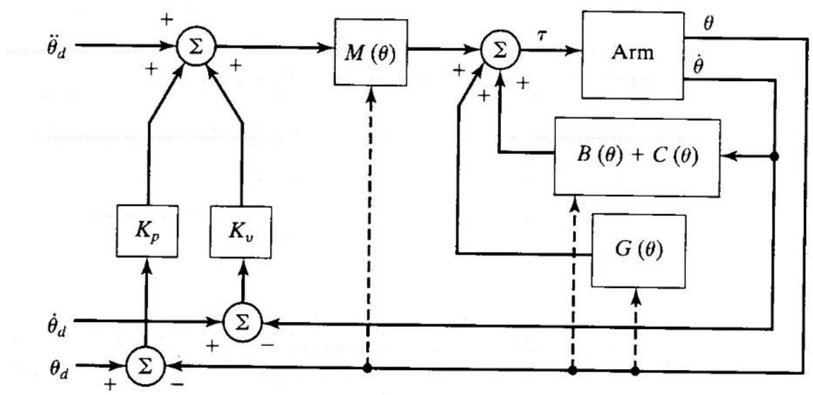
$$\ddot{e}_i + k_{vi} \dot{e} + k_{pi} e = 0$$

- Reservations – The ideal performance is unattainable in practice due the many reasons including:
  - Discrete nature of a digital computer
  - Inaccuracy of the manipulator model

Practical Considerations – Time Requirements

- Time required to compute the model
  - Model based control requires to predict joint toques based on the **dynamic equation** of the manipulator
  - Digital control / Sampling rate – For every time interval
    - Read sensor
    - Calculate feedback command
    - Send command to the actuator

Practical Considerations – Time Requirement - Dual Rate Computed Torque



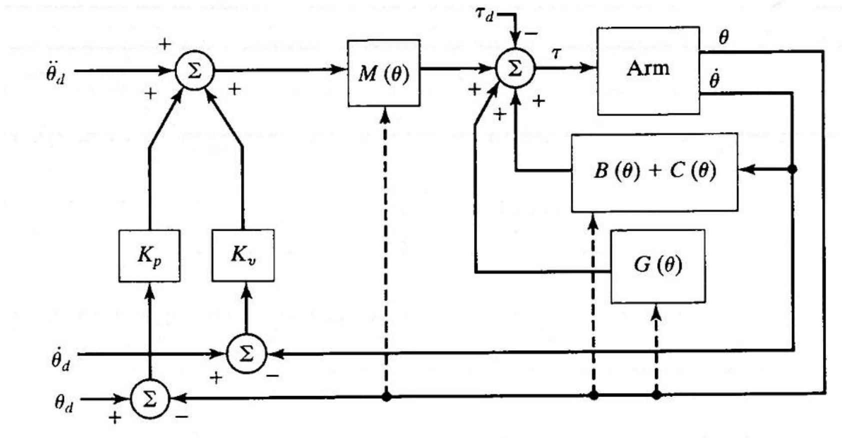
Solid Line – High rate Servo (e.g 250 Hz)
Dashed line – Low rate dynamic model (e.g. 60 HZ)

$$\tau = \boxed{M(\Theta)}\ddot{\Theta} + \boxed{V(\Theta,\dot{\Theta})} + \boxed{G(\Theta)} + F(\Theta,\dot{\Theta})$$

- Compute the joint angle based elements of the equation of motion
  - Lower rate (then the servo)
  - Pre-compute (look-up table)

Practical Considerations – Lack of Knowledge of the Parameters

- The manipulator dynamics is often not known accurately in particular
  - Friction (parameter & model)
  - Time dependent dynamics (robot joint wear)
  - Unknown external load (mass & inertia) – e.g. grasping a tool or a port by the end effector
- Summing up all the the disturbance and unknown parameters



Practical Considerations – Lack of Knowledge of the Parameters

- Error equation
  - Ideal Case $$\ddot{E} + K_v \dot{E} + K_p E = 0$$
  - Practical case $$\ddot{E} + K_v \dot{E} + K_p E = M^{-1}(\Theta)\tau_d$$
- Steady state Error $$E = K_p^{-1} M^{-1}(\Theta)\tau_d$$
- Expressing the disturbance explicitly results in

$$\ddot{E} + K_v \dot{E} + K_p E = \hat{M}^{-1}[(M - \hat{M})\ddot{\Theta} + (V - \hat{V}) + (G - \hat{G}) + (F - \hat{F})]$$

- If the model was exact the right hand side would be zero and so is the error.

Current Industrial Robot Control Systems
The Harsh Reality

- Most industrial robots nowadays have a PID control scheme

$$\tau' = K_v \ddot{E} + K_p E + K_i \int E dt$$

- **Control law** - No use of a model–based component at all
- **Separate control system for each joint** (by a separate micro controller)
- **No decoupling** – the motion of each joint effects the others joints
- **Error-driven control laws** – suppress joint error
- **Fixed Average gains** - approximate critical damping in the middle of the robot workspace (extreme conditions under-damped or over damped)
- **High gains** (as high as possible) – suppress disturbance quickly

Current Industrial Robot Control Systems
The Harsh Reality

- **Gravity terms cause static positioning errors –** Gravity compensation (simplest example of model-based controller)

$$\tau' = K_v \ddot{E} + K_p E + K_i \int E dt + \hat{G}(\Theta)$$

- **Disadvantage -** Gravity terms are coupled. The controller can no longer implemented on a strictly joint-by joint basis. The controller architecture must allow communicating between the joint controllers or must make a use of a central processor rather then individual-joint processors.

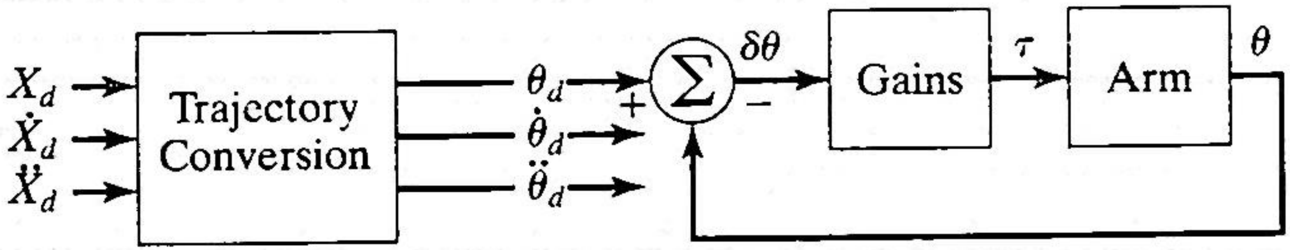
Current Industrial Robot Control Systems
The Harsh Reality

- Approximation of decoupling control (simplifying the dynamic equations)

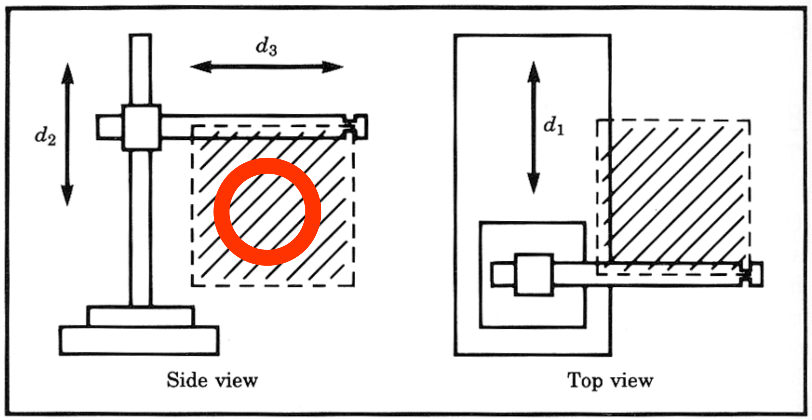
$$\tau = M(\Theta)\ddot{\Theta} + V(\Theta,\dot{\Theta}) + G(\Theta) + F(\Theta,\dot{\Theta})$$

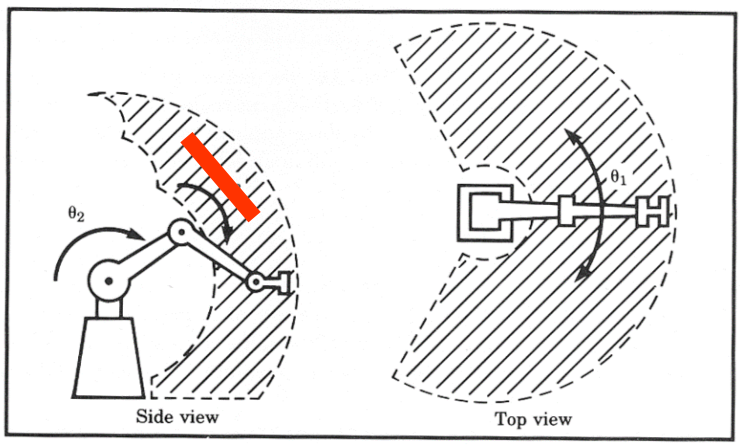
  - Ignore $V(\Theta,\dot{\Theta})$ and $F(\Theta,\dot{\Theta})$
  - Include $G(\Theta)$
  - Simplify $M(\Theta)$ by including only for major coupling between axis but not minor cross coupling effects

Cartesian –Based Control Systems



- Joint Based Control
- Cartesian based control





Cartesian –Based Control Systems

- Trajectory conversion – difficult in terms of computational expense. The computation that are required are

$$\Theta_d = Inv\_Kin(X_d)$$
$$\dot{\Theta}_d = J^{-1}(\Theta_d)\dot{X}_d$$
$$\ddot{\Theta}_d = \dot{J}^{-1}(\Theta_d)\dot{X}_d + J^{-1}(\Theta_d)\ddot{X}_d$$

- Simplified computations (in present day systems)

$$\Theta_d = Inv\_Kin(X_d)$$
$$\dot{\Theta}_d = d\Theta_d / dt$$
$$\ddot{\Theta}_d = d\dot{\Theta}_d / dt$$

Cartesian –Based Control Systems

- Numerical differentiations
  - Problem: Amplify noise

$$\dot{\Theta}_d = \frac{d\Theta_d}{dt} = \frac{\Theta_d(t) - \Theta_d(t-1)}{\Delta t}$$

  - Solution 1: When the trajectory is not known
    - causal filters (past present values)

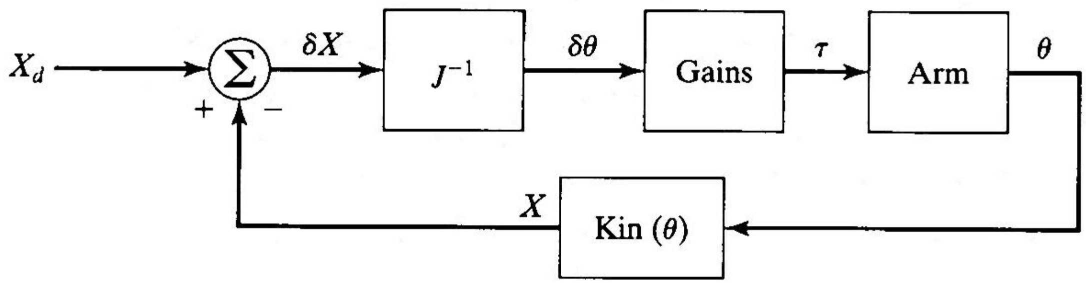
$$y(t) = f(x(t), x(t-1) \ldots x(t-n))$$

  - Solution 2: When the trajectory is known (path preplanned)
    - Non-causal filters (past present and future values)

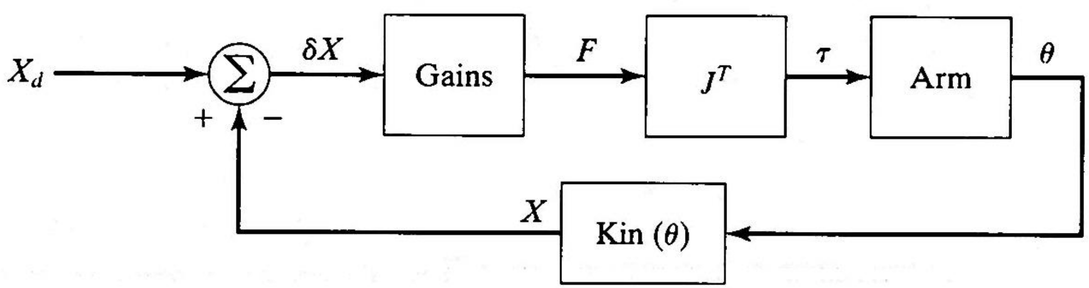
$$y(t) = f(x(t-n) \ldots (t+1), x(t), x(t-1) \ldots x(t-n))$$

Cartesian –Based Control Systems – Intuitive Schemes
Inverse or transpose Jacobian Controller

- Inverse Jacobian Controller



- Transpose Jacobian Controller



Cartesian –Based Control Systems – Intuitive Schemes
Inverse or transpose Jacobian Controller

- The exact dynamic performance of such systems is very complicated
- Both scheme can be made stable, but the same performance is not guaranteed over the entire workspace.
- We can not choose fixed gains that will result in fixed close loop poles.
- The dynamic response of such controllers will vary with arm configuration.

Cartesian –Based Control Systems – Cartesian decoupling Scheme

- Dynamic equations expressed in **joint space**

$$\tau = M(\Theta)\ddot{\Theta} + V(\Theta,\dot{\Theta}) + G(\Theta)$$

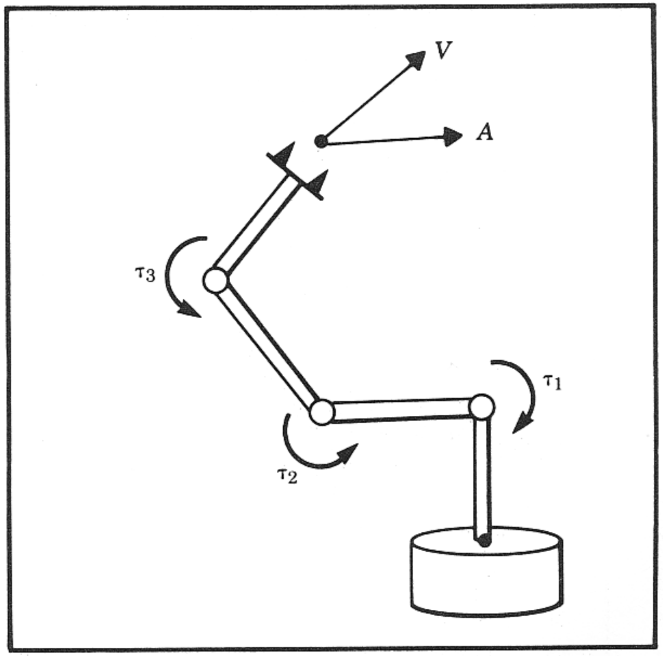
$$\tau = [\tau_1, \tau_2, ..., \tau_n]^T$$

- Dynamic equations expressed in **Cartesian state space (end effector space)**

$$F = M_x(\Theta)\ddot{X} + V_x(\Theta,\dot{\Theta}) + G_x(\Theta)$$

$$F = [f_x, f_y, f_z, \tau_x, \tau_y, \tau_z]^T$$

$$X = [x, y, z, \theta_x, \theta_y, \theta_z]^T$$



Cartesian –Based Control Systems – Cartesian decoupling Scheme

- Mapping between joint space and cartesian space (end effector)

$$\tau = J^T(\Theta)F \qquad \dot{X} = J(\Theta)\dot{\Theta}$$

$$F = J^{-T}(\Theta)\tau \qquad \ddot{X} = \dot{J}(\Theta)\dot{\Theta} + J\ddot{\Theta} \quad \Rightarrow \quad J^{-1}\ddot{X} - J^{-1}\dot{J}(\Theta)\dot{\Theta} = \ddot{\Theta}$$

$$\tau = M(\Theta)\ddot{\Theta} + V(\Theta,\dot{\Theta}) + G(\Theta)$$

- Multiply both sides by J^{-T}

$$J^{-T}\tau = J^{-T}M(\Theta)\ddot{\Theta} + J^{-T}V(\Theta,\dot{\Theta}) + J^{-T}G(\Theta)$$

$$F = J^{-T}M(\Theta)J^{-1}\ddot{X} - J^{-T}M(\Theta)J^{-1}\dot{J}\dot{\Theta} + J^{-T}V(\Theta,\dot{\Theta}) + J^{-T}G(\Theta)$$

Cartesian –Based Control Systems – Cartesian decoupling Scheme

$$F = J^{-T}M(\Theta)J^{-1}\ddot{X} - J^{-T}M(\Theta)J^{-1}\dot{J}\dot{\Theta} + J^{-T}V(\Theta,\dot{\Theta}) + J^{-T}G(\Theta)$$

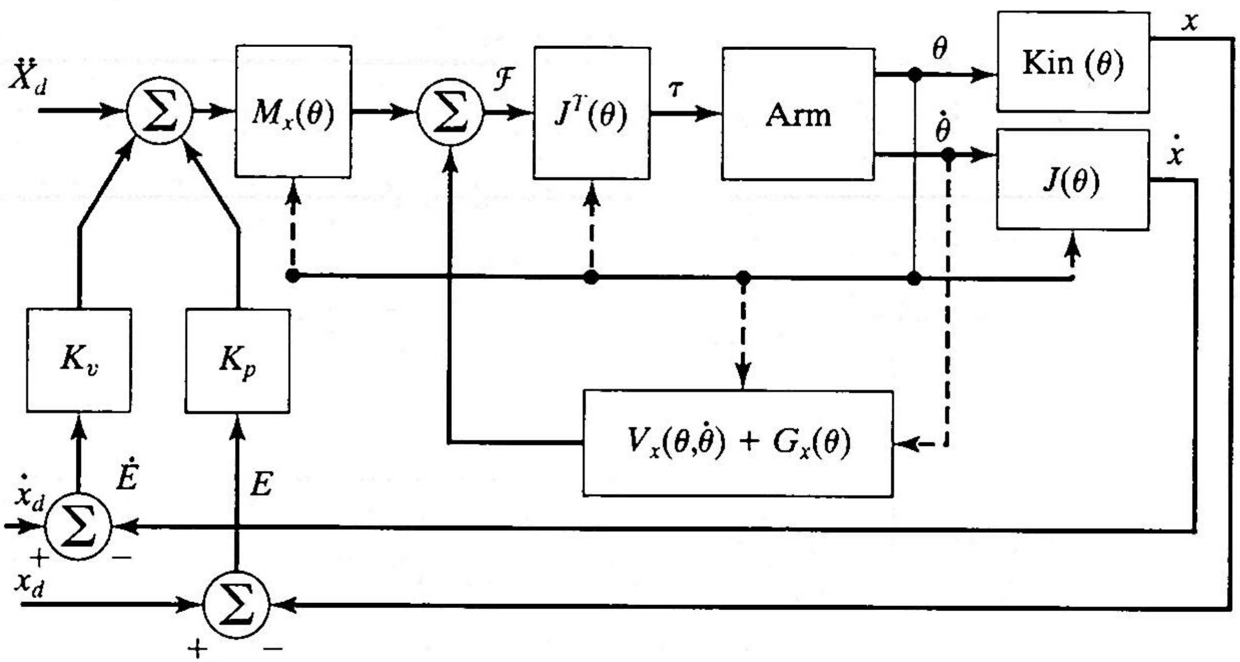
$$F = M_x(\Theta)\ddot{X} + V_x(\Theta,\dot{\Theta}) + G_x(\Theta)$$

$$M_x(\Theta) = J^{-T}M(\Theta)J^{-1}$$

$$V(\Theta,\dot{\Theta})_x = -J^{-T}M(\Theta)J^{-1}\dot{J}\dot{\Theta} + J^{-T}V(\Theta,\dot{\Theta})$$

$$G(\Theta)_x = J^{-T}G(\Theta)$$

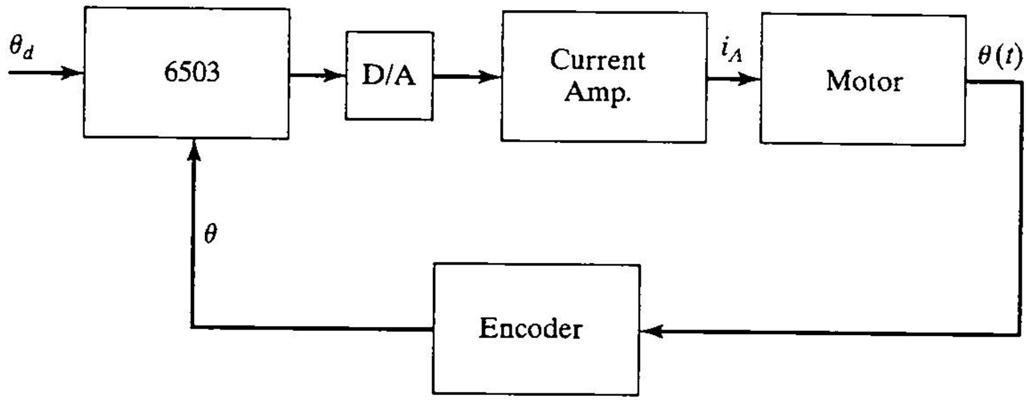
Cartesian –Based Control Systems – Cartesian decoupling Scheme

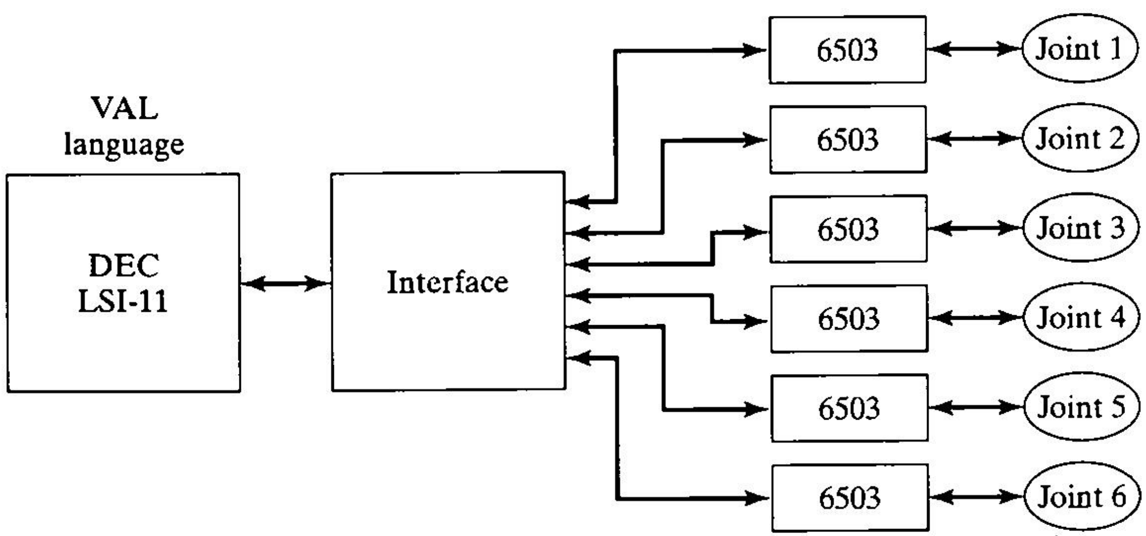


$$F = M_x(\Theta)\ddot{X} + V_x(\Theta,\dot{\Theta}) + G_x(\Theta)$$
$$\tau = J^T(\Theta)F$$
$$\dot{X} = J(\Theta)\dot{\Theta}$$

Solid Line – High rate Servo (e.g 500 Hz)
Dashed line – Low rate dynamic model (e.g. 100 HZ)

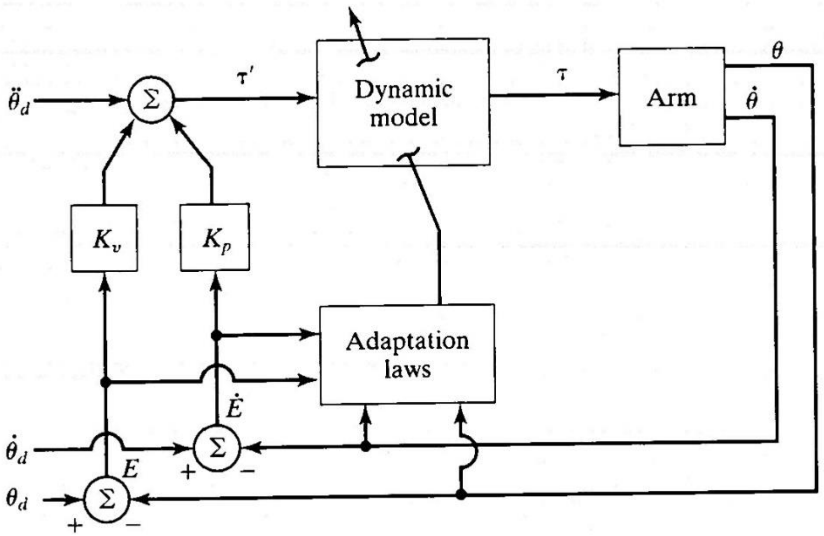
Hierarchical Computer Architecture
PUMA 560





Adaptive Control

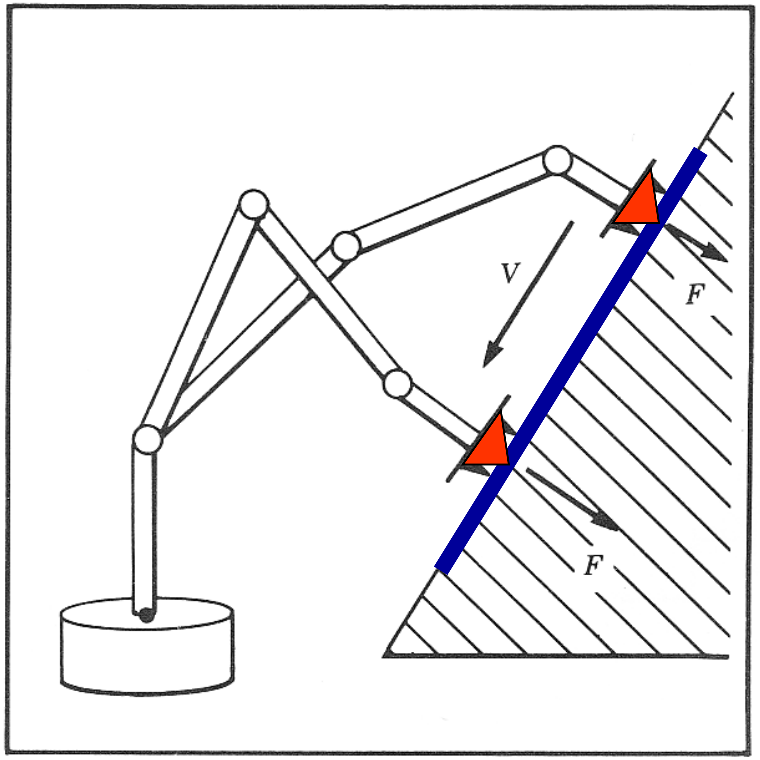
- The parameters of the manipulator are not known exactly
- Mismatch between real and estimated dynamic model parameters leads to servo errors.
- Servo errors may be used to adjust the model parameters based on adaptive laws until the errors disappear.
- The system learns its own dynamic properties



Example 1
Scraping a pint from a surface

Control type: Hybrid Control

Note: It is possible to control position (velocity) **OR** force (torque), but not both of them simultaneously along a given DOF. The environment impedance enforces a relashionship between the two



Assumption:

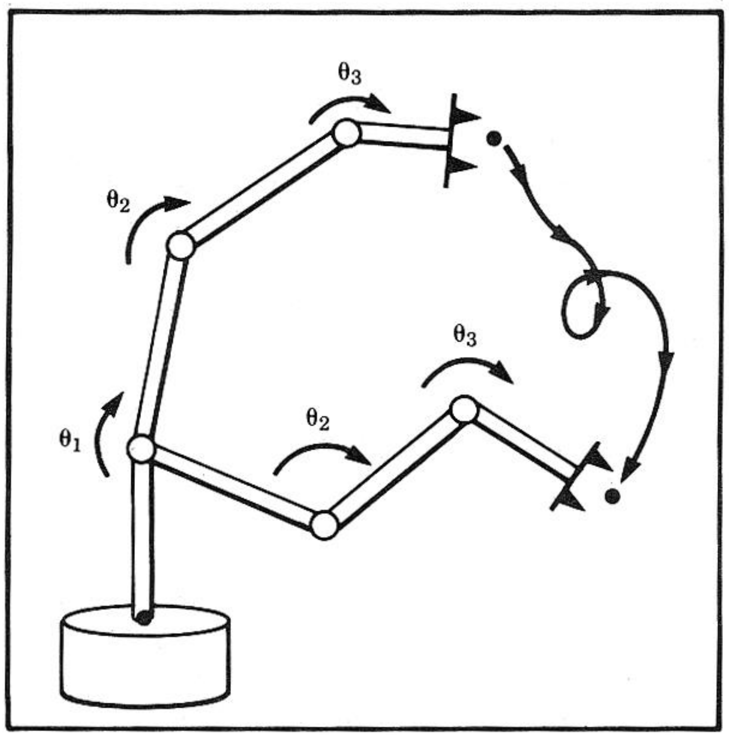
(1) The tool is stiff
(2) The position and orientation of the window is NOT known with accurately respect to the robot base.
(3) A contact force normal to the surface transmitted between the end effector and the surface is defined
(4) Position control - tangent to the surface
(5) Force control – normal to the surface

Hybrid Control of Manipulators

Introduction – Problem Definition

Position control

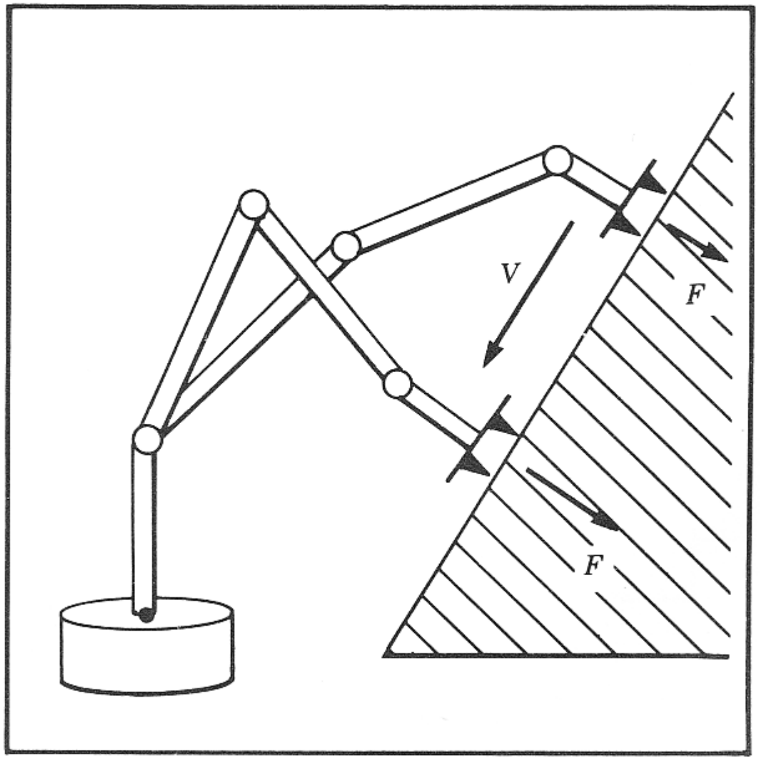
Position control is appropriate when a manipulator is following a trajectory through space



Introduction – Problem Definition

Hybrid Control

Fore control or hybrid control (position/force) may be required whenever the end effector comes in contact with the environment



Introduction – Problem Definition

Example 1

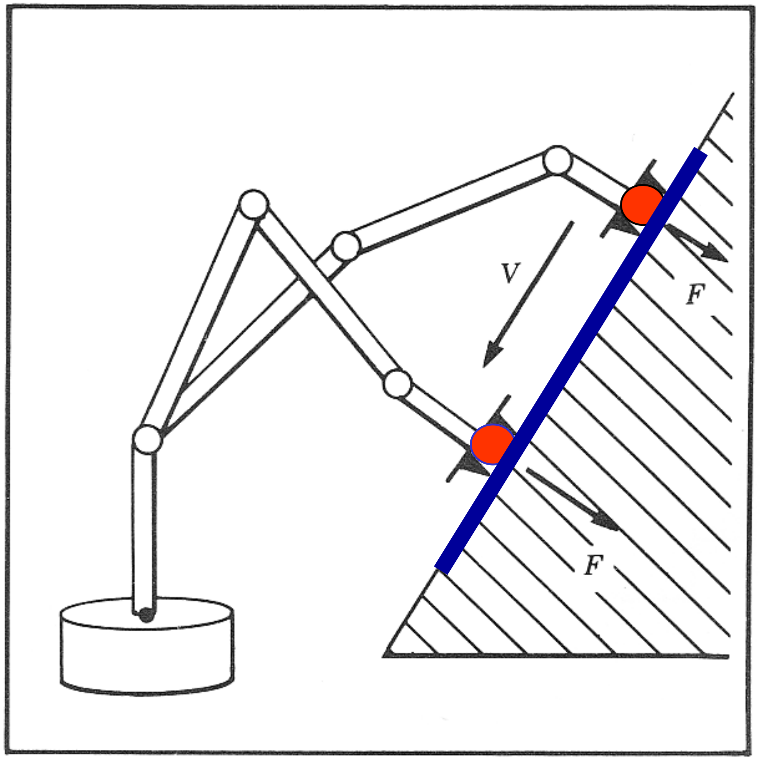
Washing a window with a sponge

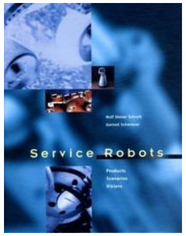
Control type: Position Control

Assumption:

(1) The sponge is compliant

(2) The position and orientation of the window is known with respect to the robot base.





Robotic Systems - Cleaning

SKYWASH

AEG, Dornier, Fraunhofer Institute, Putzmeister - Germany

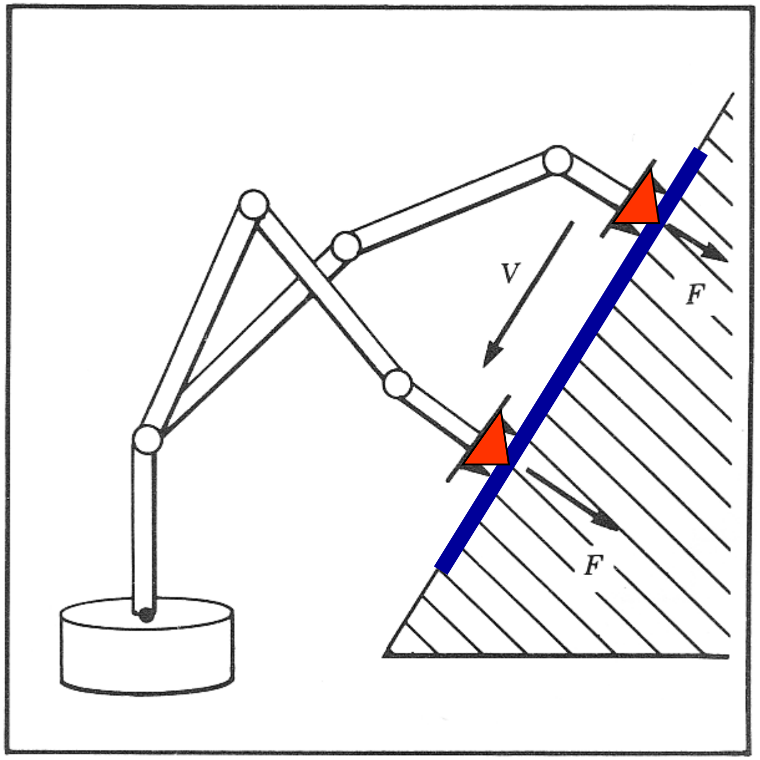
Using 2 Skywash robots for cleaning a Boeing 747-400 jumbo jet, its grounding time is reduced from 9 to 3.5 hours. The world´s largest cleaning brush travels a distance of approximately 3.8 kilometers and covers a surface of around 2,400 m² which is about 85% of the entire plane´s surface area. The kinematics consist of ***5 main joints*** for the robot´s arm, and an additional ***one for the turning circle*** of the rotating washing brush.The Skywash includes ***database that contains the aircraft-specific geometrical data***. A ***3-D distance camera*** accurately positions the mobile robot next to the aircraft. The 3-D camera and the computer determine the aircraft´s ideal positioning, and thus the cleaning process begins.

Introduction – Problem Definition

Example 2
Scraping a pint from a surface

Control type: Hybrid Control

Note: It is possible to control position (velocity) **OR** force (torque), but not both of them simultaneously along a given DOF. The environment impedance enforces a relashionship between the two



Assumption:

(1) The tool is stiff

(2) The position and orientation of the window is NOT known with accurately respect to the robot base.

(3) A contact force normal to the surface transmitted between the end effector and the surface is defined

(4) Position control - tangent to the surface

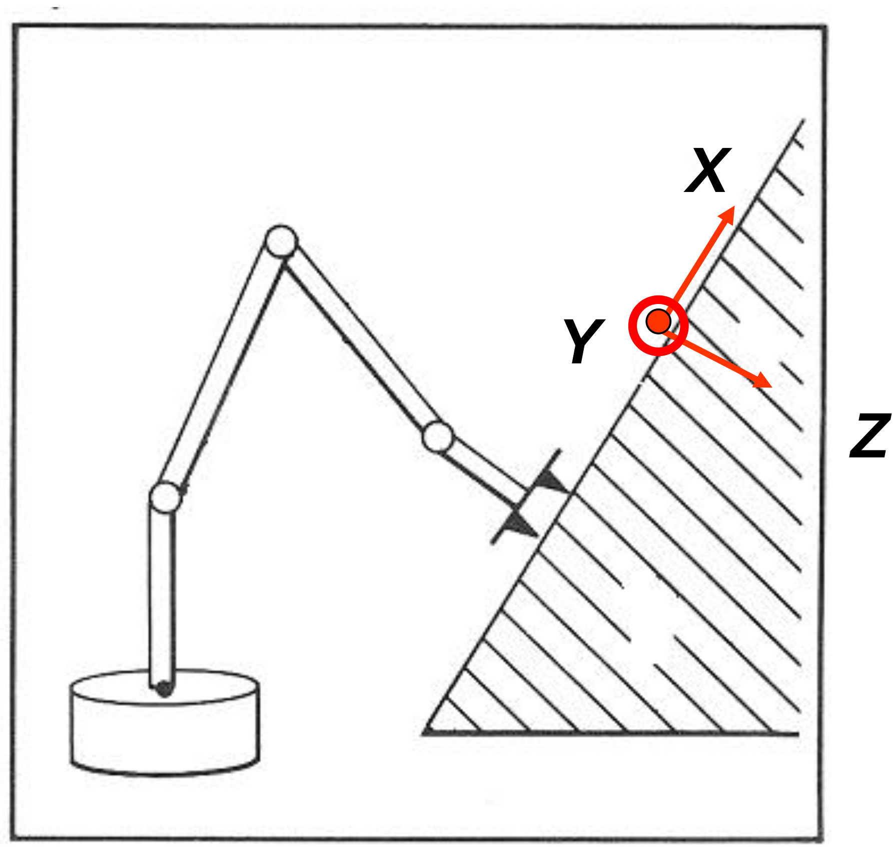
(5) Force control – normal to the surface

Hybrid Control – Strategy

- A hybrid control strategy consists of three elements:
  - Compliance Frame
  - Selection Matrix
  - Force and velocity commands
- Notes:
  - Assumption must be made about the environment
  - A given strategy may work only over a limited range of conditions

Hybrid Control – Compliance Frame

- Raibent & Craig
- We define a compliance frame so that ***X*** and ***Y*** are tangent to the surface (ignoring for a moment the orientation DOF)
- The task is to control the force in the ***Z*** direction and to control the velocity in the ***X*** and ***Y*** directions.
- Assumption – no friction – control only velocity along ***X*** and ***Y*** but not force



Hybrid Control – Selection Matrix

- Diagonal matrix
- Along the diagonal place
  - A Value of 1 for velocity control
  - A value of 0 for force control

$$s = \begin{bmatrix} s_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & s_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & s_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & s_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & s_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & s_{66} \end{bmatrix} \qquad v = \begin{bmatrix} v_x \\ v_y \\ v_z \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} \qquad f_d = \begin{bmatrix} f_x \\ f_y \\ f_z \\ \tau_x \\ \tau_y \\ \tau_z \end{bmatrix}$$

  - Velocity and force selection $[S]v_d$

$$[I - S]\, f_d$$

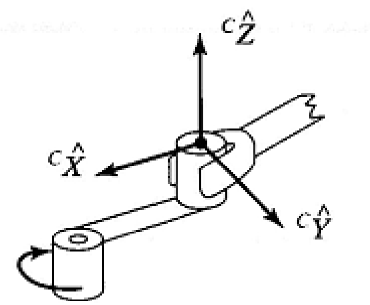
Hybrid Control – Environment Modeling

- **Natural Constraints**
  - Along each DOF of the task space, the **environment** imposes either a position or a force constraint to the manipulator end effector. Such constraints are termed natural constraints since they are determined directly by the **task geometry**.

- **Artificial Constrains**
  - Along each DOF of the task space, the manipulator can control only the variables that are not subject to natural constraints. The reference values for those variables are termed artificial constraints since they are imposed with regard to the **strategy of executing the given task**.
  - Artificial constraints are the desired trajectories (motion) or forces **specified by the user** and associated with the task

- Conditions
  - Artificial constrains must be compatible with the natural constrains since one can not control force and position along the same DOF
  - The number of natural and artificial constrains must be equal to the number of DOF of the constraint space space (6 in general)

Hybrid Control – Environment Modeling - Example



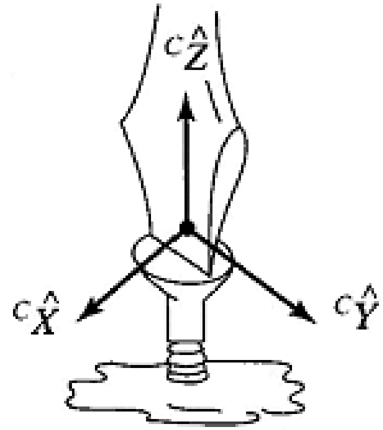
$$X : F_x = 0$$

$$Y : V_y = \omega r$$

$$Z : F_z = 0$$

$$S = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad v_d = \begin{bmatrix} * \\ \omega r \\ * \end{bmatrix} \qquad f_d = \begin{bmatrix} 0 \\ * \\ 0 \end{bmatrix}$$

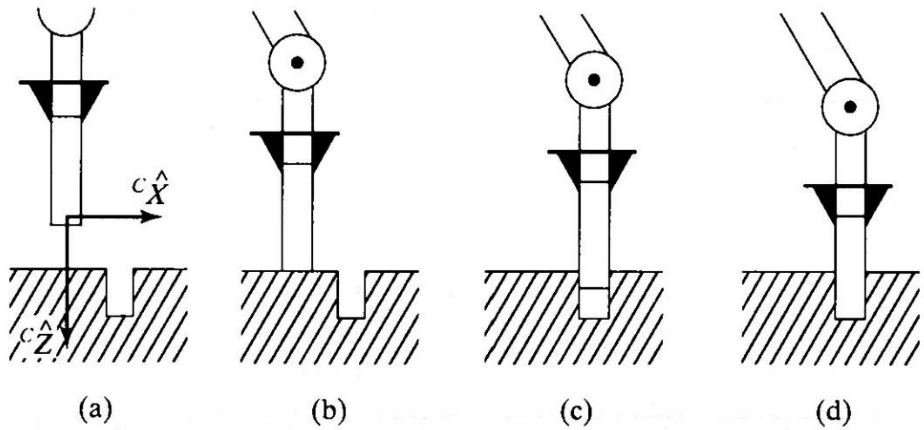
Hybrid Control – Environment Modeling - Example



$$X : V_x = 0$$
$$Y : V_y = 0$$
$$Z : F_z = -f_z$$
$$\theta_x : \omega_x = 0$$
$$\theta_y : \omega_y = 0$$
$$\theta_z : \omega_z = -\omega$$

$$S = \left[\begin{array}{ccc|ccc} 1 & & & & & \\ & 1 & & & & \\ & & 0 & & & \\ \hline & & & 1 & & \\ & & & & 1 & \\ & & & & & 1 \end{array}\right] \begin{array}{c} x \\ y \\ z \\ \theta_x \\ \theta_y \\ \theta_z \end{array} \qquad v_d = \begin{bmatrix} 0 \\ 0 \\ * \\ 0 \\ 0 \\ -\omega \end{bmatrix} \qquad f_d = \begin{bmatrix} * \\ * \\ -f_z \\ * \\ * \\ * \end{bmatrix}$$

Hybrid Control – Environment Modeling



$X : f_x = 0$	$X : v_x = v_{slide}$	$X : f_x = 0$	$X : f_x = 0$
$Y : fy = 0$	$Y : f_y = 0$	$Y : f_y = 0$	$Y : f_y = 0$
$Z : v_z = v_{approuch}$	$Z : f_z = f_{contact}$	$Z : v_z = v_{insert}$	$Z : f_z = f_{\max}$
$\theta_x : \tau_x = 0$	$\theta_x : \tau_x = 0$	$\theta_x : \tau_x = 0$	$\theta_x : \tau_x = 0$
$\theta_y : \tau_y = 0$	$\theta_y : \tau_y = 0$	$\theta_y : \tau_y = 0$	$\theta_y : \tau_y = 0$
$\theta_z : \tau_z = 0$	$\theta_z : \tau_z = 0$	$\theta_z : \tau_z = 0$	$\theta_z : \tau_z = 0$

Hybrid Control – Environment Modeling

$$X : F_x = 0$$

$$Y : F_y = 0$$

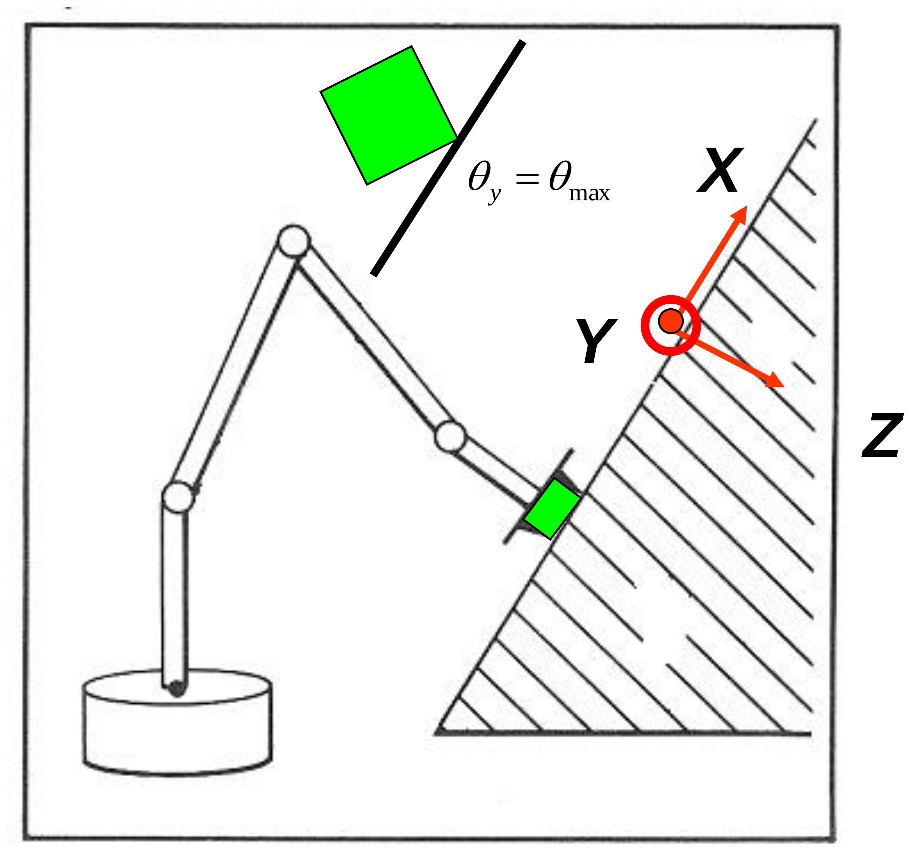
$$Z : z \le z_{surface}$$

$$\theta_x : \theta_x \le \theta_{x\max}$$

$$\theta_y : \theta_y \le \theta_{y\max}$$

$$\theta_z : \tau_z = 0$$

$$S = \begin{array}{c c} \left[\begin{array}{ccc|ccc} 1 & & & & & \\ & 1 & & & & \\ & & 0 & & & \\ \hline & & & 0 & & \\ & & & & 0 & \\ & & & & & 1 \end{array}\right] & \begin{array}{c} x \\ y \\ z \\ \theta_x \\ \theta_y \\ \theta_z \end{array} \end{array}$$

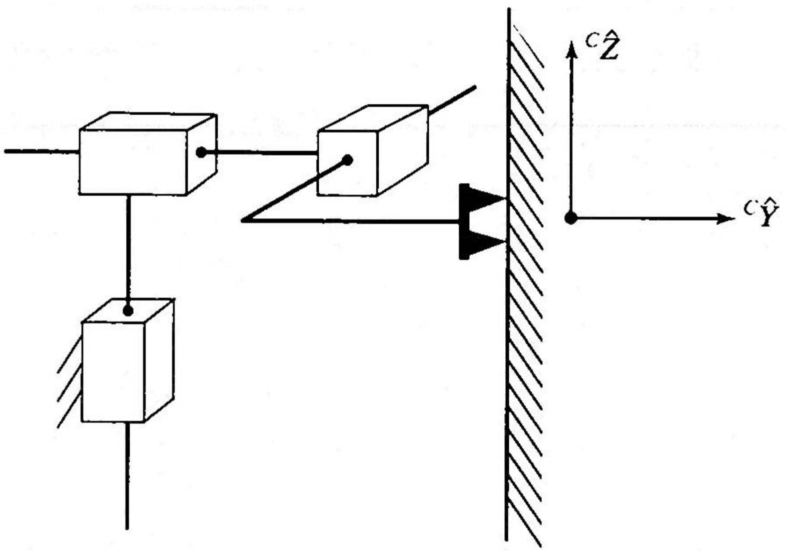


Hybrid Position/Force Control Scheme

- Manipulator
  - Cartesian
  - 3 DOF
  - End Effector frame is aligned with the compliance frame

- Control approach
  - Joints: x, z – position control
  - Joint y – force control

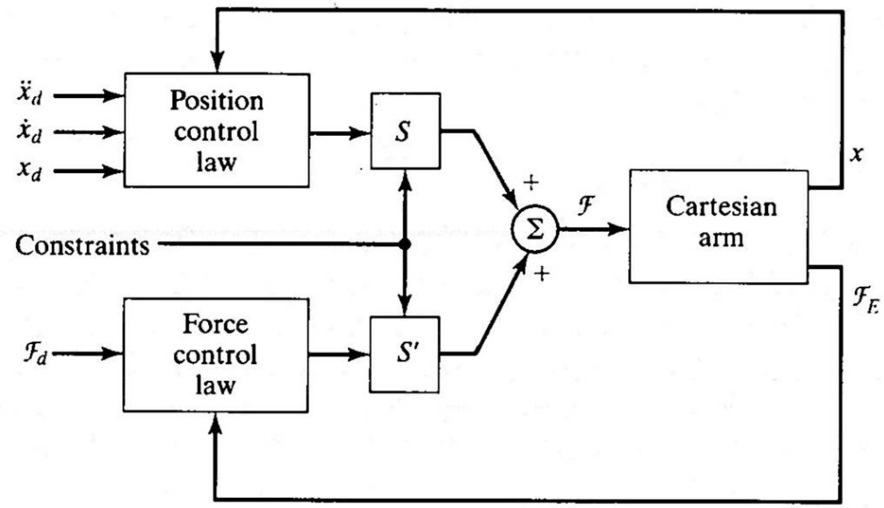
- Inputs
  - Joints: x, z – trajectory
  - Joint y – contact force



Hybrid Position/Force Control Scheme

- Robot control design (General)
  - Position control in 3 DOF
  - Force control 3 DOF
  - The mix between the DOF is arbitrary and depends on the task

- Constraints
  - Providing the constraints based on the task

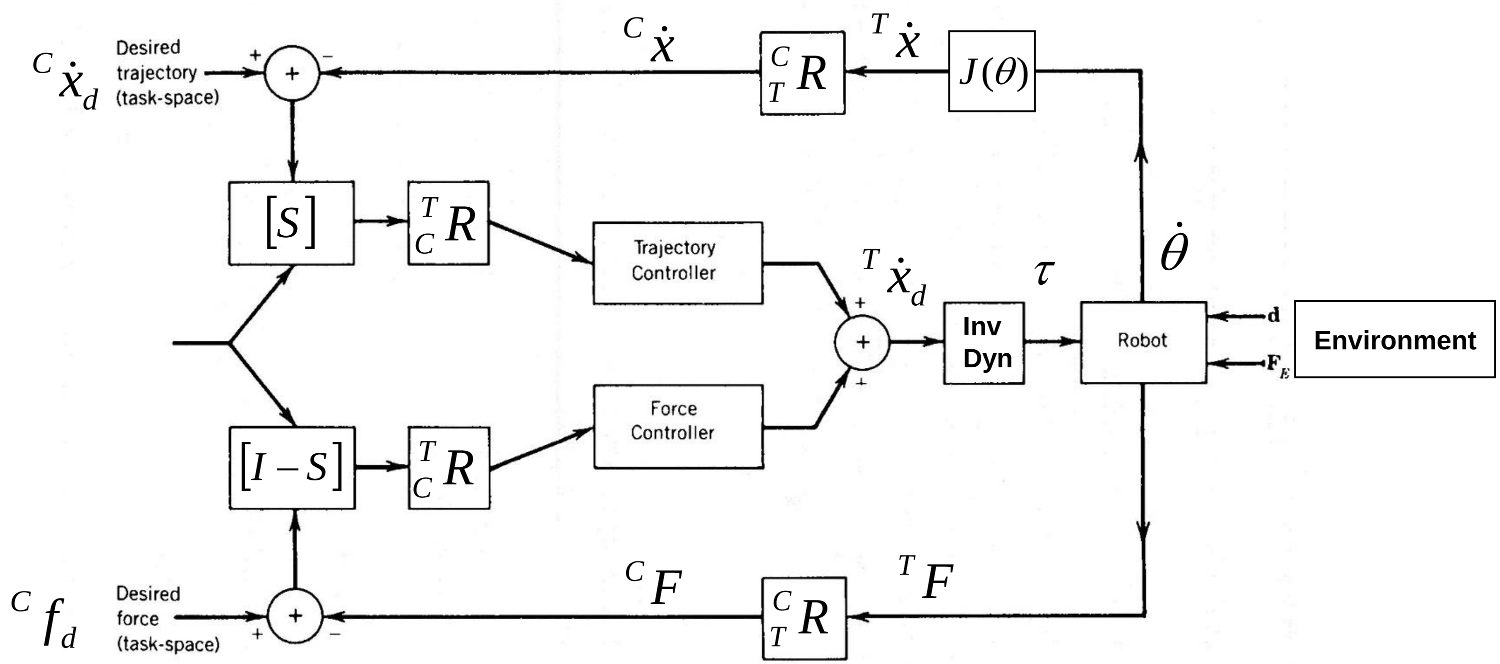


$$S = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$S' = [I - S] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

  - DOF with 0 along the diagonal of [S] are ignored

Hybrid Position/Force Generalized Control Scheme



$[S]$ – Compliance Matrix

${}^{C}_{T}R$ – Rotataion of Task frame to Compliance Frame

Hybrid Position/Force Control with Industrial Robot
The Harsh Reality

- **Industrial Robotic Control Status** - True hybrid position/force control does not exist in industrial robot

- **Practical Implementation Problems**
  - Large amount of computation
  - Lack of accurate parameters for the dynamic model
  - Lack of rugged force sensor
  - Difficult definition of position/force strategy by the user

- **Common Practice**
  - Passive Compliance
  - Compliance through softening position gains

Industrial Robot - Passive Compliance
The Harsh Reality

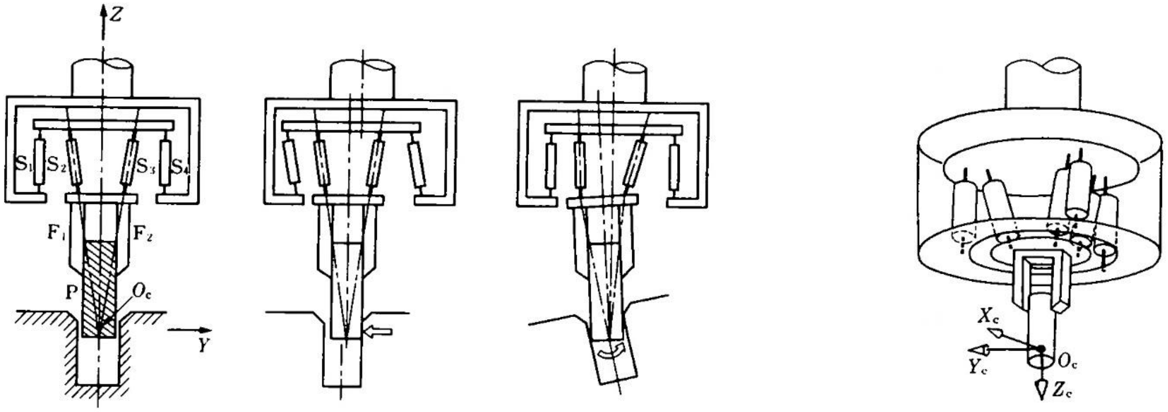
- Extremely **rigid manipulators** with **stiff position servos** are **ill-suited** to tasks in which parts come into contact and **contact forces** are generated.

- Typical Problems
  – Jamming
  – Damaged

- Successful assembly (mating parts) is achieve due to compliance
  – The parts themselves
  – The fixture
  – Compliant passive element mounted on the robot (between the end effector and the griper / part)

Industrial Robot - Passive Compliance
The Harsh Reality

- Remote Center Compliance Device (RRC) – Drapers Lab



- RRC – 6 DOF spring inserted between the robot and the end effector (gripper)
- Global Stiffness is selected by the adjusting the individual springs S1-S6 that can only bend but not expend or compressed.
  - Cased 1 - S1, S4 – Cartesian misalignment
  - Cased 2 – S2, S3 – Rotational misalignment

Industrial Robot – Compliance though softening Position Gains

The Harsh Reality

- Concept (Salisbury) – Position gains in the joint-based servo system are modified in a way that the end effector appears to have a certain stiffness along the Cartesian DOF

- Consider a general spring with a 6 DOF

$$F = K_{px}\delta X$$

$$\begin{bmatrix} f_x \\ f_y \\ f_z \\ \tau_x \\ \tau_y \\ \tau_z \end{bmatrix} = \begin{bmatrix} k_x & & & & & \\ & k_y & & & 0 & \\ & & k_z & & & \\ & & & k_{\theta x} & & \\ & 0 & & & k_{\theta y} & \\ & & & & & k_{\theta z} \end{bmatrix} * \begin{bmatrix} \delta x \\ \delta y \\ \delta z \\ \delta\theta_x \\ \delta\theta_y \\ \delta\theta_z \end{bmatrix}$$

Industrial Robot –
Compliance though softening Position Gains
The Harsh Reality

- The definition of the manipulator Jacobian

$$\delta X = J(\theta)\delta\theta$$

- Combining with the stiffness eq.

$$F = K_{px} J(\theta)\delta\theta$$

- For static forces

$$\tau = J^T(\theta)F$$

- Combing with the previous eq.

$$\tau = J^T(\theta) K_{px} J(\theta)\delta\theta$$

Industrial Robot –
Compliance though softening Position Gains
The Harsh Reality

$$\tau = J^T(\theta) K_{px} J(\theta) \delta\theta$$

- Express the Jacobian in the tool's frame.

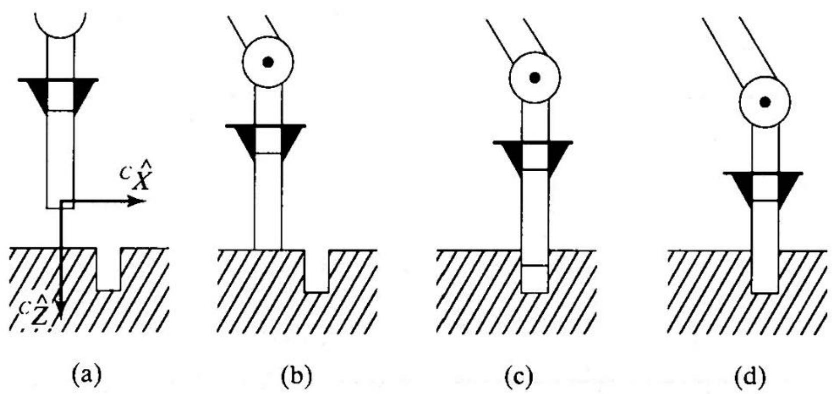
- The equation define how joint torque should be generated as a function of small changes in the joint angles $\delta\theta$, in order to make the manipulator end-effector behave as a Cartesian spring with 6 DOF
- Typical PD control ($E = \Theta_d - \Theta$) $\quad \tau = K_p E + K_v \dot{E}$

- Modified PD Controller $\quad \tau = J^T(\theta) K_{px} J(\theta) E + K_v \dot{E}$

- Through use of the Jacobian, a Cartesian stiffness has been transformed to a joint-space stiffness

Industrial Robot –
Force Sensing – Guarded Move
The Harsh Reality

- Some commercial robot include **force sensors**
- Force sensing allows a manipulator to **detect contact** with a surface and using this sensation to take some action
- **Guarded Move Strategy** – move under position control until a specific value of force is felt, then halt motion



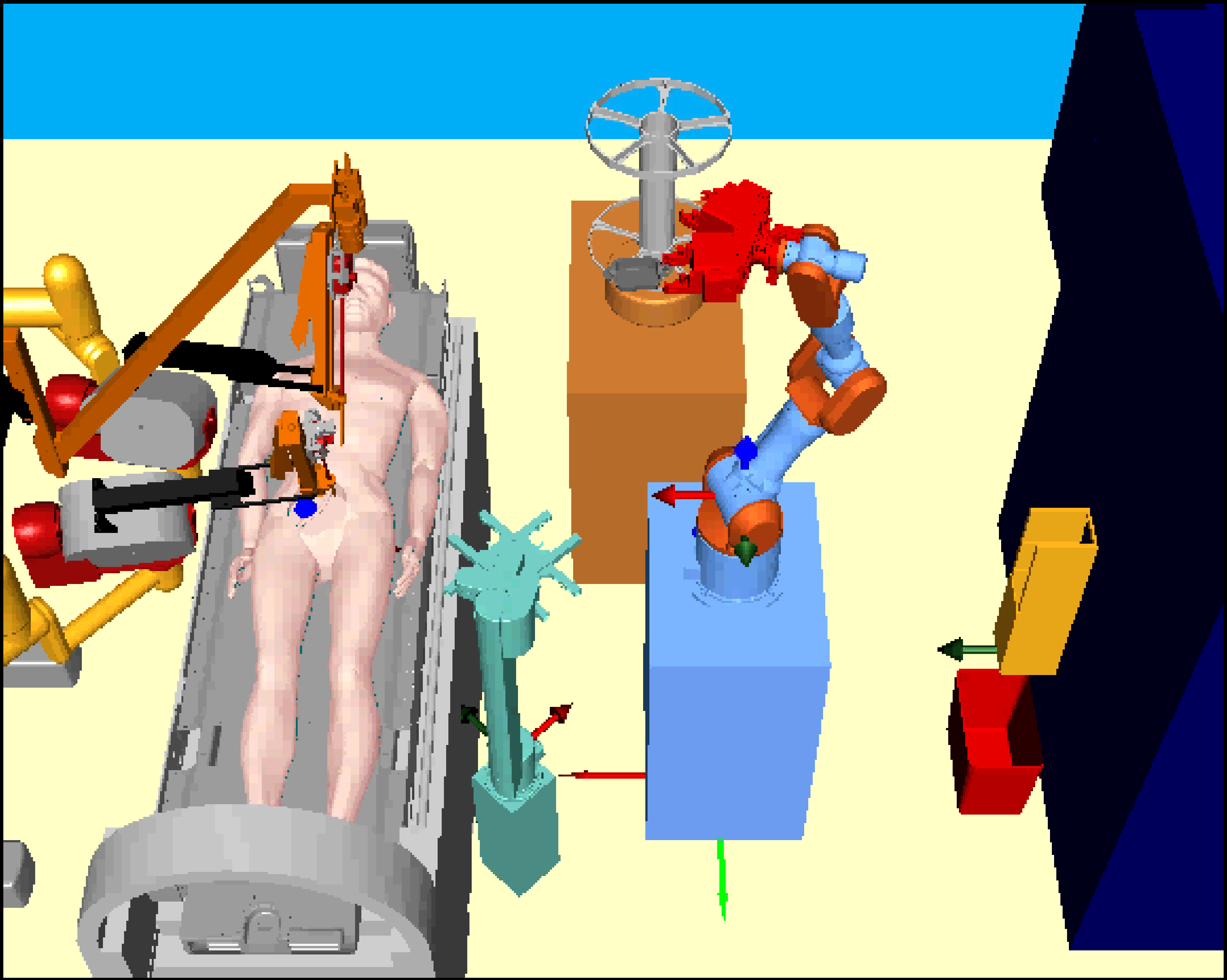
- Measure the weight of the object during part handling to ensure that the appropriate part was acquired.

Impedance Control

- Neville Hogan MIT 1980’s

- Controlling a DOF in strict position or force control represent control at two ends of the servo stiffness
  - **Ideal position servo** is infinitely stiff $K = dF / dX = \infty$ and reject all force disturbance acting on the system
  - **Ideal force servo** exhibits zero stiffness $K = dF / dX = 0$ and maintain a desired force application regardless of the position disturbance.

- **Objective**: Control a manipulator to achieve a specified mechanical impedance - a generalization of position force and hybrid control.



Trauma Pod
Position / Force Control implementation

Force Control of Mass –Spring System

Problem

The mass must maintain a desired contact force f_d with the environment.

f_e - Measured contact force

f_{dist} - Disturbance force

$$f_e = k_e x$$

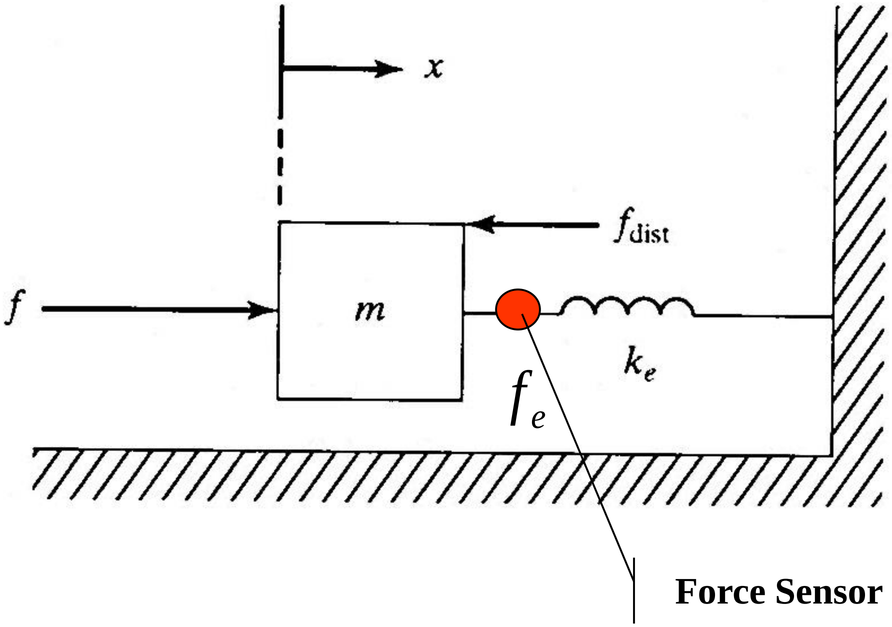
$$x = k_e^{-1} f_e$$

The equation of motion (EOF) of the system

$$f = m\ddot{x} + k_e x + f_{dist}$$

The EOM can be written in terms of the variable we wish to control

$$f = m k_e^{-1} \ddot{f}_e + f_e + f_{dist}$$



Force Control of Mass –Spring System

- Using the partitioned-controller concept

$$f = \alpha \; f' + \beta = m k_e^{-1} \ddot{f}_e + f_e + f_{dist}$$

$$\alpha = m k_e^{-1}$$

$$\beta = f_e + f_{dist}$$

$$f' = \ddot{f}_e$$

Force Control of Mass –Spring System

- Define a control law that will cause force following

$$f' = \ddot{f}_d + k_{vf}\dot{e} + k_{pf}e$$

$$e = f_d - f_e$$

$$\ddot{e} = \ddot{f}_d - \ddot{f}_e$$

$$f' = \ddot{f}_e$$

$$\ddot{f}_e = \ddot{f}_d + k_{vf}\dot{e} + k_{pf}e$$

$$\ddot{e} + k_{vf}\dot{e} + k_{pf}e = 0$$

Force Control of Mass –Spring System

- Define a control law that will cause force following

$$f' = \ddot{f}_d + k_{vf}\dot{e} + k_{pf}e$$

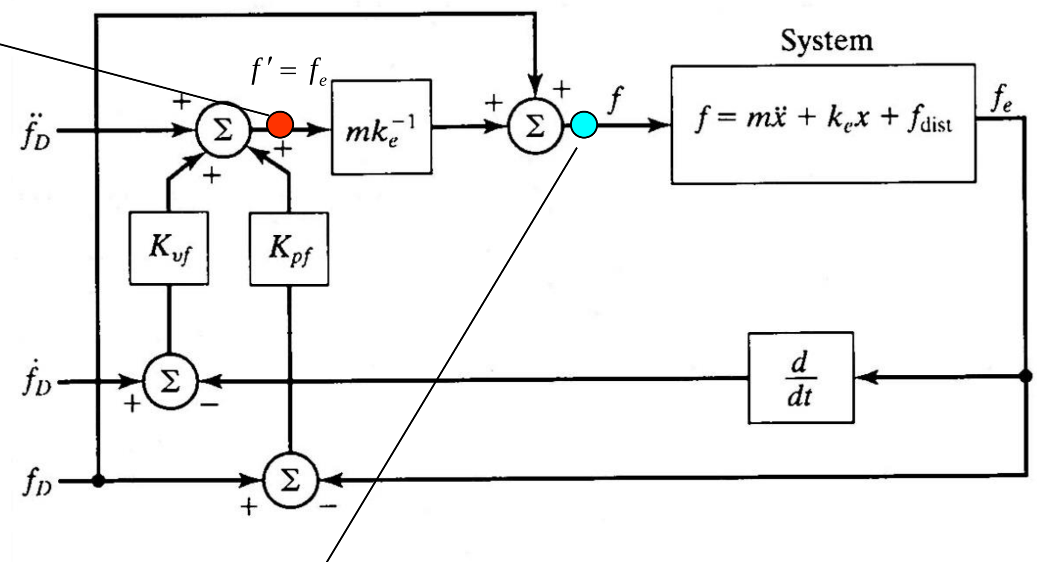
$$e = f_d - f_e$$

$$\ddot{e} = \ddot{f}_d - \ddot{f}_e$$

$$f' = \ddot{f}_e$$

$$\ddot{f}_e = \ddot{f}_d + k_{vf}\dot{e} + k_{pf}e$$

$$\ddot{e} + k_{vf}\dot{e} + k_{pf}e = 0$$



$$f = mk_e^{-1}[\ddot{f}_d + k_{vf}\dot{e} + k_{pf}e] + f_d$$

Force Control of Mass –Spring System

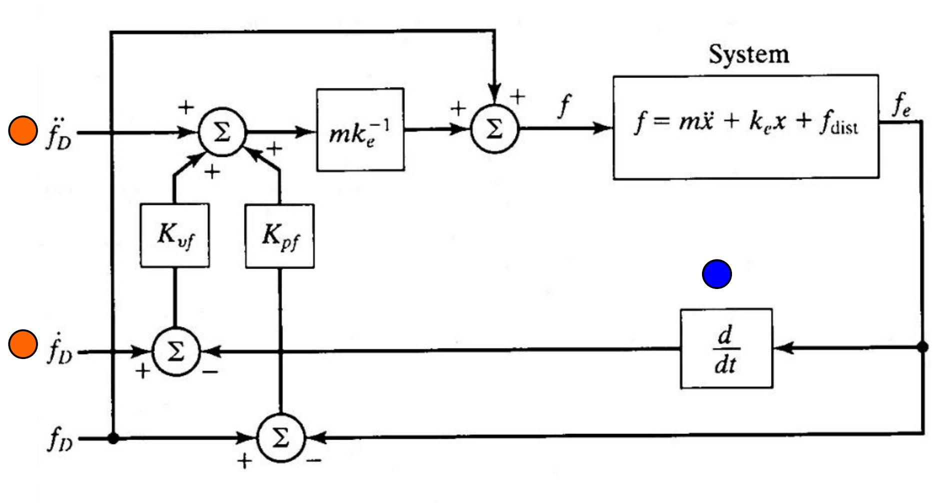
- Practical Implementations:
  - Controlling constant force

$$\ddot{f}_D = \dot{f}_D = 0$$

  - Force signals – “noisy”

$$f_e = k_e x$$

$$\dot{f}_e = \frac{df_e}{dx} = k_e \dot{x}$$

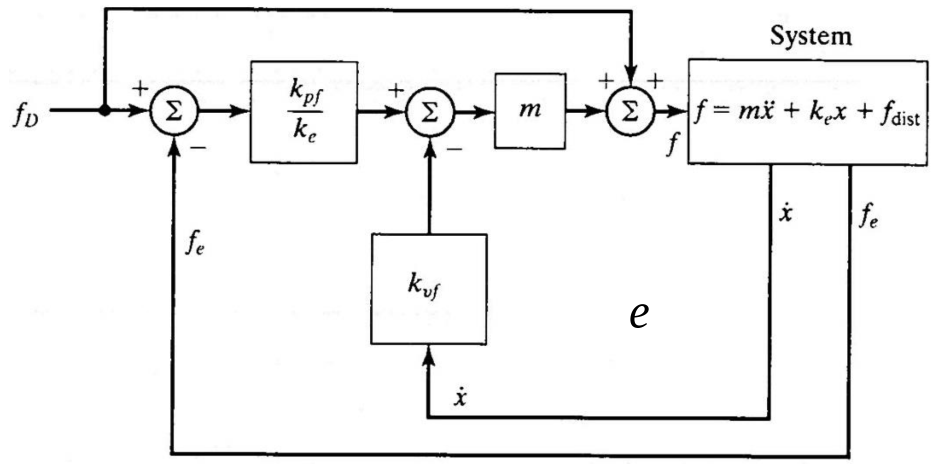


- Simplifying the control low

$$f = mk_e^{-1}[\ddot{f}_d + k_{vf}\dot{e} + k_{pf}e] + f_d$$

$$f = m[k_{pf}k_e^{-1}e - k_{vf}\dot{x}] + f_d$$

Force Control of Mass –Spring System



- Simplified control law

$$f = m[k_{pf} k_e^{-1} e - k_{vf} \dot{x}] + f_d$$

- Interpretation

  Force errors generate a set point for an inner velocity control loop with gain k_{vf} . Some control laws also include integrator to improve steady-state performance.

Force Control of Mass –Spring System

- Remaining problem -
  - – The stiffness of the environment k_e is part of the control law
  - – The stiffness k_e is unknown or changing

- Assumption - Assembly robot – rigid environment
- The gains are chosen such that the system is robust with respect to the environment

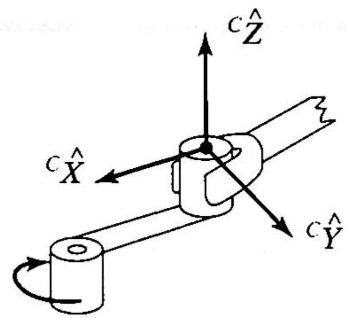
A Framework of Control in Partially Constrained Task

- **Partially Constrained Task**
  - Part mating (assembly task)
  - Peg in the hole
  - Turning a crank
  - Turning a screwdriver

- **Natural Constraints**
  - Natural constraints in position or force are **defined by the geometry** of the task that result from particular mechanical or geometrical characteristics of the task configuration

- **Artificial Constrains**
  - Artificial constraints are the desired trajectories (motion) or forces **specified by the user** and associated with the task

(a) Turning crank



Natural constraints

$v_x = 0$ $\quad f_y = 0$

$v_z = 0$ $\quad n_z = 0$

$\omega_x = 0$

$\omega_y = 0$

Artificial constraints

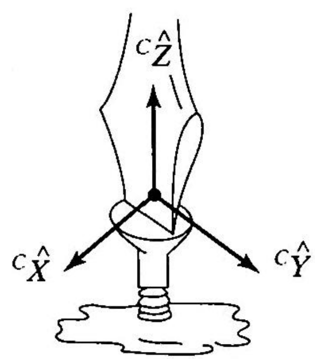
$v_y = r\alpha_1$ $\quad f_x = 0$

$\omega_z = \alpha_1$ $\quad f_z = 0$

$n_x = 0$

$n_y = 0$

(b) Turning screwdriver



Natural constraints

$v_x = 0$ $\quad f_y = 0$

$\omega_x = 0$ $\quad n_z = 0$

$\omega_y = 0$

$v_z = 0$

Artificial constraints

$v_y = 0$ $\quad f_x = 0$

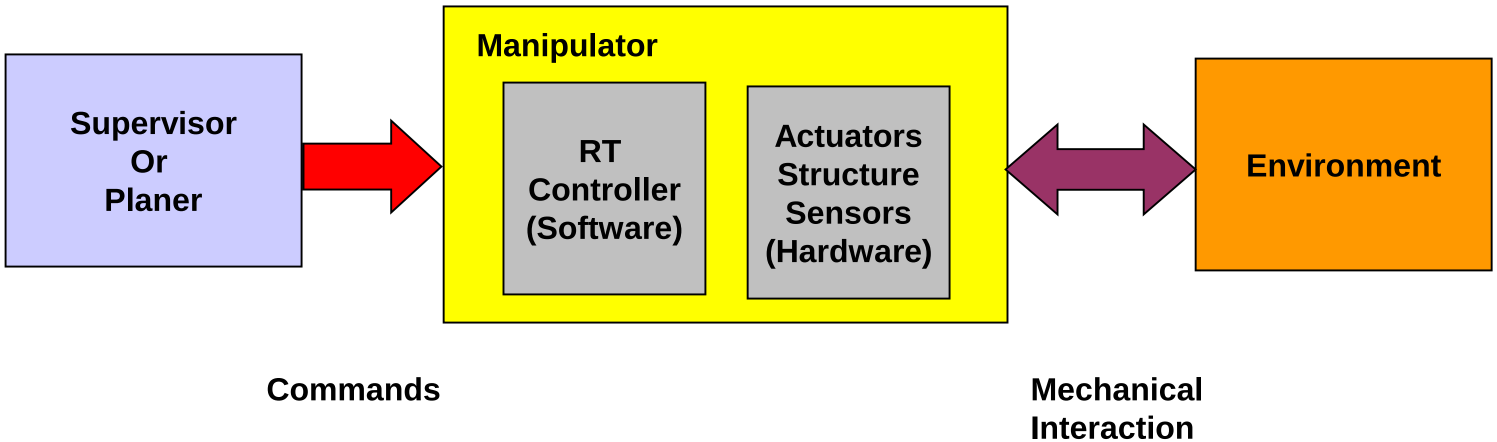
$\omega_z = \alpha_2$ $\quad n_x = 0$

$n_y = 0$

$f_z = \alpha_3$

Impedance Control of Manipulators

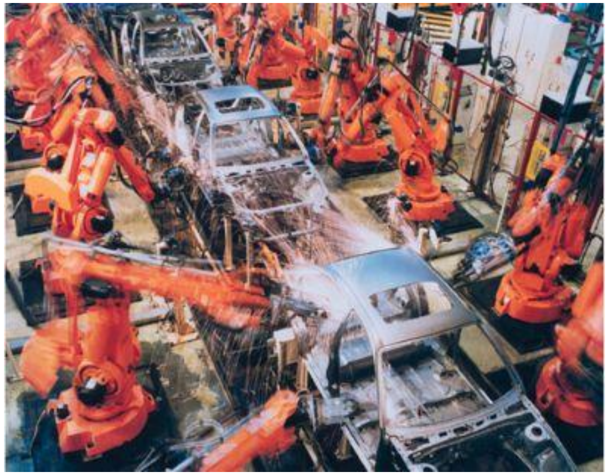
Manipulation



- Neville Hogan MIT 1980’s
- **Manipulation** – Mechanical ***interaction*** with object(s) being manipulated
- **Manipulator Task Classification** – magnitude of the mechanical ***work exchanged*** between the manipulator and its environment.

Manipulation

- Manipulation - Case 1
  - Interaction force – negligible $F = 0$
  - Interaction mechanical work – negligible $dW = F \bullet dX = 0$
  - Control variables – motion $X, \dot{X}, \ddot{X}$
  - Control implementation – Position control
  - Application: spray painting and welding



Manipulation

- Manipulation - Case 2
  - Environmental constrains
    - Tangent $F = 0$
    - Normal $X = 0$
  - Interaction mechanical work – negligible $dW = F \bullet dX = 0$
  - Control variables – Motion control (tangent) / Force control (normal)
  - Control implementation – Hybrid control
  - Application: Washing a window

Manipulation

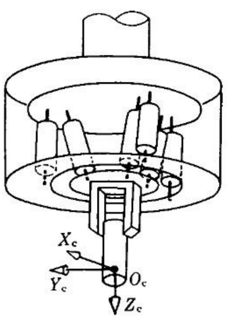
- Manipulation - Case 3 (general case)
- Environmental constrains
  - Dynamic interaction $dW = F \bullet dX \neq 0$
  - Applications (industrial): Tasks that require work to be done on the environment. Drilling, reaming, counter boring, grinding
  - Control strategy
    - Problem: Impossible to control individual vectors of position, velocity, force – in sufficient to control the mechanical work exchange
    - Solution: control the dynamic behaviors of the manipulator (the relationship between the quantities)

Impedance Control

- Environment – The environment is regarded as a disturbance to the manipulator

- Control Strategy – modulate the the disturbance response of the manipulator will allow to control of the dynamic interaction

- Modulate dynamic behavior
  - Passively (e.g. RCC)



  - Actively Modulate the controlled variables (servo gains)

Impedance Control

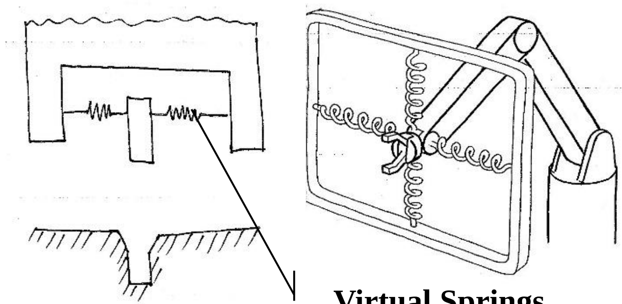
- Controlling a DOF in strict position or force control represent control at two ends of the servo stiffness
  - **Ideal position servo** is infinitely stiff $K = dF / dX = \infty$ and reject all force disturbance acting on the system
  - **Ideal force servo** exhibits zero stiffness $K = dF / dX = 0$ and maintain a desired force application regardless of the position disturbance.

Controlling variable		Stiffness
Position (P)	$P_d - P = 0$	$K = dF / dX = \infty$
Force (F)	$F_d - F = 0$	$K = dF / dX = 0$

Impedance Control

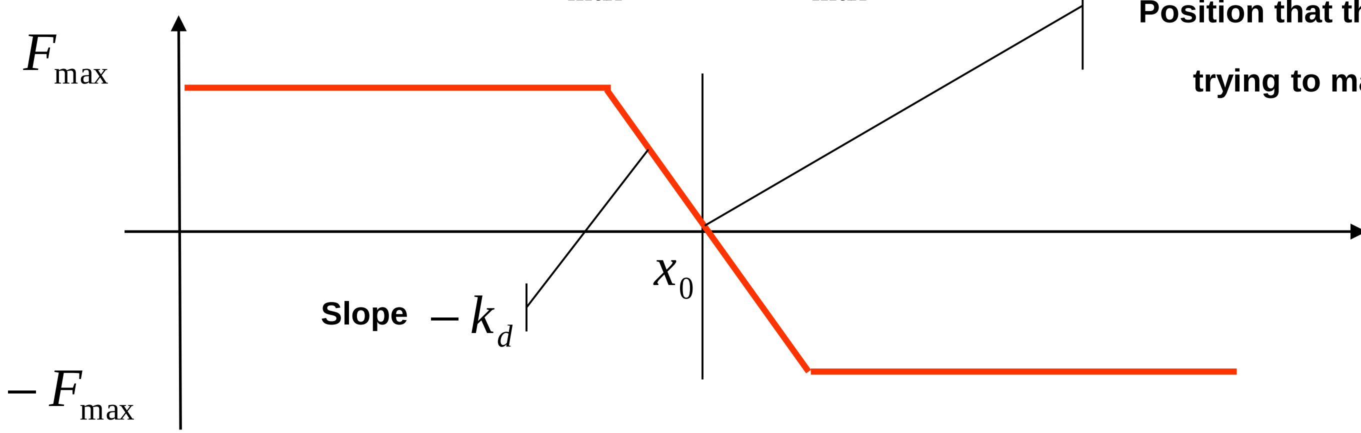
- Consider a relationship of a position controlled robot, with a control law of

$$F = k_d (x - x_0)$$



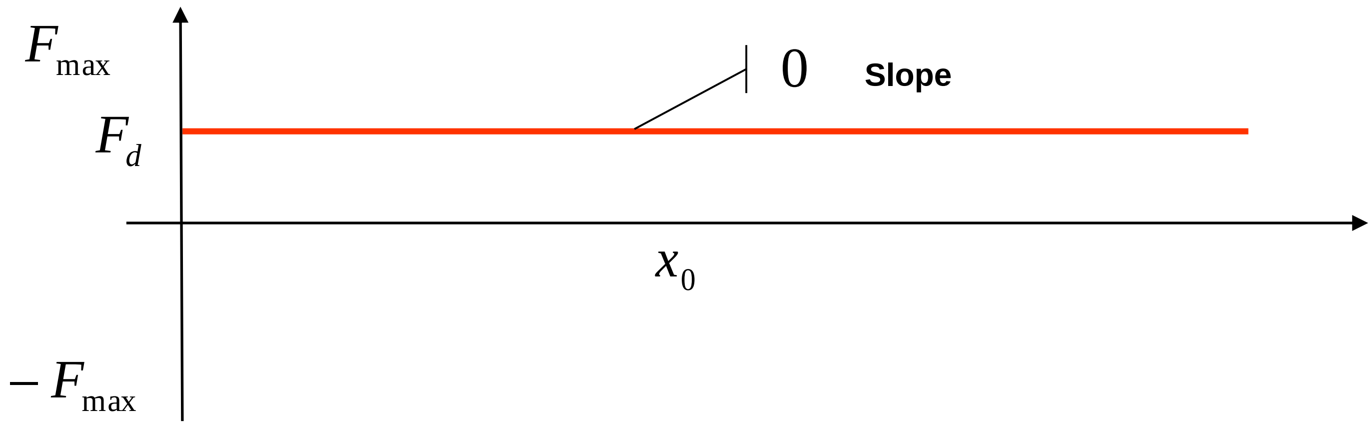
- Due to actuator limits

$$F_{\max} < F < F_{\max}$$

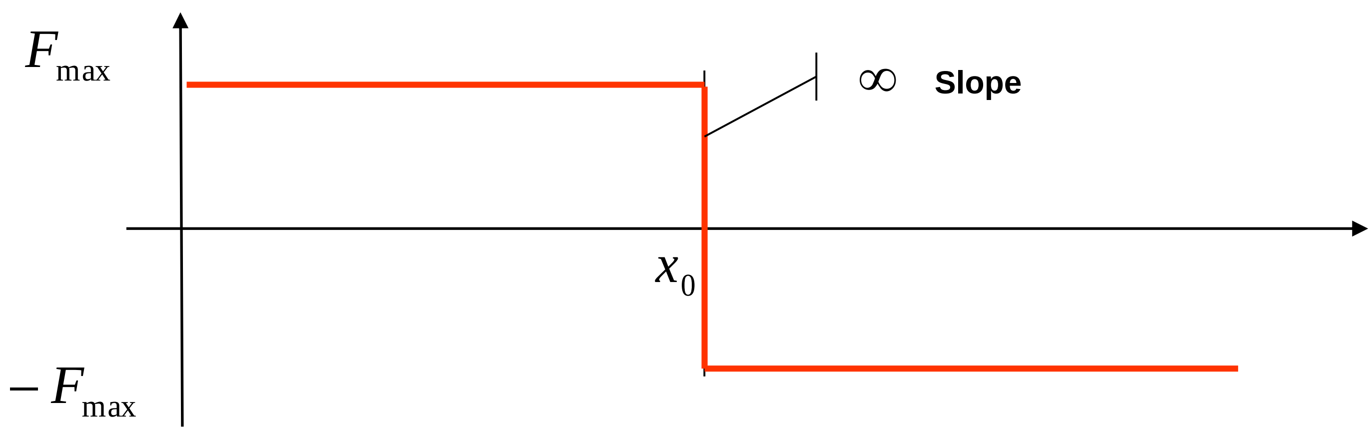


Impedance Control

- Force Control

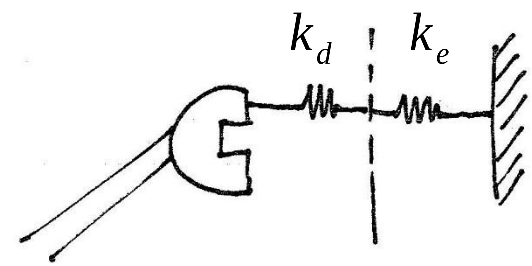


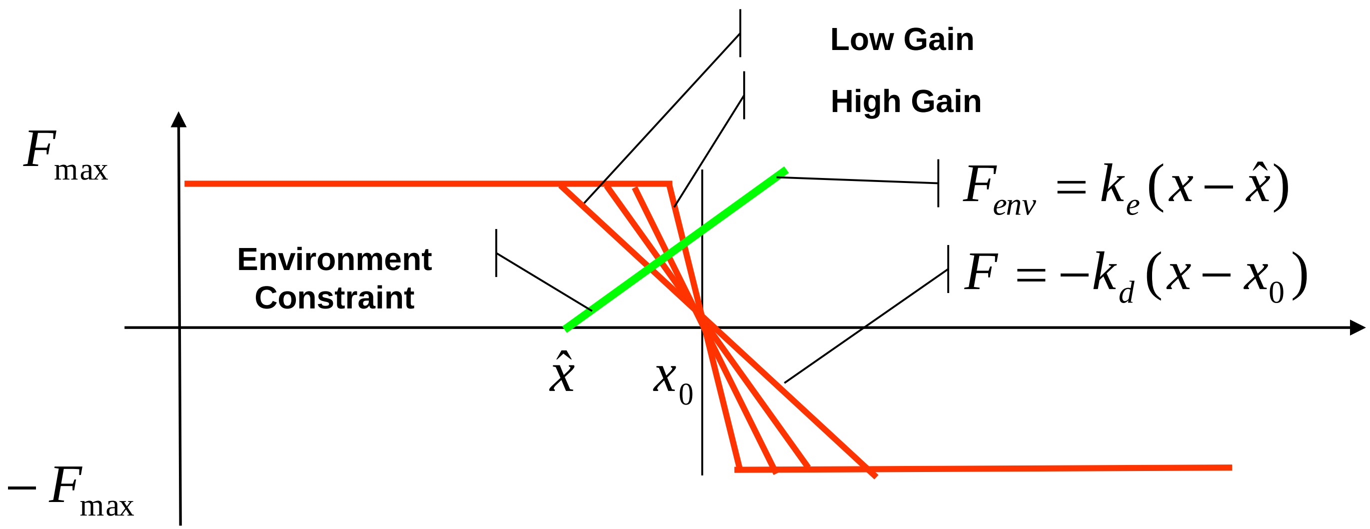
- Position Control



Impedance Control

- Another possible case is stiffness control
  - Control law $F = -k_d(x - x_0)$
  - Environment $F_{env} = k_e(x - \hat{x})$

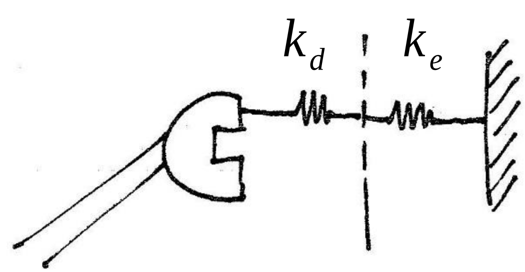




Impedance Control

- Case 1 (free motion)
  - If the external force is $F_{env} = 0$
  - Then the position is $x = x_0$

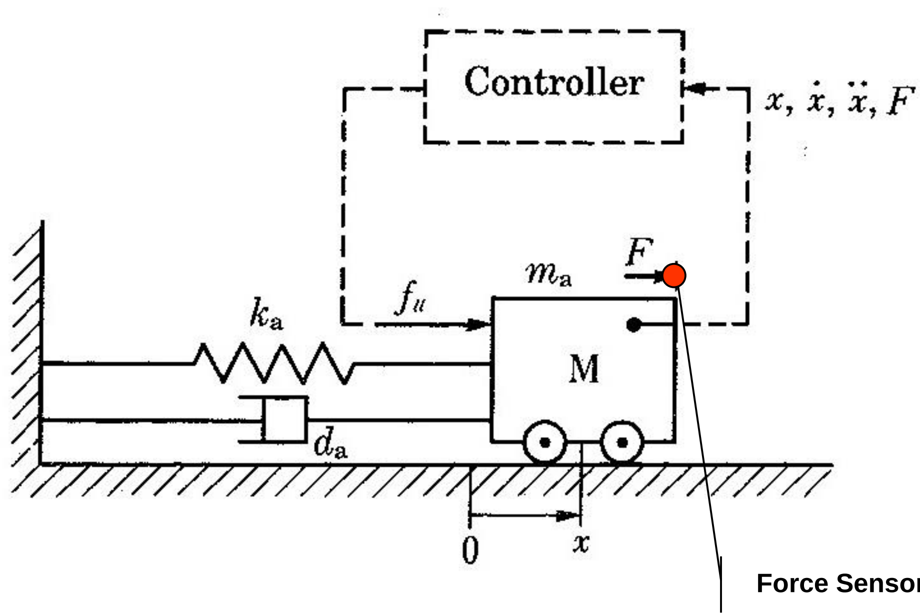
- Case 2 (interaction)
  - If in contact with a compliant environment $F_{env} = k_e(x - \hat{x})$
  - Both force and position depend on k_d k_e



Active Impedance Method – 1 DOF

- 1 DOF system
- The dynamic equation

$$m_a \ddot{x} + b_a \dot{x} + k_a x = f_u + F$$



- Where

m_a – Mass of the body

b_a – Damping coefficient

k_a – Spring constant

f_u – Driving force (servo)

F – External force

x – Displacement form equilibrium

Active Impedance Method – 1 DOF

- In equilibrium

$$x = 0 \qquad \Rightarrow \qquad f_u = F = 0$$

- We also assume that the desired impedance of the body to the external force is expressed by

$$m_d \ddot{x} + b_d(\dot{x} - \dot{x}_d) + k_d(x - x_d) = F$$

- Where

m_d – Desired mass

b_d – Desired damping coefficient

k_d – Desired spring constant

x_d – Desired position trajectory

Active Impedance Method – 1 DOF

- When $\ddot{x}, \dot{x}, x$ are measurable we can use the control law

$$m_d \ddot{x} + b_d (\dot{x} - \dot{x}_d) + k_d (x - x_d) = F$$

$$m_a \ddot{x} + b_a \dot{x} + k_a x = f_u + F$$

$$f_u = (m_a - m_d)\ddot{x} + (b_a - b_d)\dot{x} + (k_a - k_d)x + b_d \dot{x}_d + k_d x_d$$

- Let $m_a = m_d$ the control law is reduced to position and velocity feedback laws

$$f_u = (b_a - b_d)\dot{x} + (k_a - k_d)x + b_d \dot{x}_d + k_d x_d$$

- We have developed a control law to achieve the desire impedance

$$m_d \ddot{x} + b_d (\dot{x} - \dot{x}_d) + k_d (x - x_d) = F$$

Active Impedance Method – 1 DOF

- A remaining problem is to determine the coefficients b_d, k_d

$$m_d \ddot{x} + b_d (\dot{x} - \dot{x}_d) + k_d (x - x_d) = F$$

- Consider one of the two cases
  - the system makes no contact with other object

    OR
  - We can regard the external force $F = 0$ because there is small perturbing force acting, if any.

- Set the natural frequency to be as large as possible for better transient response

$$\omega_c = \sqrt{\frac{k_d}{m_d}}$$

Active Impedance Method – 1 DOF

- Let the damping coefficient be around 0.7-1.0 (critical to over damping)

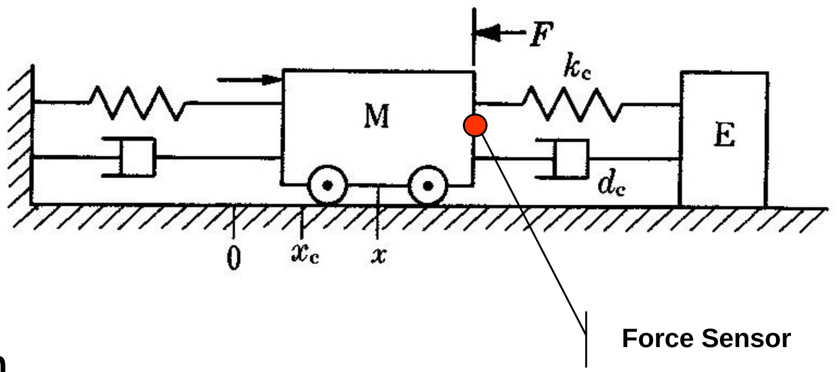
$$\zeta = \frac{b_d}{2\sqrt{m_d k_d}}$$

- As long as m_d, b_d, k_d are positive, the steady –state position error and velocity error converge to zero for any desired trajectory x_d

Active Impedance Method – 1 DOF

- 1 DOF system
- The body M is in contact with a fixed body E (environment)
- The interaction with the environment described as

$$b_c \dot{x} + k_c (x - x_c) = -F$$



- Where x_c is the equilibrium position for which $F = 0$

Active Impedance Method – 1 DOF

- Substituting

$$b_c \dot{x} + k_c (x - x_c) = -F$$

$$m_d \ddot{x} + b_d (\dot{x} - \dot{x}_d) + k_d (x - x_d) = F$$

- Yields

$$m_d \ddot{x} + (b_d + b_c)\dot{x} + (k_d + k_c)x = b_d \dot{x}_d + k_d x_d + k_c x_c$$

- The natural frequency and the damping coefficient are

$$\omega_c = \sqrt{\frac{k_d + k_c}{m_d}} \qquad \zeta = \frac{b_d + b_c}{2\sqrt{m_d (k_d + k_c)}}$$

Active Impedance Method – 1 DOF

$$\omega_c = \sqrt{\frac{k_d + k_c}{m_d}} \qquad \zeta = \frac{b_d + b_c}{2\sqrt{m_d(k_d + k_c)}}$$

- Given k_c, b_c determine k_d, b_d for acceptable ω_c, ζ
- Problem: k_c, b_c are unknown
- Solution: Active impedance – Adjust k_d, b_d
  - A set of k_d, b_d for non-contact
  - A set of k_d, b_d for contact

Active Impedance Method – 1 DOF

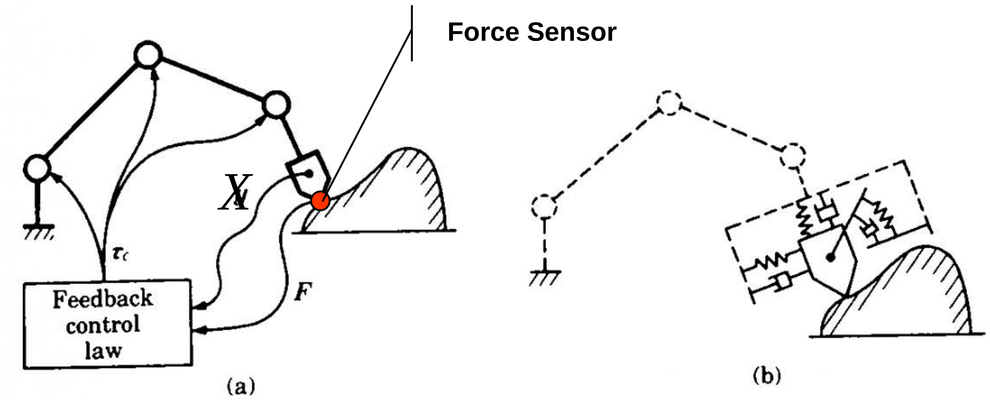
$$\omega_c = \sqrt{\frac{k_d + k_c}{m_d}} \qquad \zeta = \frac{b_d + b_c}{2\sqrt{m_d (k_d + k_c)}}$$

- If the real stiffness of the environment is larger then the estimated value and the damping is relatively small

$$k_{c-real} > k_{c-estimated}$$
$$b_c \to 0$$

- Result: Inadequate damping characteristics
- Solution:
  - Choosing large b_d
  - Choosing small k_d - smaller contact forces (no damage to the robot or the environment)

Active Impedance Method – General Case



- Measuring the end effector position/ orientation X and the external contact force F acting on the end effector are used to drive the actuators at the joint through feedback control law
- Select the control law such that
  - The system behaves like an end effector with desired mechanical impedance
  - The arm follows a desirable trajectory

Active Impedance Method – General Case

- Consider a 6 DOF manipulator
- Assume that the desired mechanical impedance for its end effector is described by

$$F_e = M_R \ddot{X} + B_D \dot{X}_e + K_D X_e$$

- Where X_e is the difference between the current value position/ordination vector X and its desired value X_0

$$X_e = X - X_0$$

Active Impedance Method – General Case

- Where K_D, B_D are 6x6 diagonal matrices representing the desired stiffness and damping of the manipulator

$$K_D = \begin{bmatrix} k_{11} & & & & & \\ & k_{22} & & & 0 & \\ & & k_{33} & & & \\ & & & k_{44} & & \\ & 0 & & & k_{55} & \\ & & & & & k_{66} \end{bmatrix} \qquad B_D = \begin{bmatrix} b_{11} & & & & & \\ & b_{22} & & & 0 & \\ & & b_{33} & & & \\ & & & b_{44} & & \\ & 0 & & & b_{55} & \\ & & & & & b_{66} \end{bmatrix}$$

Impedance Control – Generalized Approach for a mDOF

- Desired Behavior of the robot (M_R, K_D, B_D)

$$F_e = M_R \ddot{X} - B_D(\dot{X}_0 - \dot{X}) - K_D(X_0 - X)$$
$$\ddot{X} = M_R^{-1}[K_D(X_0 - X) + B_D(\dot{X}_0 - \dot{X})] + F_e$$

- Known kinematics

$$\dot{X} = J\dot{\theta} \qquad \tau_e = J^T F_e$$
$$\ddot{X} = \dot{J}\dot{\theta} + J\ddot{\theta} \quad \Rightarrow \quad \ddot{\theta} = J^{-1}(\ddot{X} - \dot{J}\dot{\theta})$$

- Dynamics Model of the manipulator with an external force acting on its end effector

$$M\ddot{\theta} + H(\theta, \dot{\theta}) = \tau + \tau_e = \tau + J^T F_e$$
$$H(\theta, \dot{\theta}) = V(\theta, \dot{\theta}) + G(\theta)$$

Impedance Control – Generalized Approach for a mDOF

- Desired Behavior of the robot (M_R, K_D, B_D)

$$F_e = M_R \ddot{X} - B_D(\dot{X}_0 - \dot{X}) - K_D(X_0 - X)$$

$$\ddot{X} = M_R^{-1}[K_D(X_0 - X) + B_D(\dot{X}_0 - \dot{X})] + F_e$$

- Known kinematics

$$\dot{X} = J\dot{\theta} \qquad \tau_e = J^T F_e$$

$$\ddot{X} = \dot{J}\dot{\theta} + J\ddot{\theta} \qquad \Rightarrow \qquad \ddot{\theta} = J^{-1}(\ddot{X} - \dot{J}\dot{\theta})$$

- Dynamics Model of the manipulator with an external force acting on its end effector

$$M\ddot{\theta} + H(\theta, \dot{\theta}) = \tau + \tau_e = \tau + J^T F_e$$

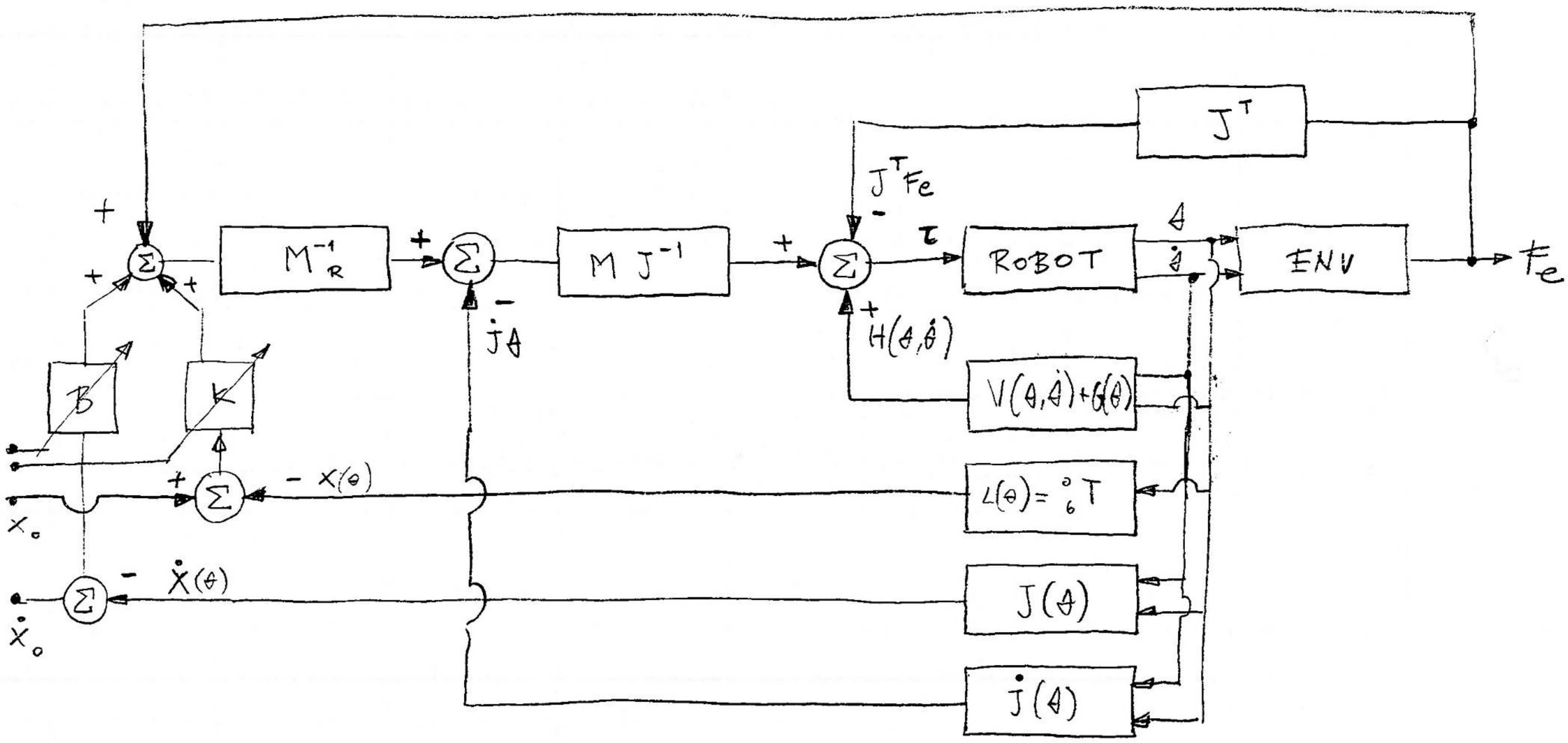
$$H(\theta, \dot{\theta}) = V(\theta, \dot{\theta}) + G(\theta)$$

Impedance Control – Generalized Approach for a mDOF

- The control law of the robot

$$\tau = \underbrace{MJ^{-1}\{M_R^{-1}[K(X_0 - X) + B(\dot{X}_0 - \dot{X}) + F_e] - \dot{J}\dot{\theta}\}}_{\text{Impedance Control Law}} + \underbrace{H(\theta,\dot{\theta}) - J^T F_e}_{\text{Dynamic Model}}$$

Impedance Control – Generalized Approach for a mDOF



Impedance Control – Generalized Approach

- Generalizing the stiffness control by adding damping

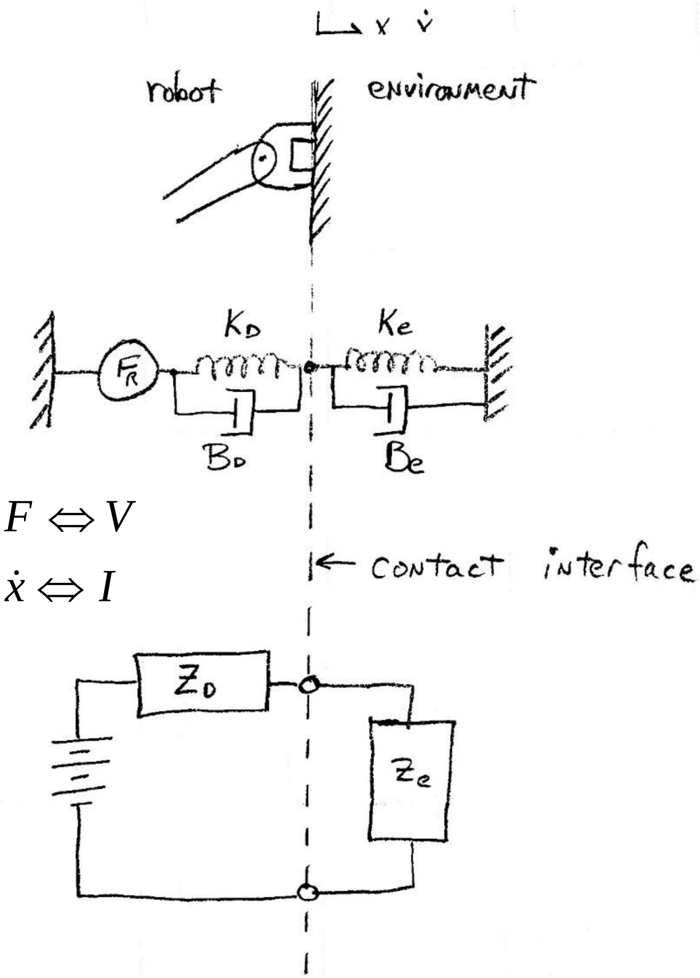
$$F_e = \dot{x}Z_e = F_R - \dot{x}Z_d$$

$$F_e = \dot{x}Z_e = F_R - K_d \int \dot{x}dt + C - B_d \dot{x}$$

$$C = -X_0$$

$$F_e = F_R - K_d(x - x_0) - B_d(\dot{x} - \dot{x}_0)$$

$$F_e = F_R + K_d(x_0 - x) + B_d(\dot{x}_0 - \dot{x})$$



$F \Leftrightarrow V$

$\dot{x} \Leftrightarrow I$

- Case 1 – Contact small velocity – Stiffness Control

$\dot{x}_0 = \dot{x} \approx 0 \qquad F_e = F_R + K_d(x_0 - x)$

- Case 2 – No contact Free motion – Velocity Control

$(x_0 - x) = 0 \qquad F_e = F_R + B_d(\dot{x}_0 - \dot{x})$

Impedance Control – Generalized Approach for a mDOF

- Assumptions
  - Ignoring dynamics
  - Compensation for gravity loads
- Joint torques (Eq. of motion of the robot)

$$\tau = J^T F + G(\Theta)$$

$$\tau = J^T [K_D (X_0 - X) + B_D (\dot{X}_0 - \dot{X})] + G(\Theta)$$

- Where K_D, B_D are 6x6 diagonal matrices representing the desired stiffness and damping of the manipulator

$$K_D = \begin{bmatrix} k_{11} & & & & & \\ & k_{22} & & & 0 & \\ & & k_{33} & & & \\ & & & k_{44} & & \\ & 0 & & & k_{55} & \\ & & & & & k_{66} \end{bmatrix} \qquad B_D = \begin{bmatrix} b_{11} & & & & & \\ & b_{22} & & & 0 & \\ & & b_{33} & & & \\ & & & b_{44} & & \\ & 0 & & & b_{55} & \\ & & & & & b_{66} \end{bmatrix}$$