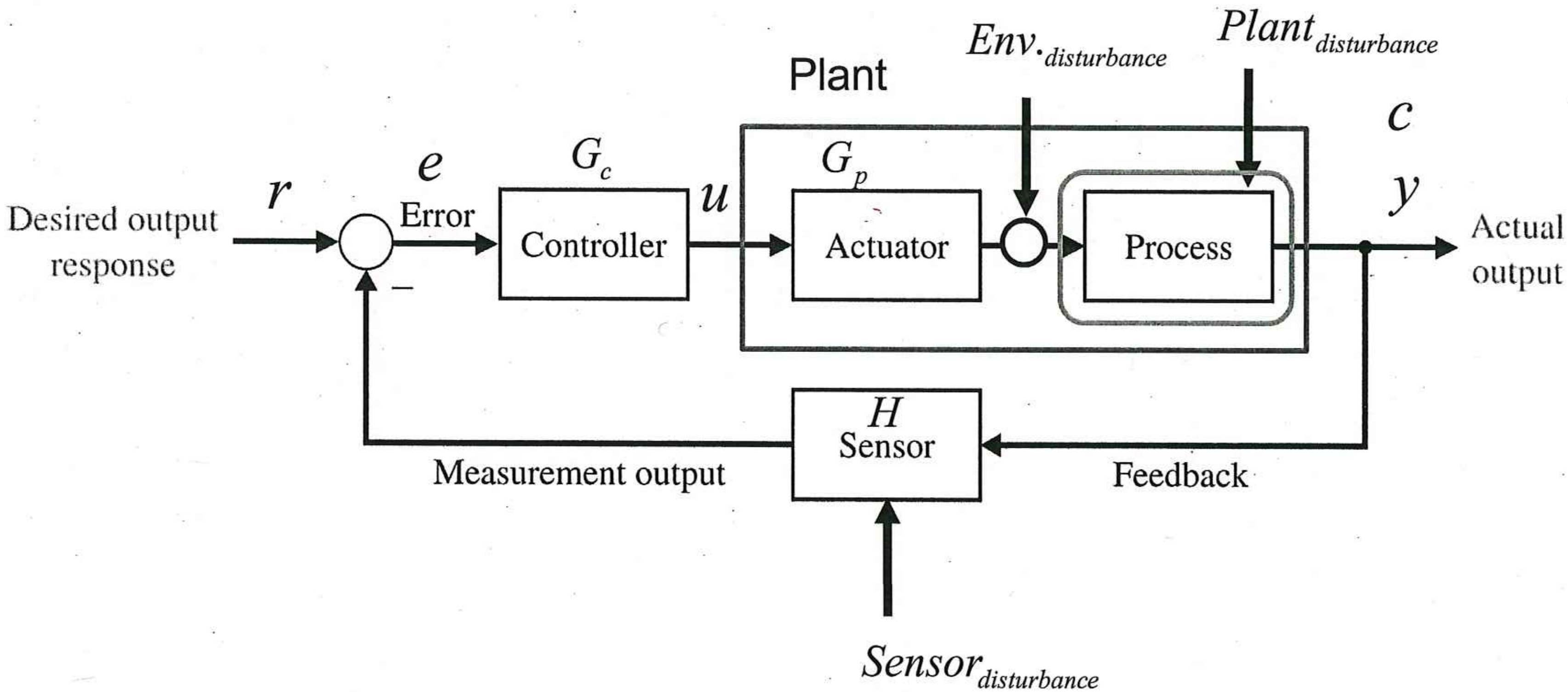
Control - Nyquist Design Method
Gain / Phase Margins

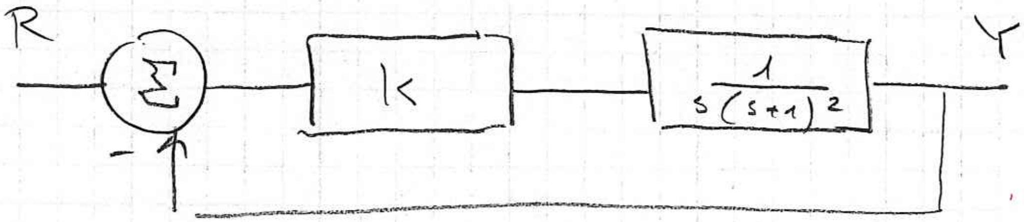
Close Loop / Feedback Control

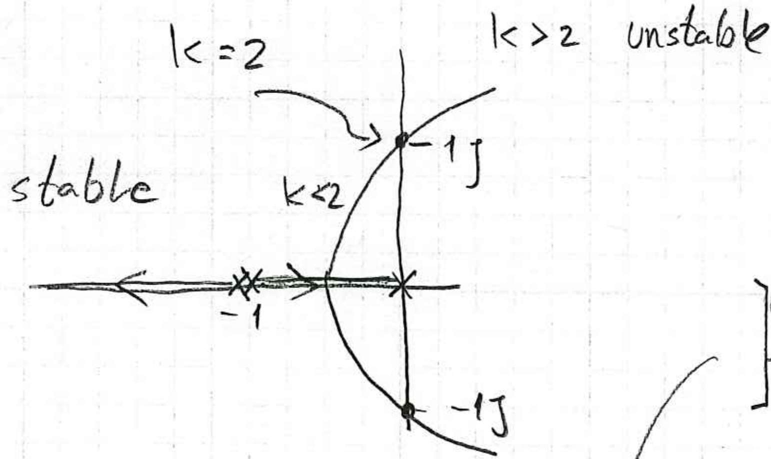


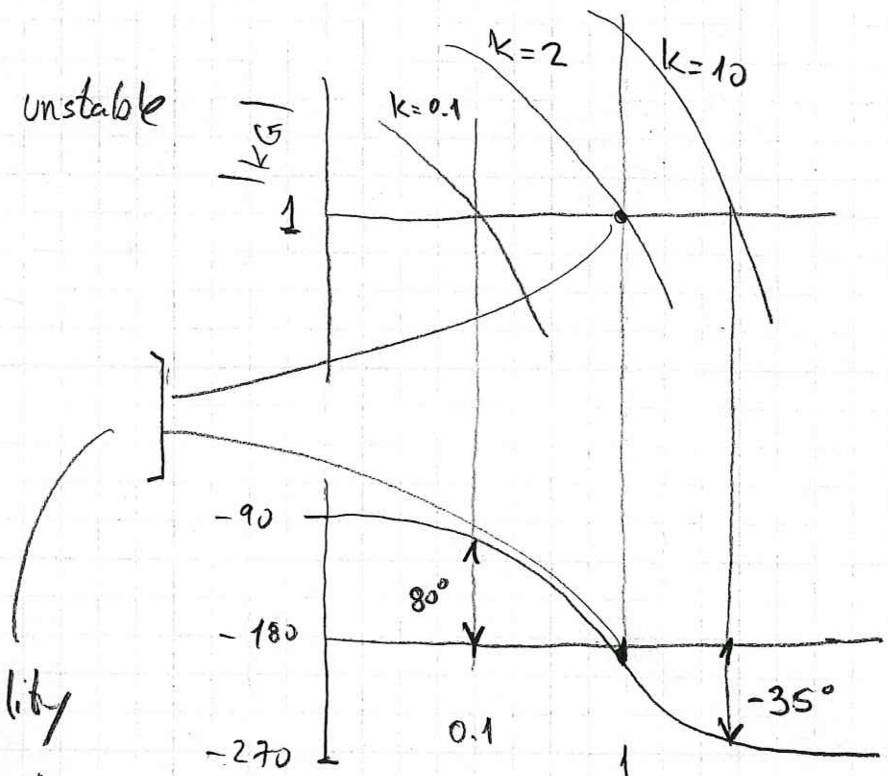
The Design Process of a Control System

- **Step 1: Goal, variables and, specifications**
  - Establish the control goals or requirements
  - Indentify the variables to be controlled
  - Write the specifications based on the requirements
- **Step 2: System definition and modeling**
  - Establish the system configuration (select sensors & actuators)
    - functional block diagram
    - Signal flow diagram
    - State space presentation
  - In case of multiple blocks, simplify the block diagram to a standard close loop system diagram
  - Obtain a model of the process, the actuator and the sensor
- **Step 3: Control system design simulation and analysis**
  - Describe the controller and select key parameters to be adjusted
  - Optimize the parameters and **analysis / simulate** the performance
- **If** the performance meets the specifications or the relaxed specifications – Finalize the design (End)
- Else iterate the configuration (go to step 2)

NEUTRAL STABILITY







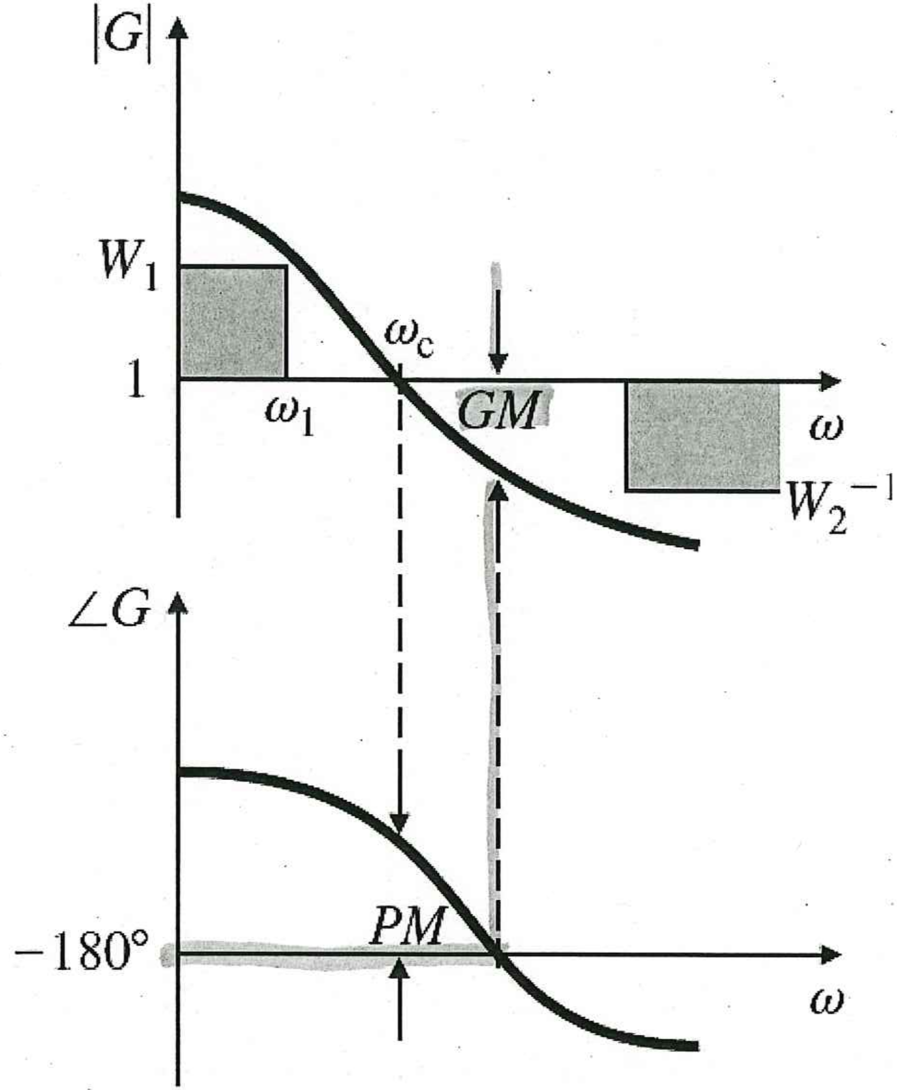
$$|KG(s)| = 1$$

$$\angle G(s) = -180$$

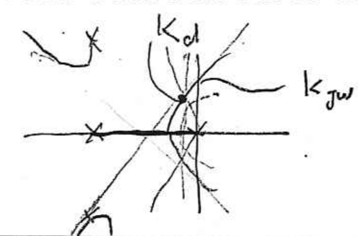
stability criteria

Gain & Phase Margin – Introduction

Gain Margin – Concept

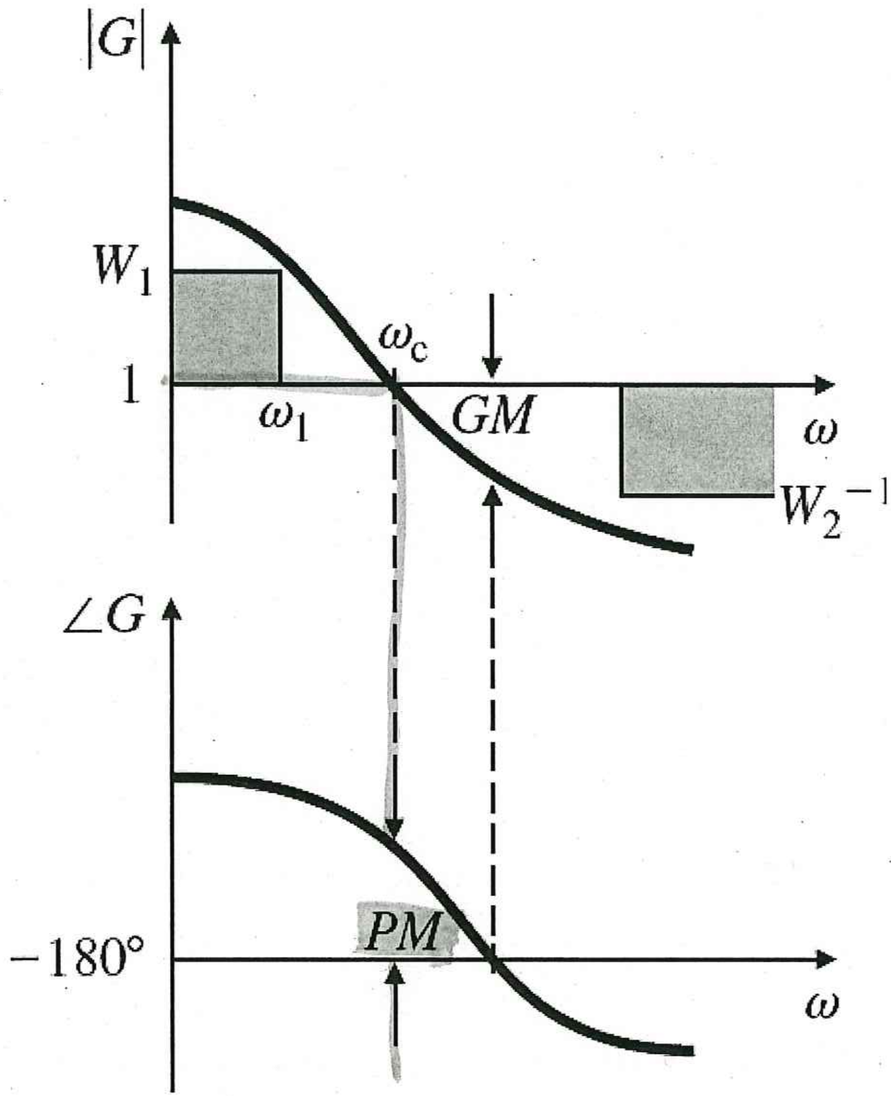


- **Gain Margin** (GM) – The factor by which the gain can be raised before instability results.
- Measure The vertical distance between curve and the $|KG(j\omega)|$ line at a frequency where $|KG(j\omega)| = 1$ $\angle KG(j\omega) = 180$
- Notes
  - GM is a factor by which the gain K can be raised before the instability results
  - The system is unstable when $|KG(j\omega)| < 1 \quad GM < 0dB$
  - The GM can also be determined from the root locus with respect to K by noting two values of K : (1) at the point where the locus crosses the jw-axis and (2) at the nominal close loop poles. The GM is the ratio between these two values.



$GM = \frac{K_{JW}}{K_d}$

Phase Margin – Concept



- **Phase Margin** (PM) – The amount by which the phase of $\angle KG(j\omega)$ exceeds –180 Deg when $|KG(j\omega)| = 1$
- How much phase lag the system can tolerate before it goes unstable.

$$PM \approx \frac{\xi}{100}$$

- **Crossover Frequency** (ω_c) – The frequency at which the gain is unity or 0 db

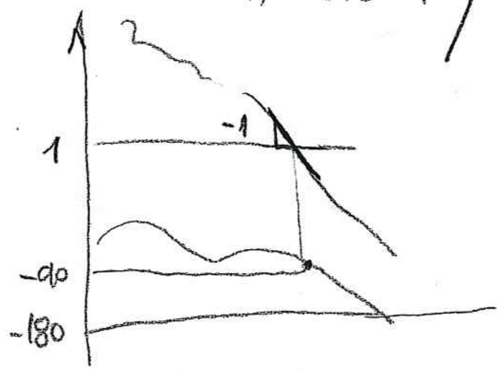
BODE THEOREM

SLOPE OF $|KG|$ $\triangleq$ PHASE $\measuredangle GH$

Slope of $\lvert KG \rvert$		Phase
0		0°
−1	⇒	−90°
−2	⇒	−180°
−3		−270°

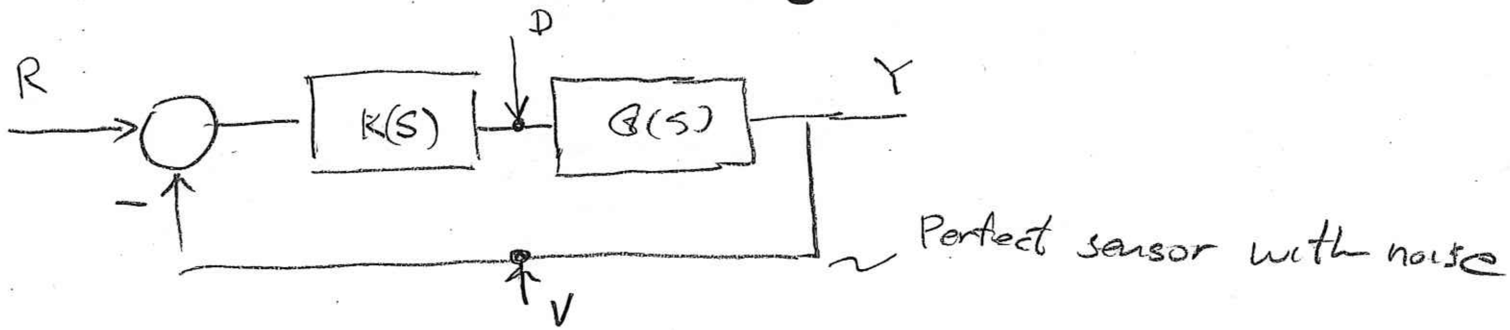
* FOR A GOOD STABILITY → CROSS $|KG| = 1$ WITH A SLOPE OF -1

⇒ $\measuredangle KG = 90°$ $\left[\zeta = \frac{90}{100} = 0.9 \right]$



* THE LEAD/LAG COMPANSATOR CAN HELP TO FIX THE SLOPE TO BE CLOSE TO -1

Bode Design

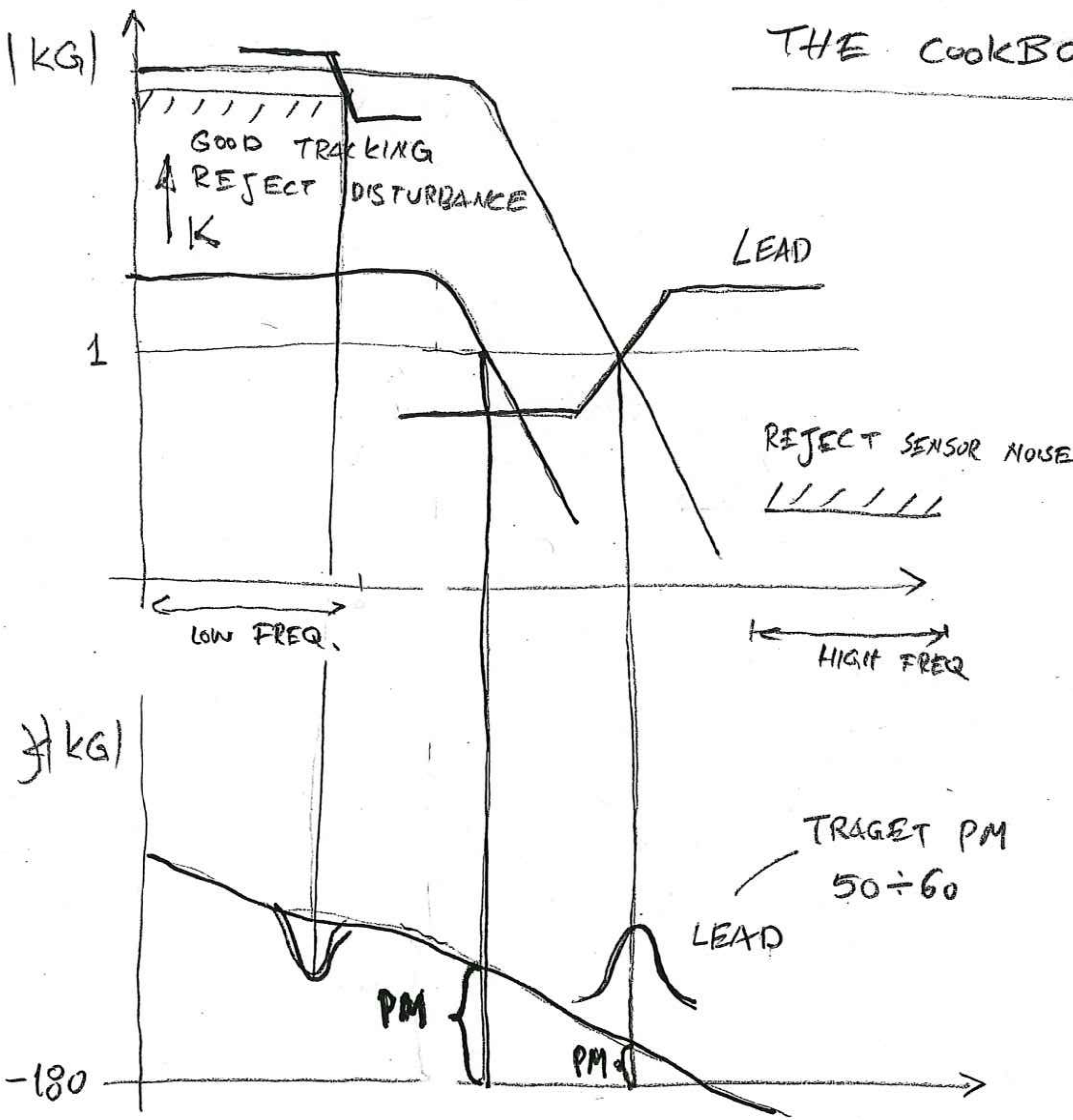


Requirment for KG

	KG	WHY	FREQ.
$\frac{Y}{V} = \frac{kG}{1+kG}$; $\frac{Y}{V} \to 0$	SMALL KG	• Reject noise from the sensor	HIGH
$\frac{Y}{D} = \frac{G}{1+kG}$; $\frac{Y}{D} \to 0$	LARGE KG	• Reject disturbance	LOW
$\frac{Y}{R} = \frac{kG}{1+KG}$; $\frac{Y}{R} \to 1$	LARGE KG	• Good Tracking	LOW

Bodе Design - GENERAL APPROUCH

THE COOKBOOK



I Increas gain k
- Meet steady state error req.
- Meet bandwith req.

II Use a Lead comp.
- Increase the PM
- Problem
  - Atten - low Freq
  - Ampl - High Freq

III Use a Lag comp. (OR LP)
- Fix the gain at low Freq.

Bode Design – Guidelines

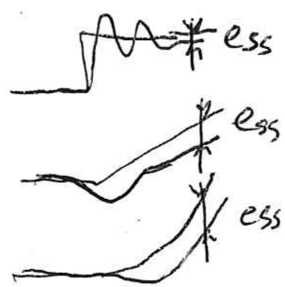
- **Step 1 – Gain K** – Determine open loop gain K to satisfy (1) **steady state error** or (2) **frequency bandwidth** requirements
  - **1.1 Steady State Error** - To meet the steady state error requirements pick K to satisfy steady error e_{ss}

$$\frac{E(s)}{R(s)} = \frac{1}{1+kG(s)} \Rightarrow E(s) = \left[\frac{1}{1+kG(s)}\right] \cdot R(s)$$

$$e_{ss} = e(t \to \infty) = \lim_{s \to 0} s \left[\frac{1}{1+kG(s)}\right] R(s)$$

FVT

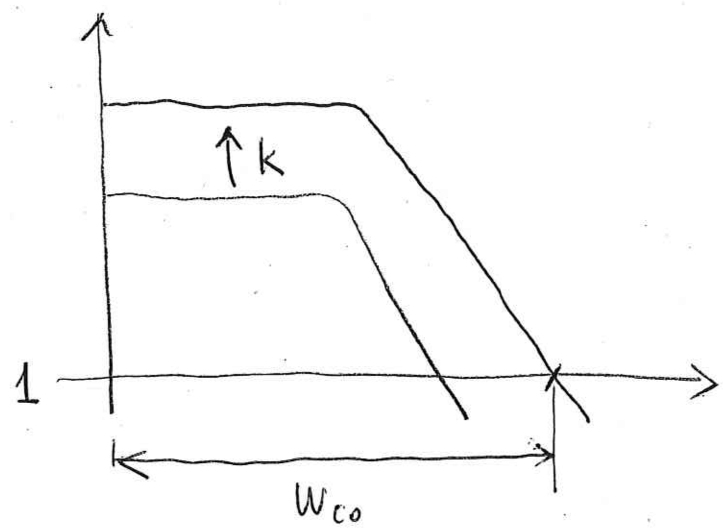
$$R(s) = \begin{cases} \frac{1}{s} \\ \frac{1}{s^2} \\ \frac{1}{s^3} \end{cases}$$



Bodе Design – Guidelines

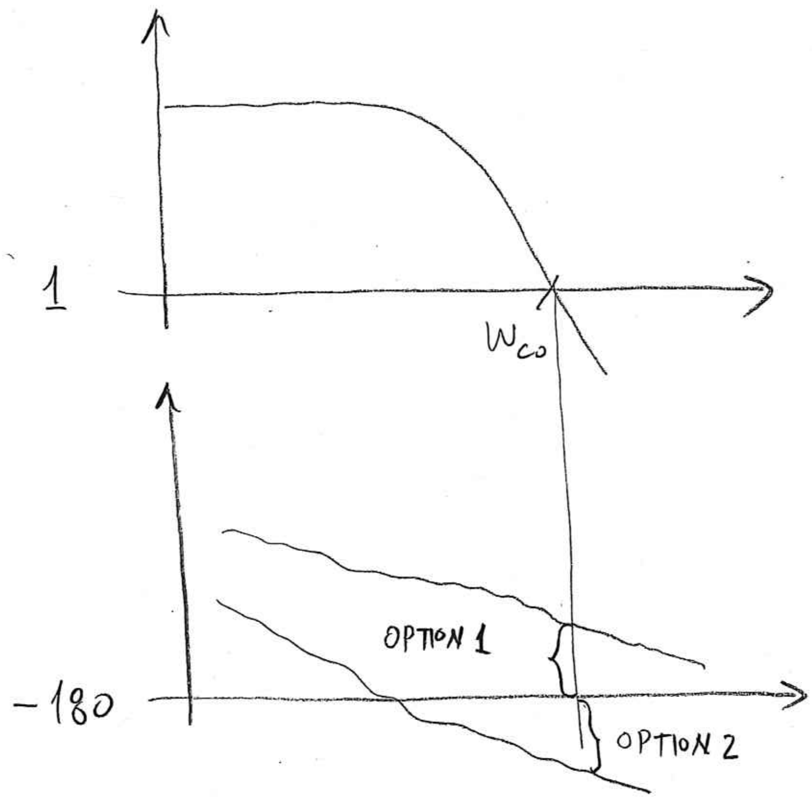
•

– **1.2 Frequency Bandwidth** - Pick K such that the open loop cross over frequency (ω_{CO}) is below the desired close loop frequency bandwidth by a factor of 2



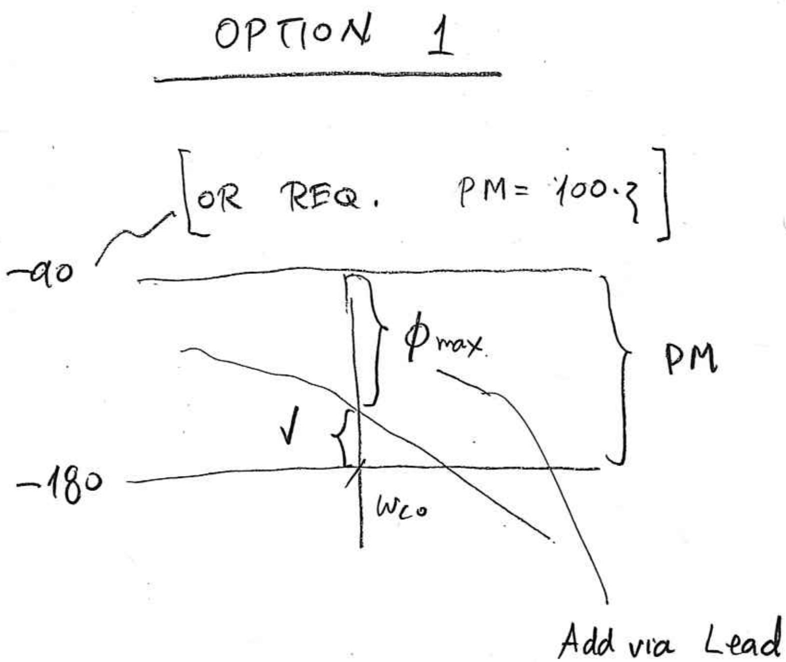
Bodе Design – Guidelines

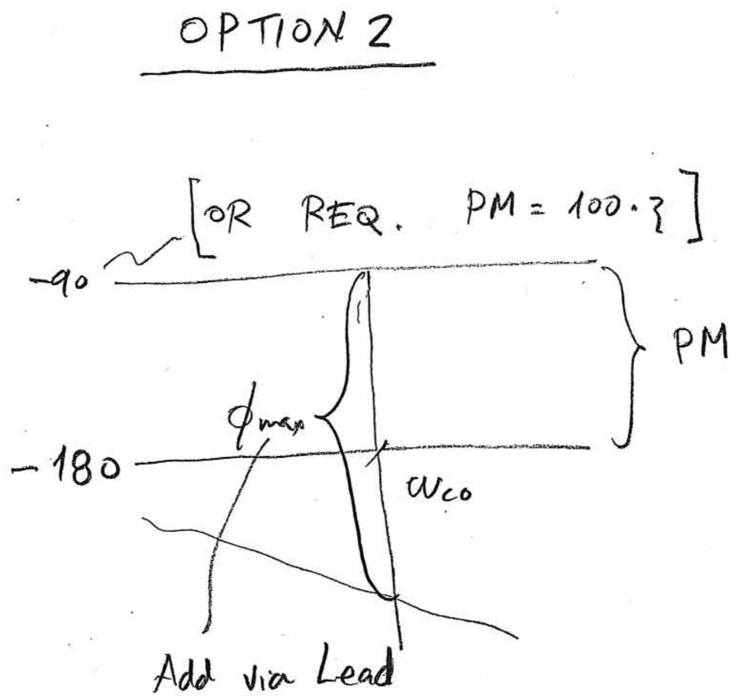
- **Step 2 – Lead Compensator –**
  - 2.1 - Evaluate the PM of the uncompensated system using the value of K obtained from Step 1.2



Bode Design – Guidelines

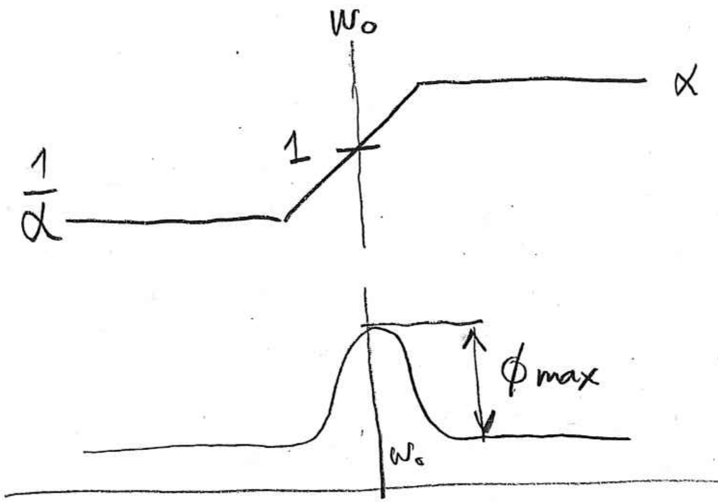
- **Step 2 – Lead Compensator –**
  - 2.2 - Allow for extra margin (~ 10 deg) and determine the needed phase lead ϕ_{max} (added by the lead compensator)





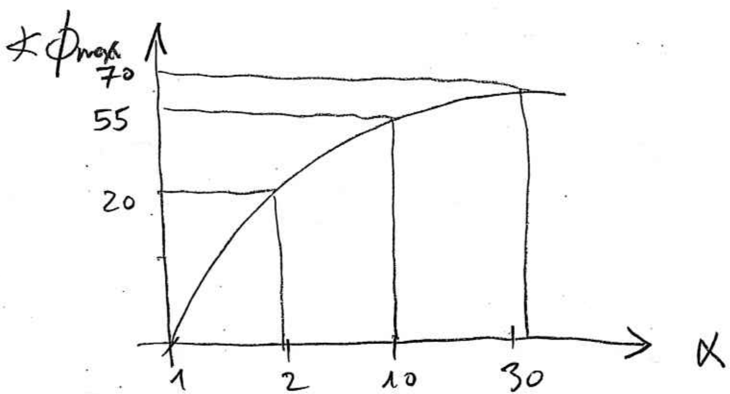
Bode Design – Guidelines

- **Step 2 – Lead Compensator –**
  - **2.3 – Adjusting the parameters of the Lead** - Determine the α of the lead compensator



$$K(s) = \sqrt{\alpha}\,\frac{s + \frac{w_o}{\sqrt{\alpha}}}{s + w_o\sqrt{\alpha}}$$

$$K(s)\Big|_{s\to 0} = \sqrt{\alpha} \cdot \frac{\frac{w_o}{\sqrt{\alpha}}}{w\sqrt{\alpha}} = \frac{1}{\sqrt{\alpha}}$$



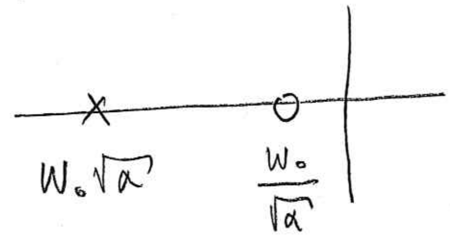
$$\rightarrow \sphericalangle\phi_{max} = \tan^{-1}\sqrt{\alpha} - \tan^{-1}\frac{1}{\sqrt{\alpha}}$$

Bode Design – Guidelines

- **Step 2 – Lead Compensator –**
  - **2.4 – Adjusting the parameters of the Lead** - Pick ω_0 to be the cross over frequency

$$W_o = W_{co}$$

$$K(s) = \sqrt{\alpha}\,\frac{s + \frac{W_{co}}{\sqrt{\alpha}}}{s + W_{co}\sqrt{\alpha}}$$



  - **2.5 – Bode Plot** - Draw the compensator frequency response and check the PM
  - **2.6 – Design Iteration** - Iterate on the design and adjust the poles , zeros and gain
  - **2.7 – Additional Leads** - Add an additional lead compensator (i.e. a double lead compensator)
    - **Note – Max PM per lead compensator is 70 Deg**

Bode Design – Guidelines

- **Step 3 – Lag Compensator –**
  - **3.1 – Gain K Adjustment**
    - Determine the open loop gain K that will meet the PM requirements without compensation.

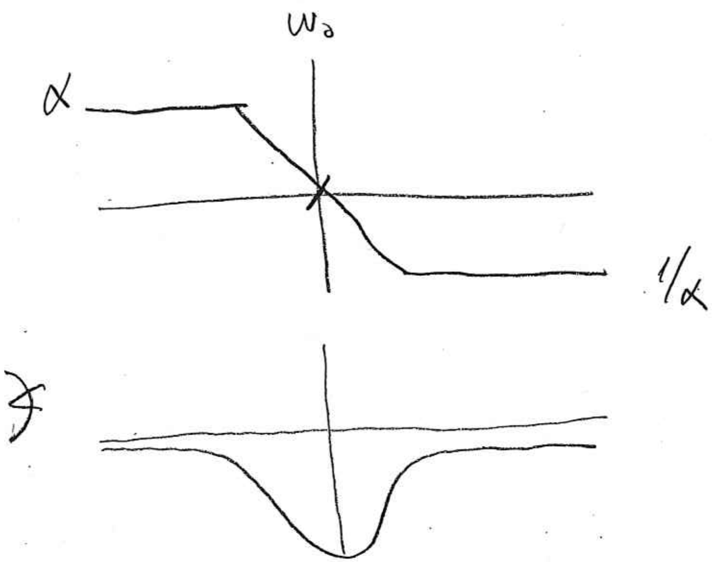
    OR

    - Determine the open loop gain K that is still needed to meet the Steady State error – e_{ss}

  - **3.2 - Bode Plot -** Draw the bode plot of the open loop system with the cross over frequency and evaluate the low frequency gain.

Bode Design – Guidelines

- **Step 3 – Lag Compensator –**
  - **3.3 – Adjusting the parameters of the lead compensator**
  - Determine the α of the lead compensator

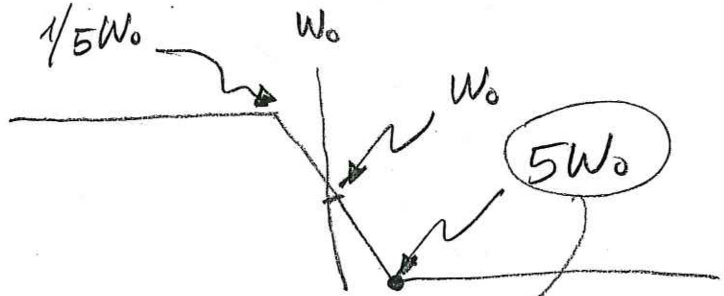


$$K(s) = \frac{s + w_o\sqrt{\alpha}}{s + \frac{w_o}{\sqrt{\alpha}}}$$

$$K(s)\Big|_{s\to 0} = \frac{w_o\sqrt{\alpha}}{\frac{w_o}{\sqrt{\alpha}}} = \alpha$$

Bode Design – Guidelines

- **Step 3 – Lag Compensator –**
  - **3.4 – Adjusting the parameters of the lead compensator**
  - Choose the cross over frequency ω_0 of the lag (zero of the lag compensator) to be 1 octave to 1 decade below the new cross over frequency



$$\frac{W_{oc}}{2} > 5W_{o\,(Lag)} > \frac{W_{co}}{10}$$

MOST CRITICAL DESIGN

SET W_o OF THE LAG ALL THE WAY TO THE LEFT FROM THE W_{co}

$$5W_o = \frac{W_{co}}{10}$$

$$W_o = \frac{W_{co}}{50}$$

  - Design Iteration – Iterate on the design and adjust the compensator parameters (pole , zero gain) in order to meet all the specs.

BODE DESIGN

(1) STEP 1 - GAIN K - Determine open loop gain k to satisfy error [or] bandwith requirements

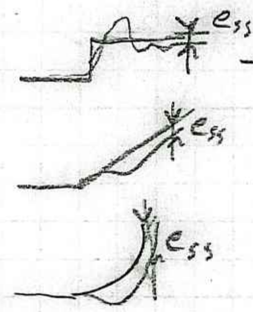
1.1 ERROR - To meet error requirment pick K to satisfy steady state error e_{ss}

$$\frac{E(s)}{R(s)} = \frac{1}{1+kG(s)} \Rightarrow E(s) = \frac{1}{1+kG(s)} R(s)$$

$$e_{ss} = e(t \to \infty) = \lim_{s \to 0} s \left[\frac{1}{1+kG} \right] R(s)$$

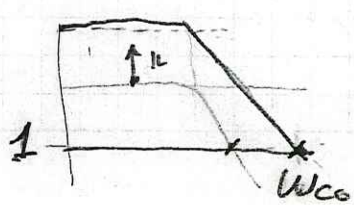
NOTE

R(s)



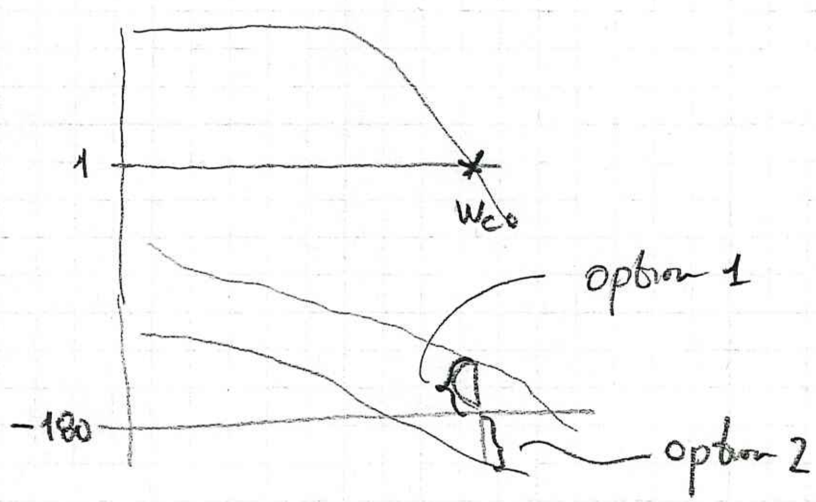
$\frac{1}{s}$, $\frac{1}{s^2}$, $\frac{1}{s^3}$

1.2 BANDWIDTH - Pick k so that the open loop croosover (w_{co}) frequancy is a factor of 2 below the desired close-loop bandwith



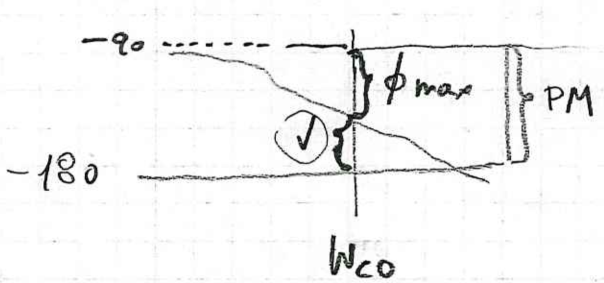
(2) STEP 2 - LEAD

2.1 Evaluate the PM of the uncompensated system using the value of K obtained from step 1.2

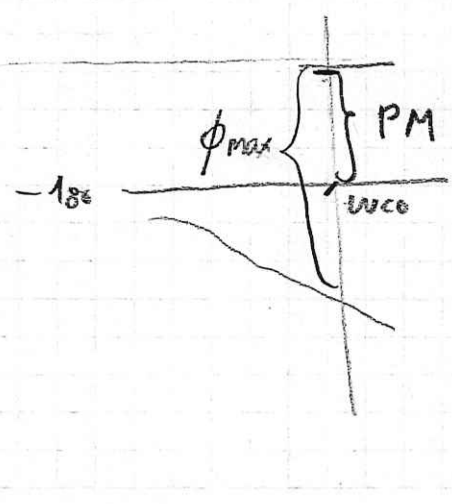


2.2 Allow for extra margin (~10°) and determine the needed phase lead ϕ_{max} (added by the lead comp.)

OPTION 1

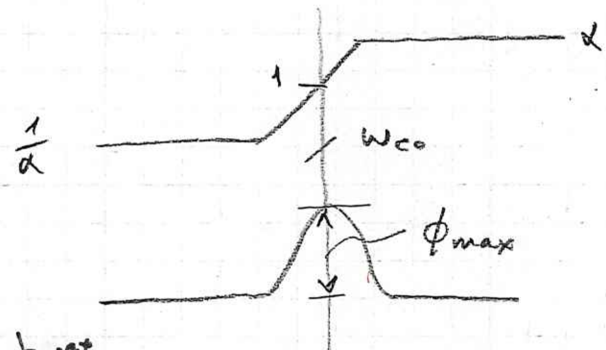


OPTION 2

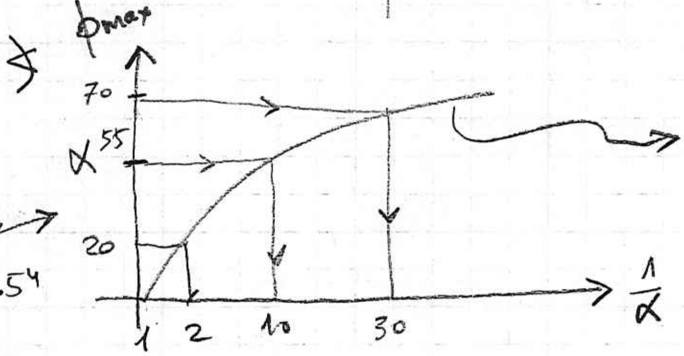


$K|_{s\to 0} = \sqrt{\alpha}\,\frac{\omega_0/\sqrt{\alpha}}{\omega_0\sqrt{\alpha}} = \sqrt{\alpha}\,\frac{\omega_0}{\sqrt{\alpha}}\,\frac{1}{\omega_0\sqrt{\alpha}} = \frac{1}{\sqrt{\alpha}}$

2.3 Determine α



$$K(s) = \sqrt{\alpha}\,\frac{s + \frac{\omega_0}{\sqrt{\alpha}}}{s + \omega_0\sqrt{\alpha}}$$



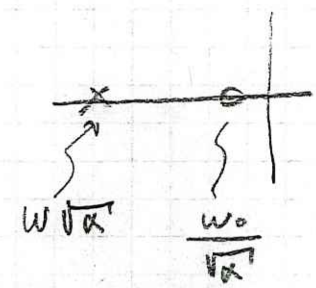
see fig 6.54 p 350

$$\measuredangle = \tan^{-1}\sqrt{\alpha} - \tan^{-1}\frac{1}{\sqrt{\alpha}}$$

2.4 Pick ω_0 to be the crossover frequency

$\omega_0 = \omega_{co}$

$$K(s) = \sqrt{\alpha}\,\frac{s + \frac{\omega_0}{\sqrt{\alpha}}}{s + \omega\sqrt{\alpha}}$$



2.5 Draw the compensated frequency response and check the PM

2.6 Iterate on the design

Adjust poles, zeros and Gain

2.7 Add an additional lead compansator (i.e. a double-lead compansator)

MAX PM PER LEAD COM. IS 70°

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(3) STEP 3 - LAG

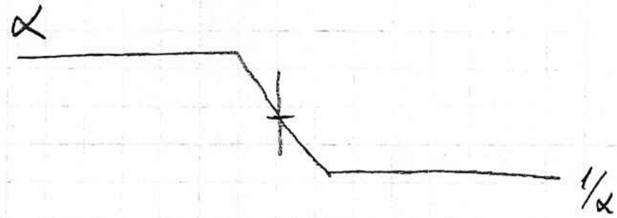
3.1 Determin the open loop gain K that will meet the PM requirment with out compansation

OR

[Determin the open loop gain K that is still needed to meet the e_{ss}

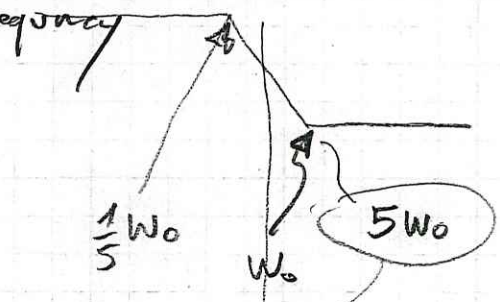
3.2 Draw the Bode plot of the open loop system with the cross over freq. and evalute the low freqvancy gain

3.3 Determine α to meet the low frequency gain based on the steady state req.



$$K(s) = \frac{s + W_o\sqrt{\alpha}}{s + \frac{W_o}{\sqrt{\alpha}}} \xrightarrow[s \to 0]{} K(0) = \alpha$$

3.4 Choose the corner frequency (ZERO of the lag compenstor to be 1 octave to 1 decade bellow the new cross over frequency



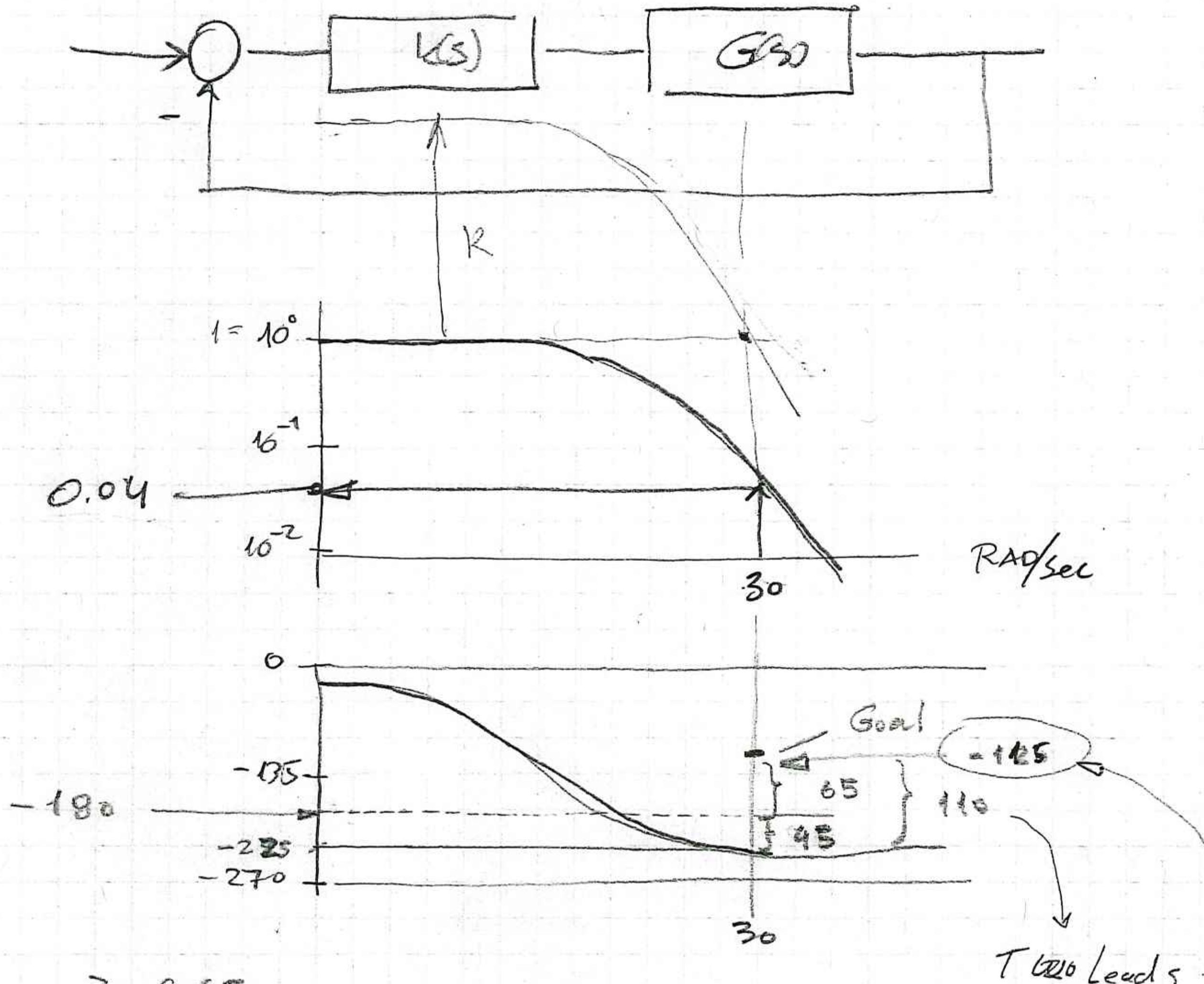
$$\frac{W_{oc}}{2} > 5W_{o(lag)} > \frac{W_{co}}{10}$$

3.5 Iterate on the design. Adjust compensator parameters (poles, zeros, gain) to meet all specs.

Review Book
P 356 ↳ LEAD } BLUE
P 361 – 362 LAG } Box

BODE DESIGN – EXAMPLE

$$G(s) = \frac{1152}{(s+9)(s^2+16s+128)}$$



$\zeta = 0.65$

$\zeta = \frac{PM}{100}$

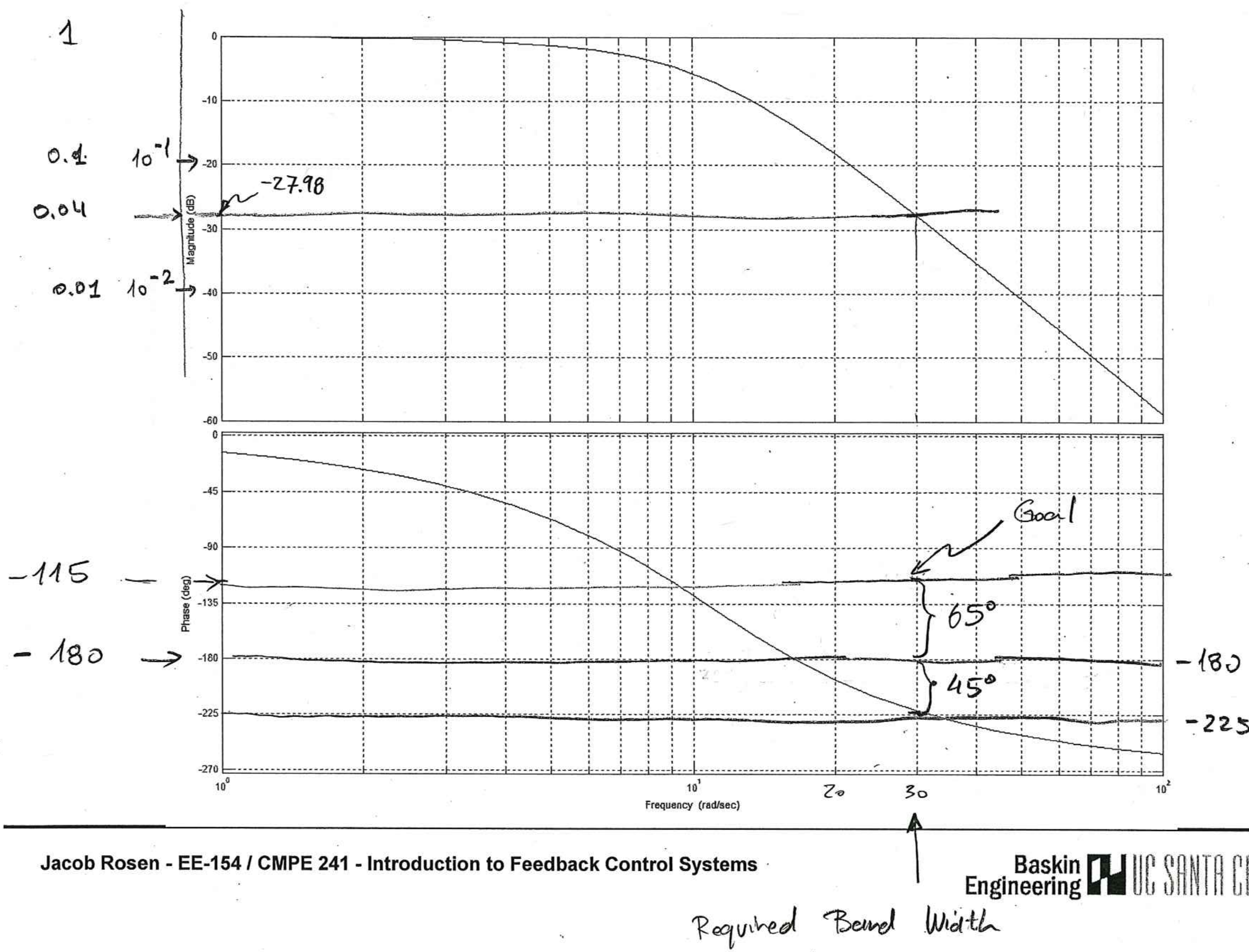
$W_{co} = 4.75$ Hz

$PM = 65 \Rightarrow \measuredangle\ 180 - 65 = 115$

$E_{ss} < 1\%$

Bode Design – Example

$$G(s) = \frac{1152}{(s+9)(s^2+16s+128)}$$



STEP 1 (K)

$W_{co} = 4.75\ Hz = 4.75 \frac{cycle}{sec} \frac{2\pi\ RAD}{1 cycle} = 29.84 \frac{RAD}{sec}$

- FROM THE BODE GRAPH
- FOR $W_{co} \simeq 30\ RAD/sec \rightarrow |G(30)| = 0.04$
- CHOOSE K_0 TO GET $W_{co} \simeq 30$ RAD

$$|K_0 G|_{W_{co}} = 1$$

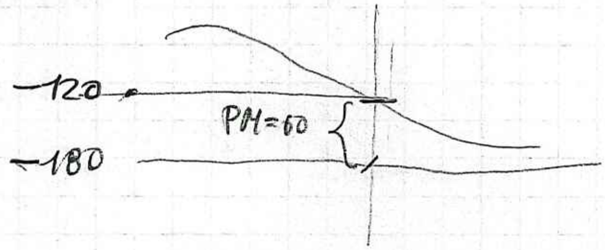
$$K_0 = \frac{1}{|G|_{W_{co}}} = \frac{1}{0.04} = 25$$

STEP 2 (LEAD)

- ADD PM WITH A LEAD

AT W_{co} $\quad \angle G|_{W_{co}} \cong -225$

- FOR $PM \simeq 65$ $\quad \angle G|_{W_{co}} \geq -115$

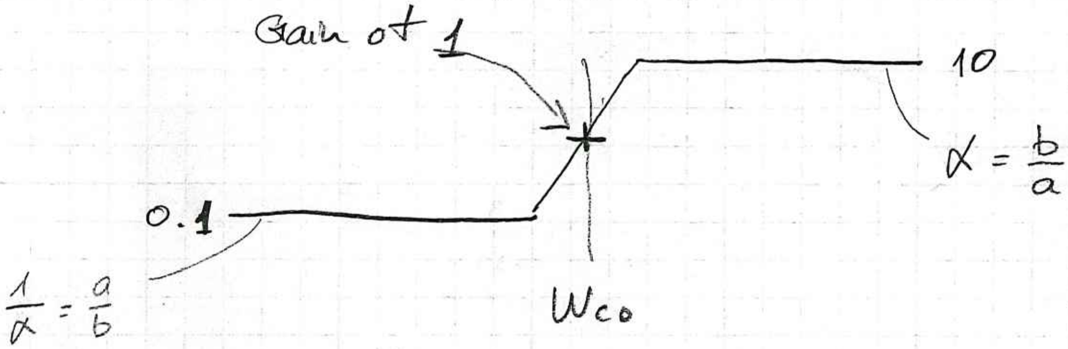


ADD 110°

- NEED TO ADD 110° $\quad \angle G|_{W_{co}} = -225 \rightarrow -115$

• TO ADD 110° (TOO MUCH FOR A SIGLE LEAD - TYPICALLY ADD UP TO 70° PM)

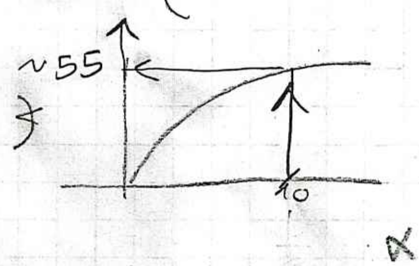
⇒ ADD TWO LEADS @ 55°



$$K(s) = \sqrt{\alpha}\,\frac{s + \frac{W_0}{\sqrt{\alpha}}}{s + W_0\sqrt{\alpha}}$$

→ SEE GRAPH (FIG 6.54)

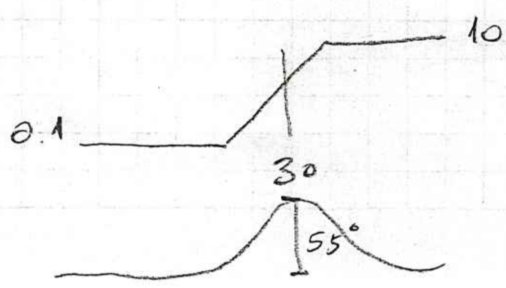
FOR PM ≃ 55°



$\alpha = 10$

$W_{co} = 30$

phase $\measuredangle = \tan^{-1}\sqrt{10} - \tan^{-1}\frac{1}{\sqrt{10}} = 72.45° - 17.58 = 54$



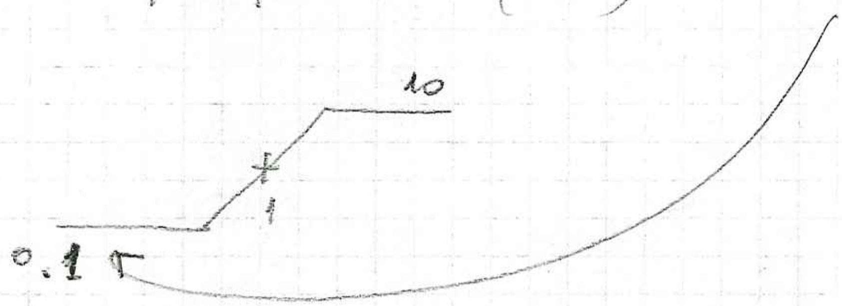
FOR ONE LEAD

$$k(s) = \sqrt{10}\ \frac{s + \frac{30}{\sqrt{10}}}{s + 30\sqrt{10}}$$

FOR TWO LEADS

$$k(s) = 10 \left(\frac{s + 9.48}{s + 94.86} \right)^2$$

FOR $s \to 0$ $\quad |k| = 10 \cdot (0.1)^2 = 0.1$



-COMBINED CONTROLER

$$k(s) = k \cdot k(\text{LEAS})$$

$$= 25 \cdot 10 \left(\frac{s + 9.48}{s + 94.86} \right)^2$$

$$= 250 \left(\frac{s + 9.48}{s + 94.86} \right)^2$$

$$e_{ss} = \lim_{s \to 0} s \left[\frac{1}{1 + k(s)G(s)} \right] R(s)$$

$$e_{ss} = \lim_{s \to 0} \cancel{s} \left[\frac{1}{1 + k(s)\,G(s)} \right] \frac{1}{\cancel{s}}$$

$$= \frac{1}{1 + k(0)\,G(0)}$$

$$G(s)\Big|_{s=0} = \frac{1152}{(s+9)(s^2+16s+128)}\Big|_{s=0} = \frac{1152}{9 \cdot 128} = 1$$

$$k(s)\Big|_{s=0} = 250 \left(\frac{s + 9.48}{s + 94.80} \right)^2 \Big|_{s=0} = 250 \cdot \left(\frac{1}{10} \right)^2 = \boxed{2.5}$$

$$e_{ss} = \frac{1}{1 + 2.5} = 0.285$$

$$e_{ss} < 0.01 \qquad \frac{1}{1+k} = 0.01$$

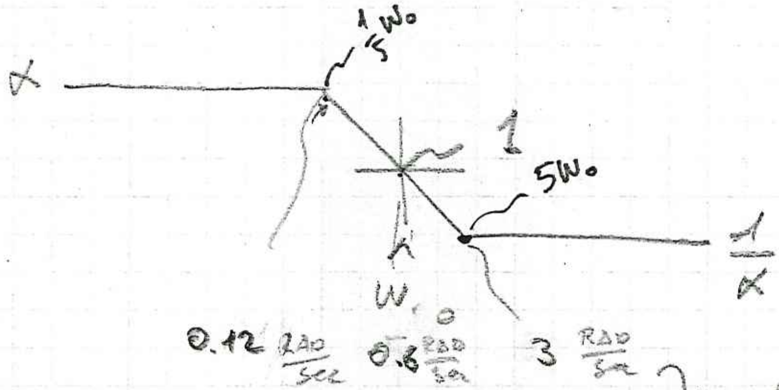
$$1 = 0.01 + 0.01k$$

$$0.99 = 0.01k$$

$$k = 99 \simeq 100$$

$$\text{NEW } k = 100 / \boxed{2.5} = 40$$

LAG



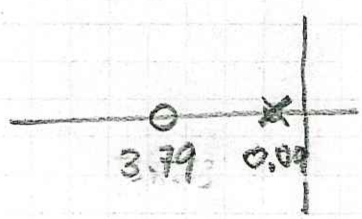
1 decade belowe the ω_{cb}

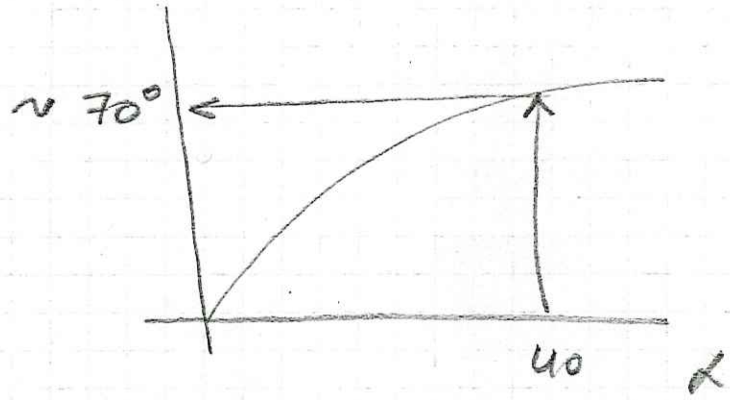
$$k(s) = \frac{s + W_0\sqrt{\alpha}}{s + \frac{W_0}{\sqrt{\alpha}}}$$

$$s \to 0 \qquad \frac{W\sqrt{\alpha}}{\frac{W_0}{\sqrt{\alpha}}} = W\sqrt{\alpha}\,\frac{\sqrt{\alpha}}{W} = \alpha$$

$$W_0 = 0.6\ \frac{RAD}{sec} \qquad \alpha = 40$$

$$k(s) = \frac{s + 0.6\overbrace{\sqrt{40}}^{6.32}}{s + 0.6/\sqrt{40}} = \frac{s + 3.79}{s + 0.09}$$





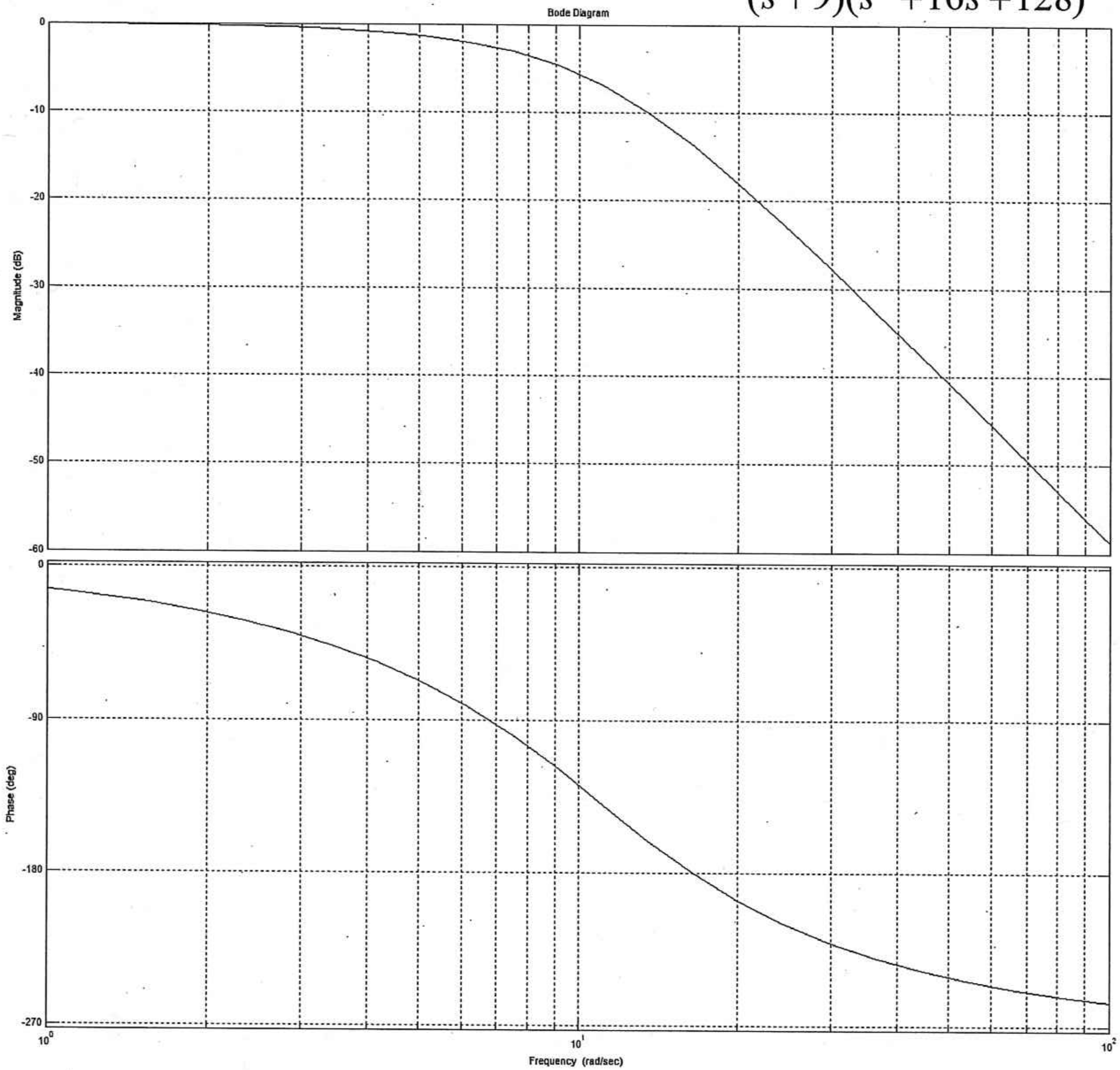
$$\vartheta = \tan^{-1}\frac{1}{\sqrt{\alpha}} - \tan^{-1}\sqrt{\alpha} = \tan^{-1}\frac{1}{\sqrt{40}} - \tan^{-1}\sqrt{40}$$

($\sqrt{40}$: 6.32; $\sqrt{40}$: 6.32)

$$= 8.98 - 81.00 \approx 71^\circ$$

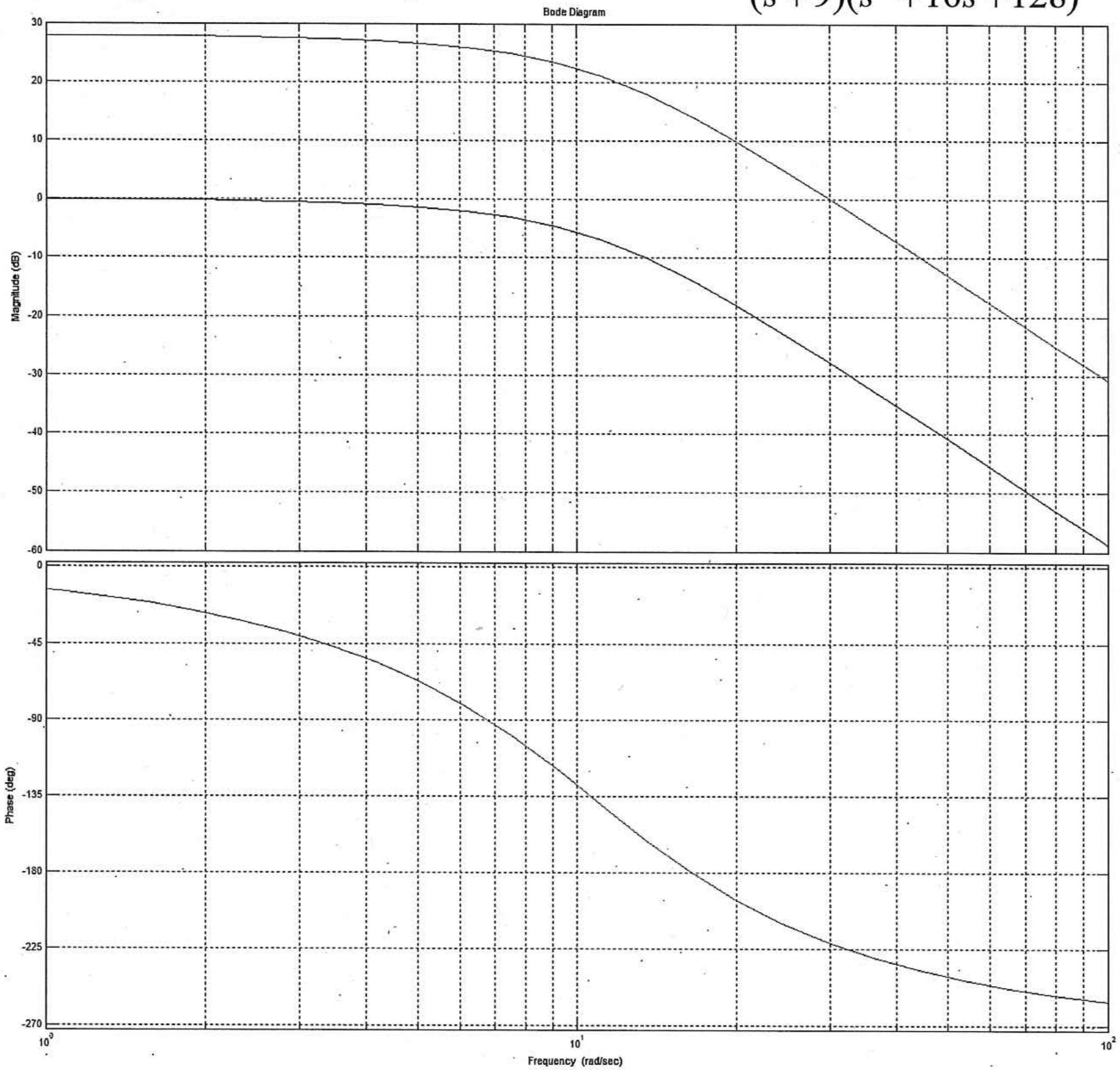
Bode Design – Example

$$G(s) = \frac{1152}{(s+9)(s^2+16s+128)}$$



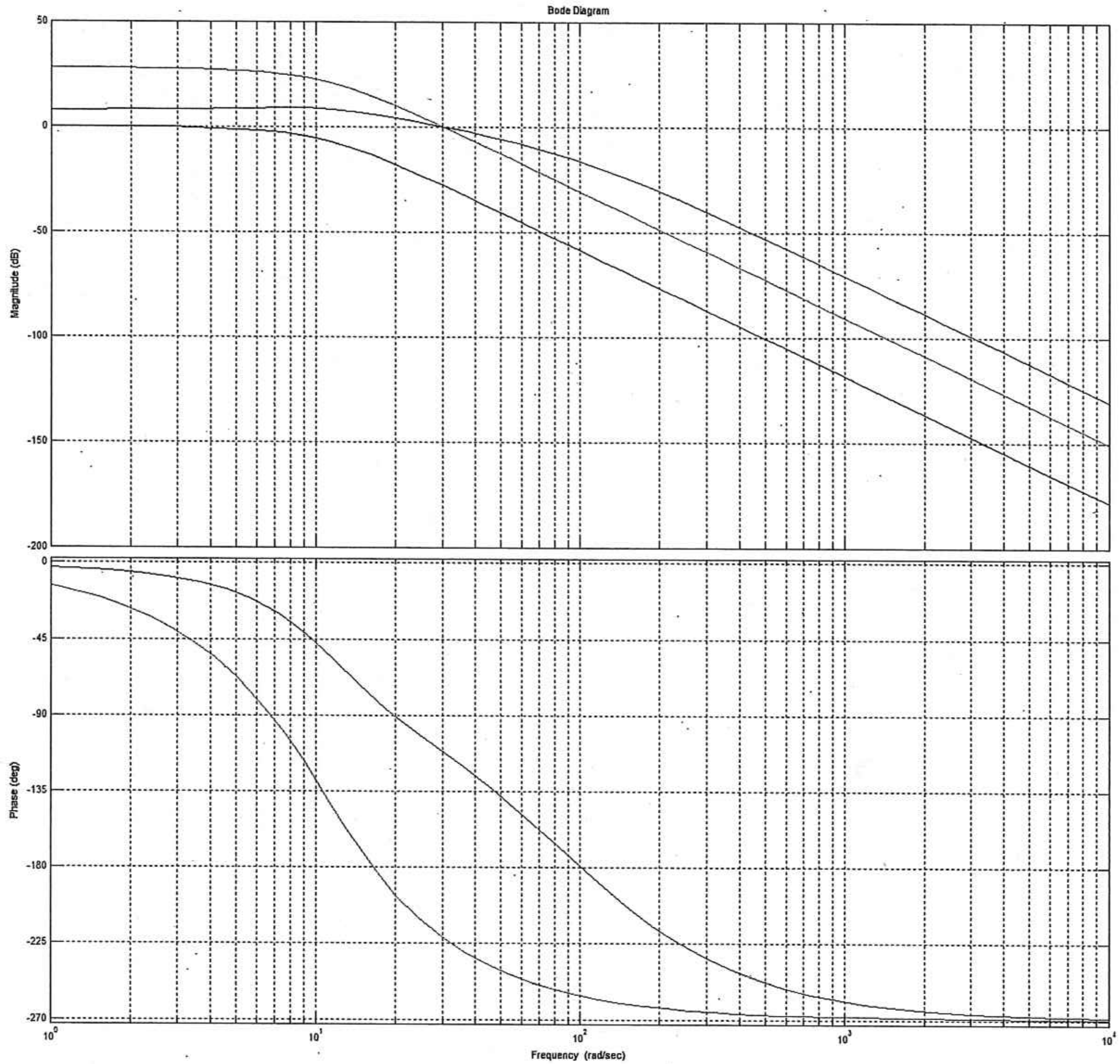
Bode Design – Example

$$G(s)=\frac{25\cdot 1152}{(s+9)(s^2+16s+128)}$$



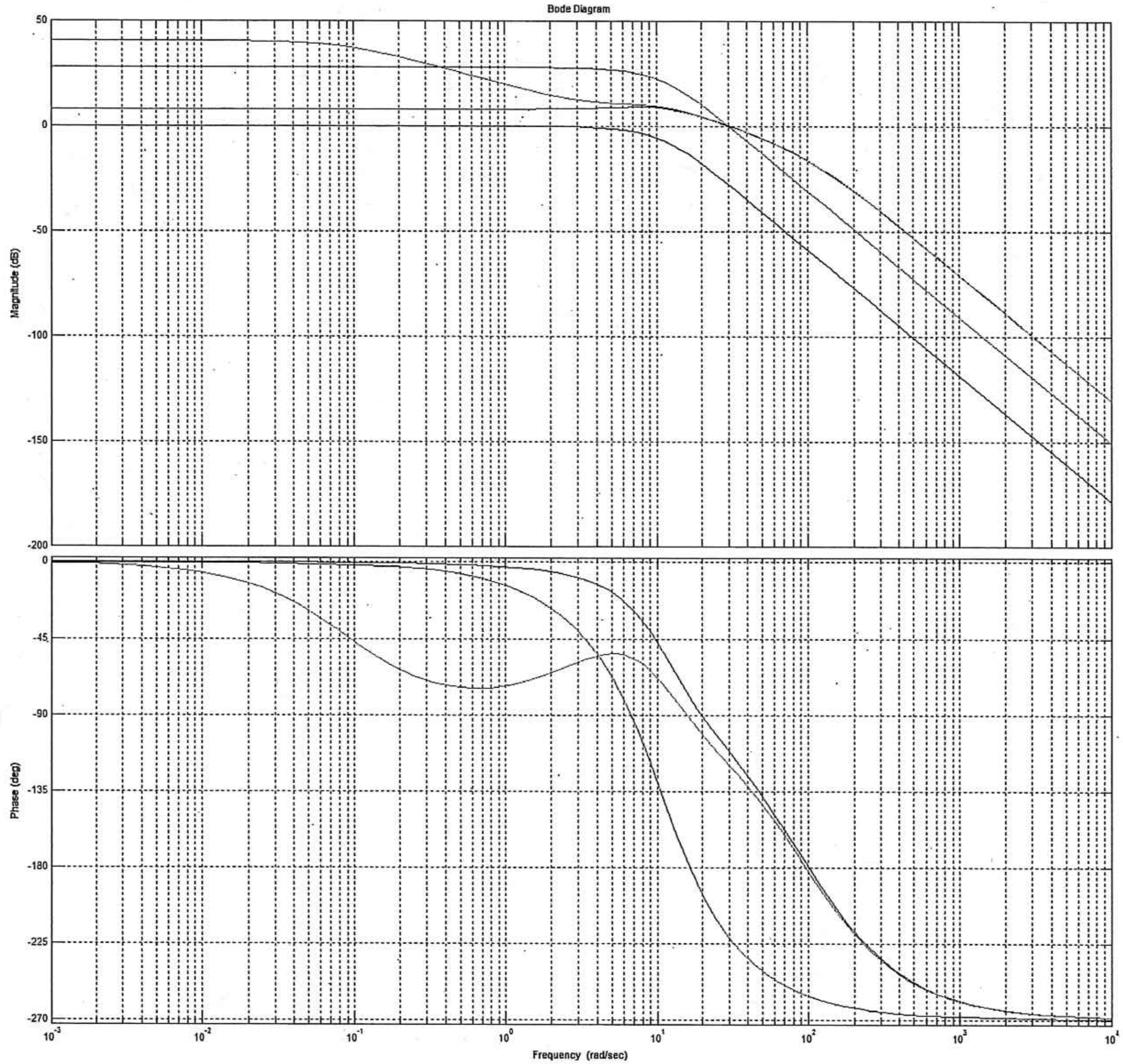
Bode Design – Example

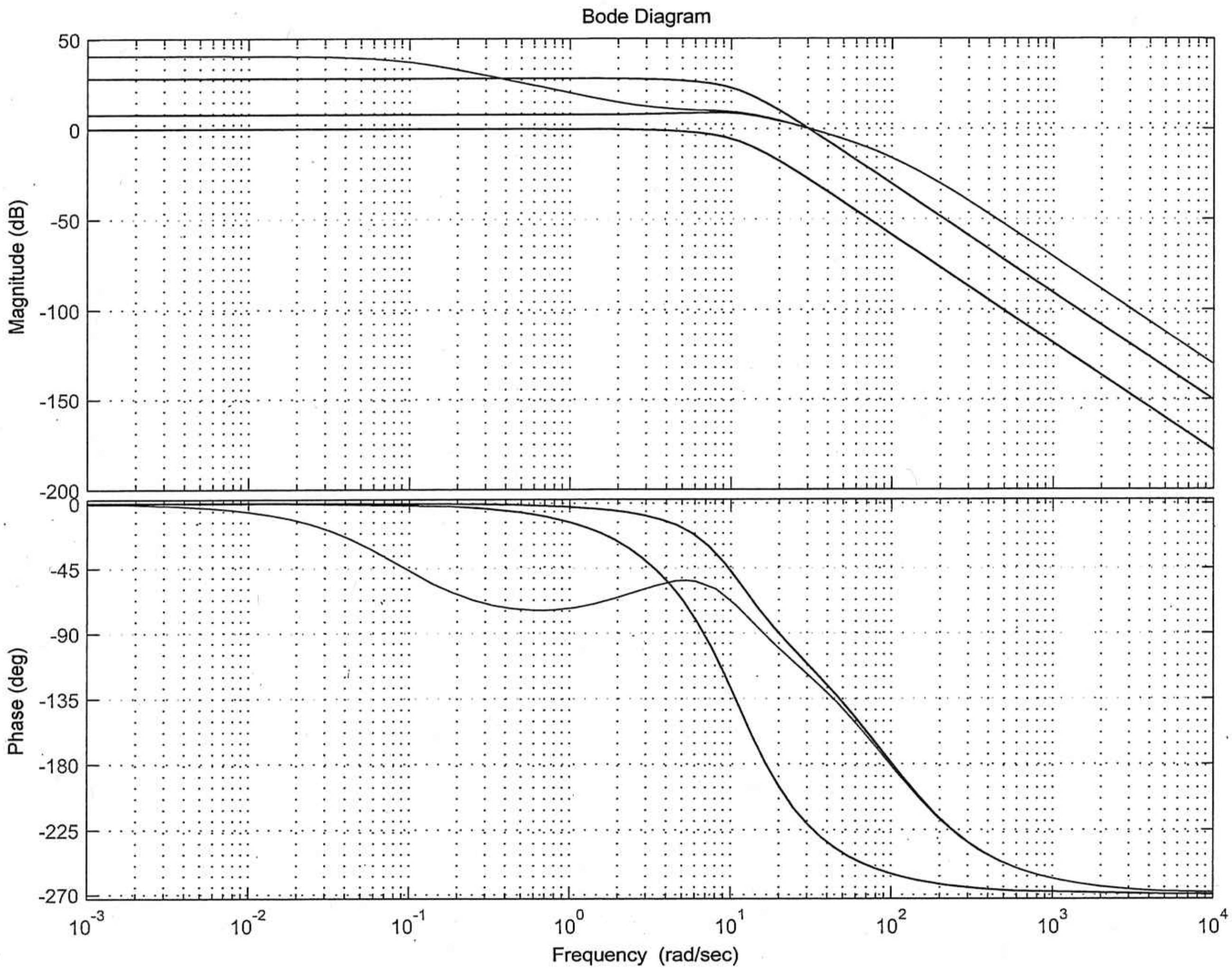
$$G(s) = \frac{25 \cdot 1152}{(s+9)(s^2+16s+128)} \cdot 10\left(\frac{s+9.48}{s+94.86}\right)^2$$



Bode Design – Example

$$G(s)=\frac{25\cdot 1152}{(s+9)(s^2+16s+128)}\cdot 10\left(\frac{s+9.48}{s+94.86}\right)^2\cdot\left(\frac{s+3.79}{s+0.09}\right)$$





Bode Diagram
Magnitude (dB)
50
0
-50
-100
-150
-200
Phase (deg)
0
-45
-90
-135
-180
-225
-270
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
10^{4}
Frequency (rad/sec)

Bode Design – Example (Matlab)

- s = tf('s');
- sys = 1152/((s+9)*(s^2 + 16*s + 128));
- bode(sys)
- grid on
- sys1 = 25*1152/((s+9)*(s^2 + 16*s + 128));
- hold on
- bode(sys1)
- sys2=sys1* 10*((s+9.48)/(s+94.86))^2
- hold on
- bode(sys2)
- sys3=sys2*((s+3.79)/(s+0.09))
- hold on
- bode(sys3)