Control - Bode Design Method
Lead / Lag

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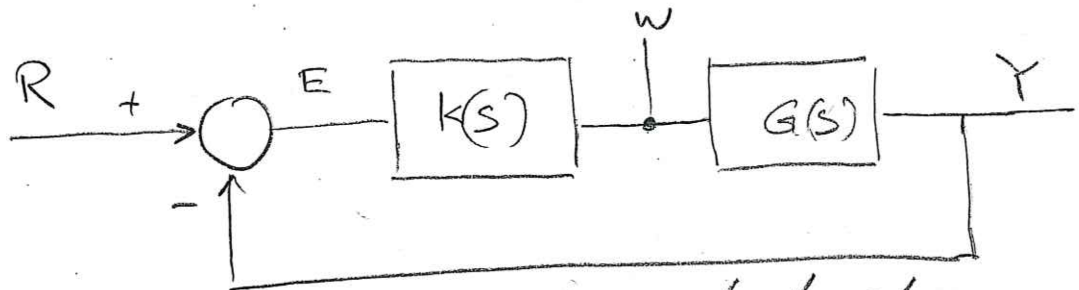
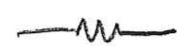
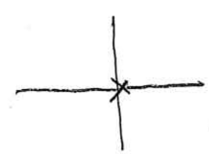
Design Philosophy

- **Problem:** By using a simple gain K we might not meet the design specifications
- **Potential Solution:** Add "zeros" and "Poles"
- **Problem with the Solution:** Were to put the "zeros" and "Poles" – Endless search
- **Class of Controllers**
  - PID
  - Lead / Leg
- **Challenge:**
  - Develop a class of controllers
  - Design Tools – Systematic approach

PID – Summary



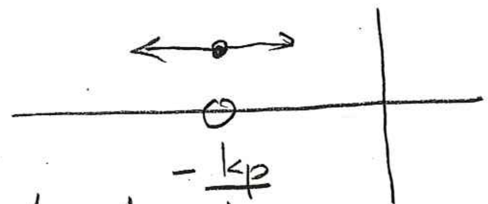
COMBINATIONS: P, PD, PI, PID, ~~D~~, ~~I~~, ~~ID~~

TYPE	FORM	S-PLANE		PROBLEM(S)
(P) PROPORTIONAL CONTROL	K_p			⊖ May not meet design requirment
(D) DERIVATIVE CONTROL	$K_d s$	 ZERO @ THE ORIGINE		⊖ Has no DC gain DC gain @ $s=0 \to 0$  ⊖ Derivative - Amplify noise
(I) INTEGRAL CONTROL	$\frac{K_I}{s}$			⊖ May make the system unstable (Root locus - RHS of the "S" Plane ⊕ Seady state error $\to 0$

Lead / Lag – PD Controller

- PD – Controller – Proportional + Derivative

$$K(s) = k_P + k_D s = k_D\left[s + \frac{k_P}{k_D}\right]$$



$-\frac{k_P}{k_D}$

- 2 DOF
  - Slide the zero Left/Right
  - Adjust the gain k_d

- Problem: Amplify high frequency noise

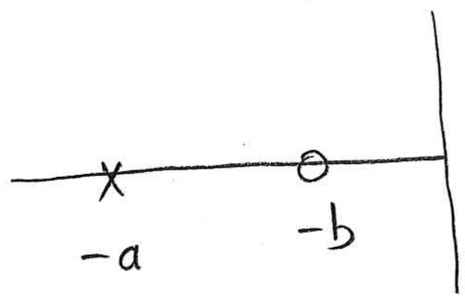
  input: $\cos\omega t$

  $\downarrow \frac{d}{dt}$

  output: $-\omega \sin \omega t$

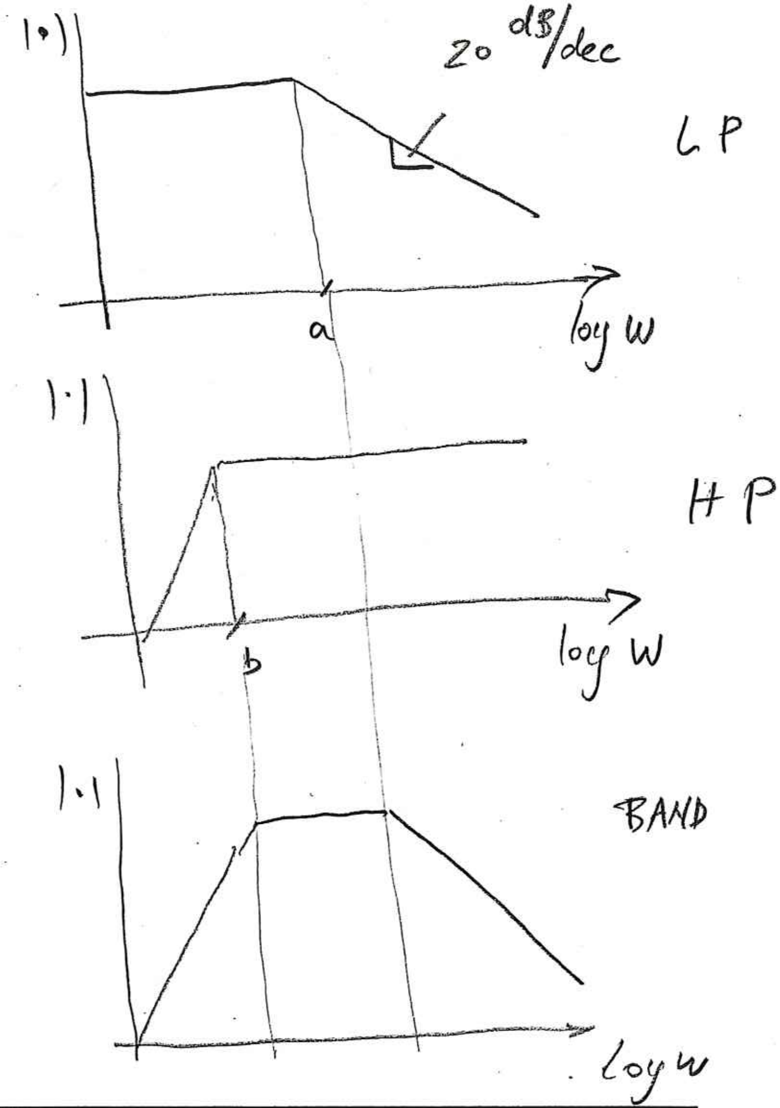
- High-pass filter with a cut-off at $\frac{k_p}{k_d}$

Lead / Lag – PD Controller



$$\frac{a}{s+a}$$

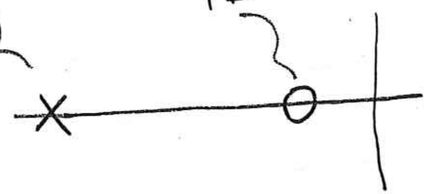
$$\frac{s+b}{b}$$



$$\boxed{\frac{a}{a+s}} - \boxed{\frac{s+b}{b}} \Rightarrow \boxed{k\,\frac{s+b}{s+a}}$$

$\boxed{\text{Lead}}$ / Lag – PD Controller

- LEAD CONTROLER → LOWPASS FILTER + PD CONTROLER



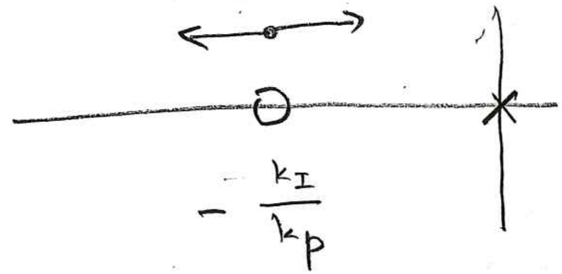
$$\underbrace{\left[k_p + k_d s\right]}_{PD}\left[\frac{a}{s+a}\right] = \frac{k_p a + k_d a s}{s+a} = \frac{\overset{k}{a k_d}\left[s + \overset{b}{\frac{k_p \cancel{a}}{k_d \cancel{a}}}\right]}{s+a}$$

$$\Rightarrow \; k\,\frac{s+b}{s+a} \; \Rightarrow \; \text{LEAD COMPENSATOR}$$

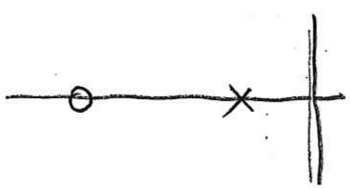
Lead / Lag – PI Controller

- PI – Controller – Proportional + Integrator

$$K(s) = k_P + \frac{k_I}{s} = \frac{k_P s + k_I}{s} = \frac{k_P\left[s + \frac{k_I}{k_P}\right]}{s}$$



- Advantage : Steady state $e_{ss} \to 0$
- Disadvantage : Distabilaizer
- Generalizing : Locate the pole (originally at the origine for the PI) at any selected point on the real axis



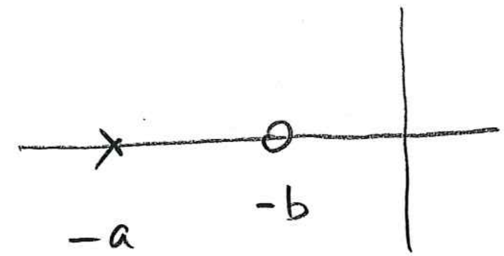
$$K \frac{s+b}{s+a} \Rightarrow \text{LAG COMPENSATOR}$$

Lead / Lag – Summary

- **Lead Compensator**
  - The Zero is closer to the origin than the Pole

$$K(s) = K\frac{s+b}{s+a}$$

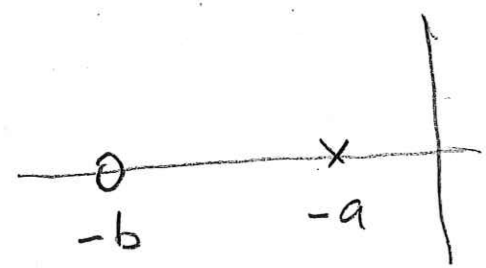
$$a > b$$



- **Lag Compensator**
  - The Pole is closer to the origin than the Zero

$$K(s) = K\frac{s+b}{s+a}$$

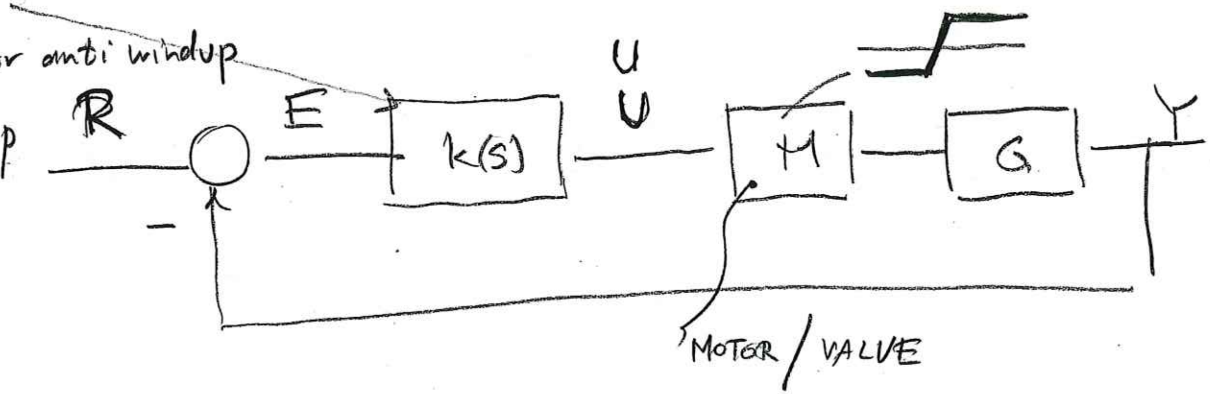
$$a < b$$

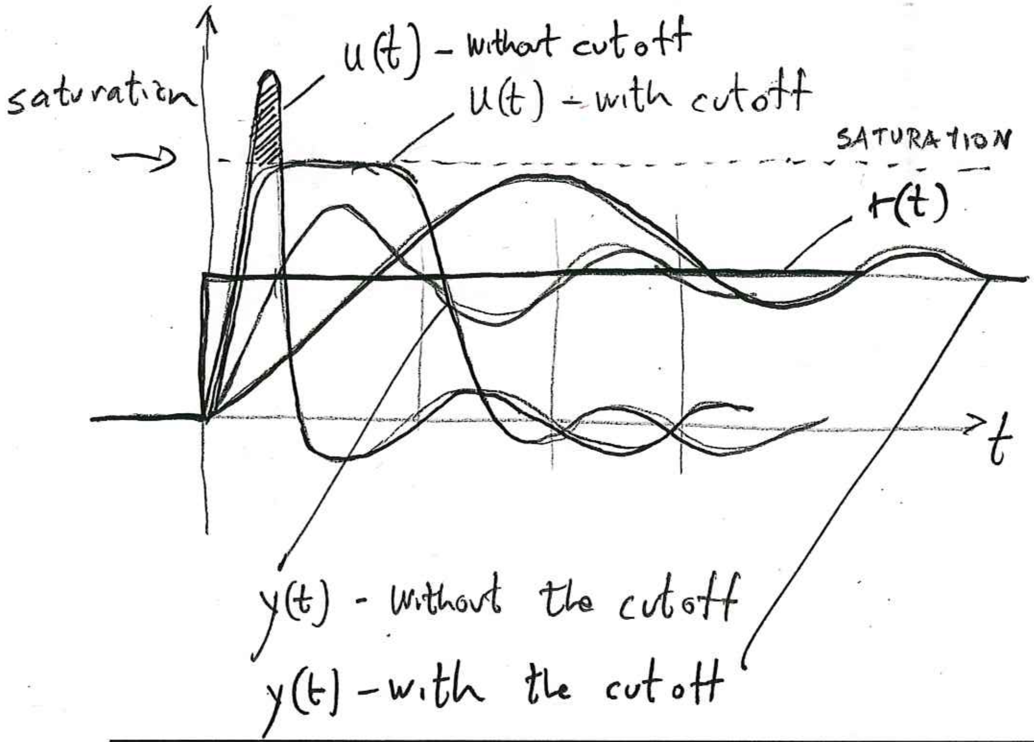


PI Controller – Integrator Problem

+ Solution

- Logic → Integrator anti windup
- If saturate stop integrate





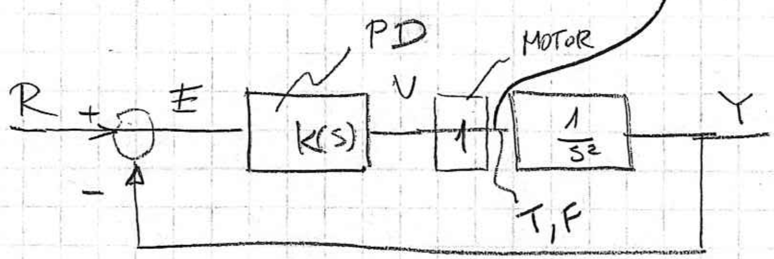
+ In Reality

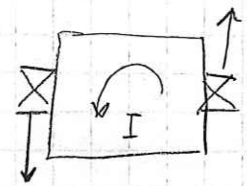
- Actuators saturate
- Value { Max open / Min close

+ Problem

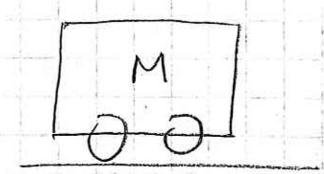
- The integrating controler will keep integrating up although the system can't produce more actuation
- Accumulating bagadge
- keep holding at saturation longer
- High overshoot → Less damping

WHY A DESIGN THAT PUT POLES FAR AWAY FROM THE ORIGIN REQUIRES LARGE MOTORS





$\tau = \ddot{\theta} I$

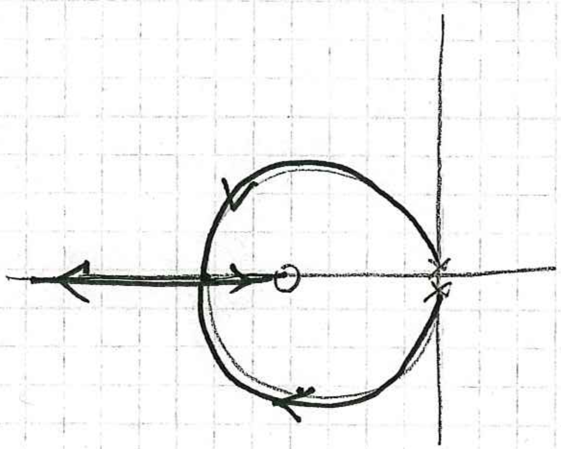


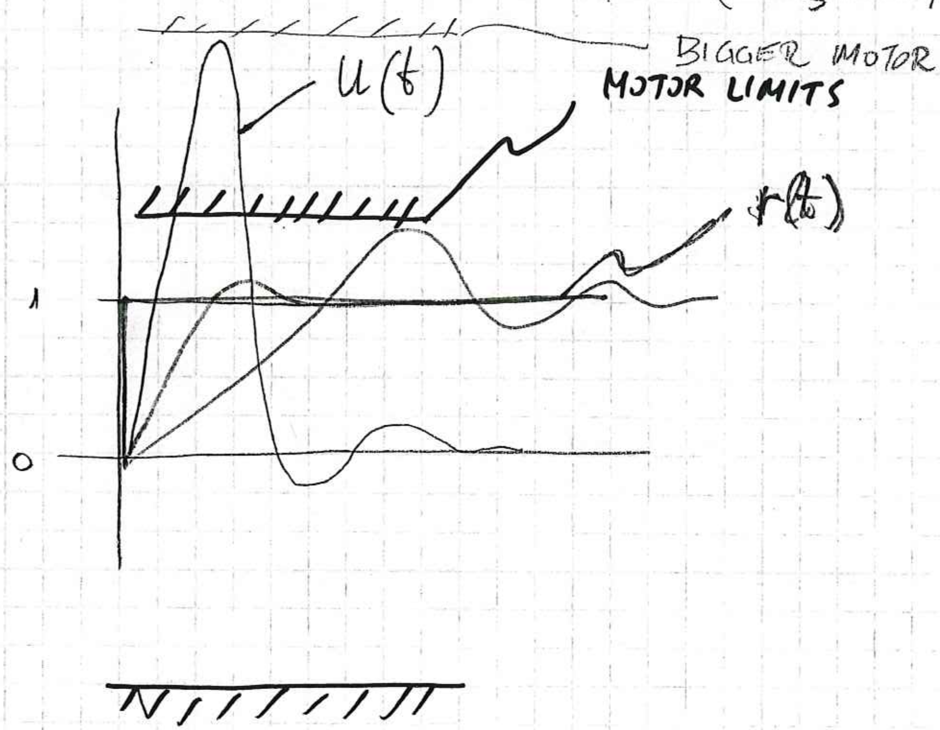
$F = \ddot{x} M$

$$1 + KG = 1 + [k_p + k_d s] \frac{1}{s^2} = 0$$

ROOT LOCUS

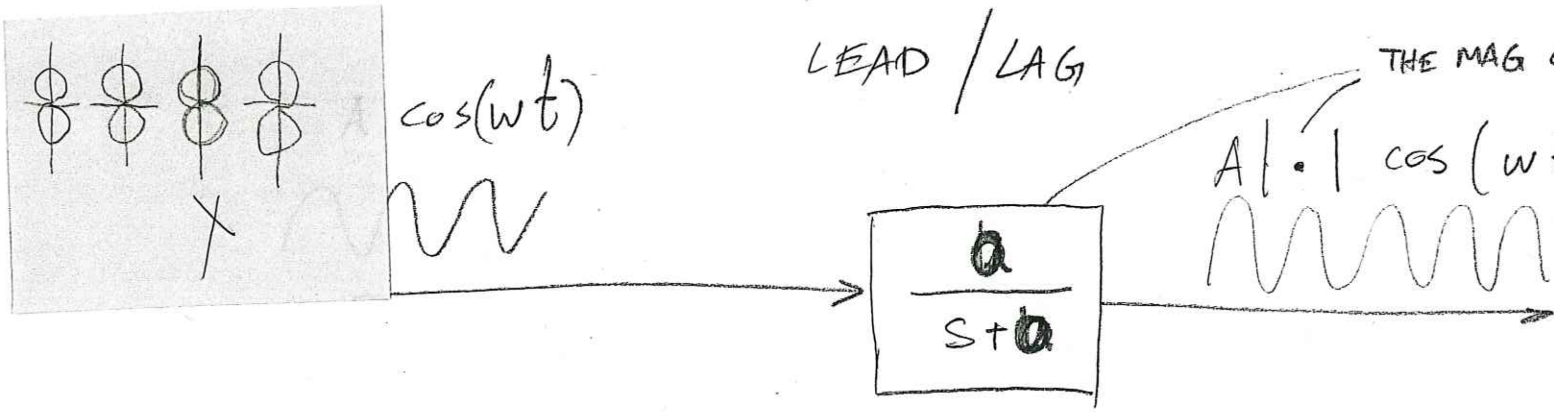
$$KG = \frac{k_p + k_d s}{s^2} = \frac{k_d \left(s + \frac{k_p}{k_d}\right)}{s^2} \rightarrow K_1 \left(\frac{s + k_2}{s^2}\right)$$





LEAD / LAG

THE MAG OF $\left|\frac{a}{s+a}\right|$ AT $s=jw$



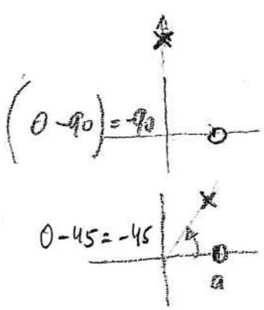
$A|\cdot|\cos(wt+\phi)$

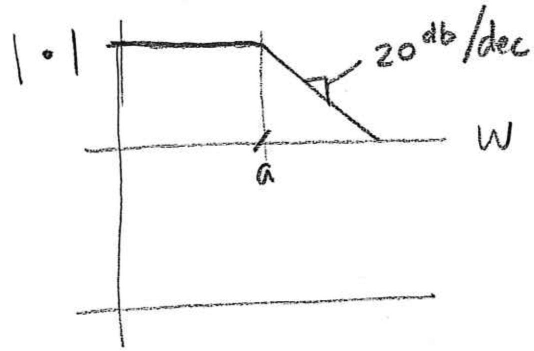
$\measuredangle\left(\frac{a}{s+a}\right)\Big|_{s=jw}$

$\text{DC GAIN} = \frac{a}{s+a} = 1$ [Filter with DC gain of 1]

$\frac{a}{jw+a} \longrightarrow$

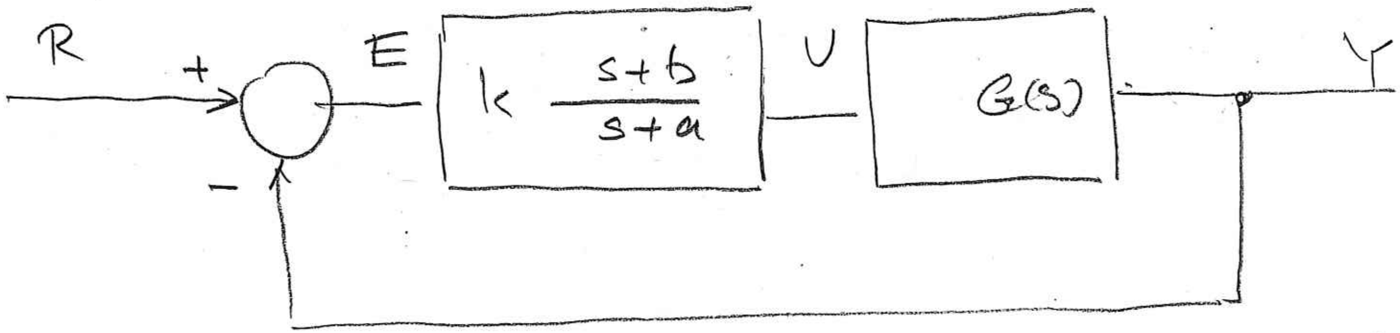
	GAIN	PHASE
@ $w=0$	1	0
@ $w\to\infty$	0	-90° $(0-90)=-90$
@ $w=a$	0.707	-45° $0-45=-45$





$\Rightarrow$ LOW PASS FILTER

Lag – Nature of the Compensator

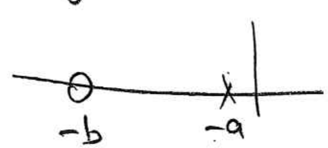


$$\frac{E}{R} = \frac{1}{1 + k\frac{s+b}{s+a}G(s)} = \frac{s+a}{(s+a) + k(s+b)G(s)} \xrightarrow[s \to 0]{\text{Steady state Error}} \frac{a}{a + k\,b\,G(s=0)}$$

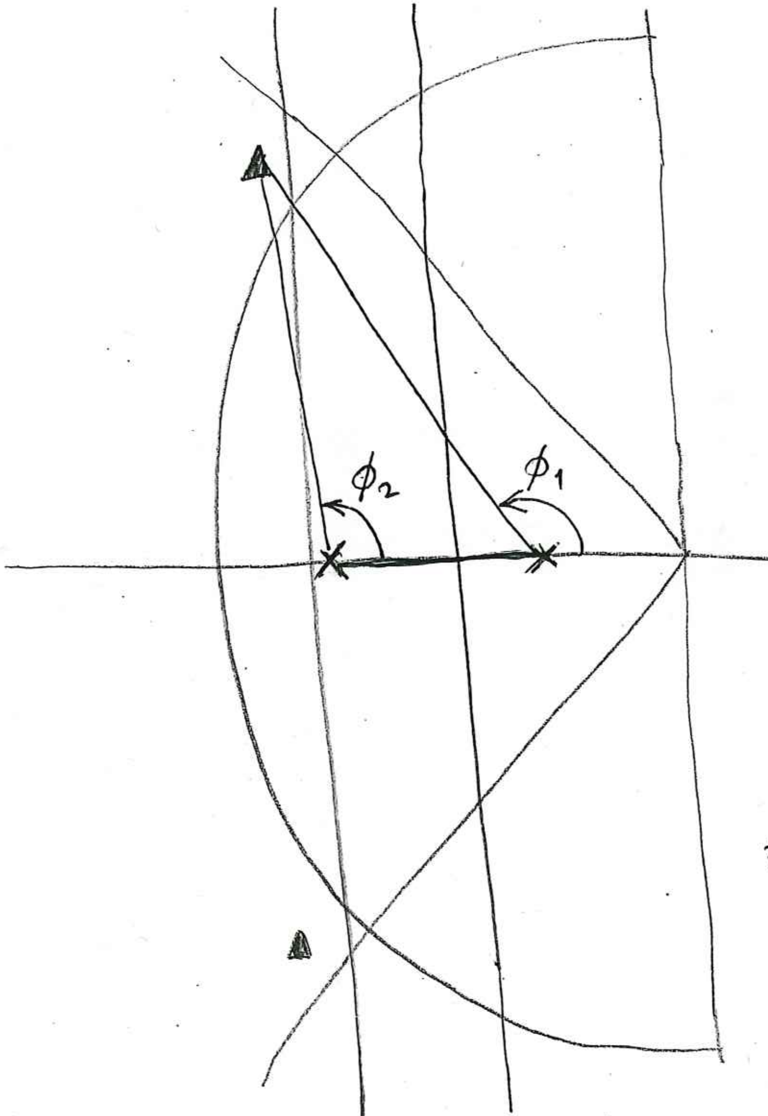
$$\uparrow$$

For $a \ll b$ (a is small relative to b)

close to an integrator → Decreasing the steady stat



Lead – Systematic Approach for Choosing Compensation



- Based on the req., choose a new locations for the poles
- Using only gain (k) it is impossible to meet the req.
- Measure the phase at the req. point ▲

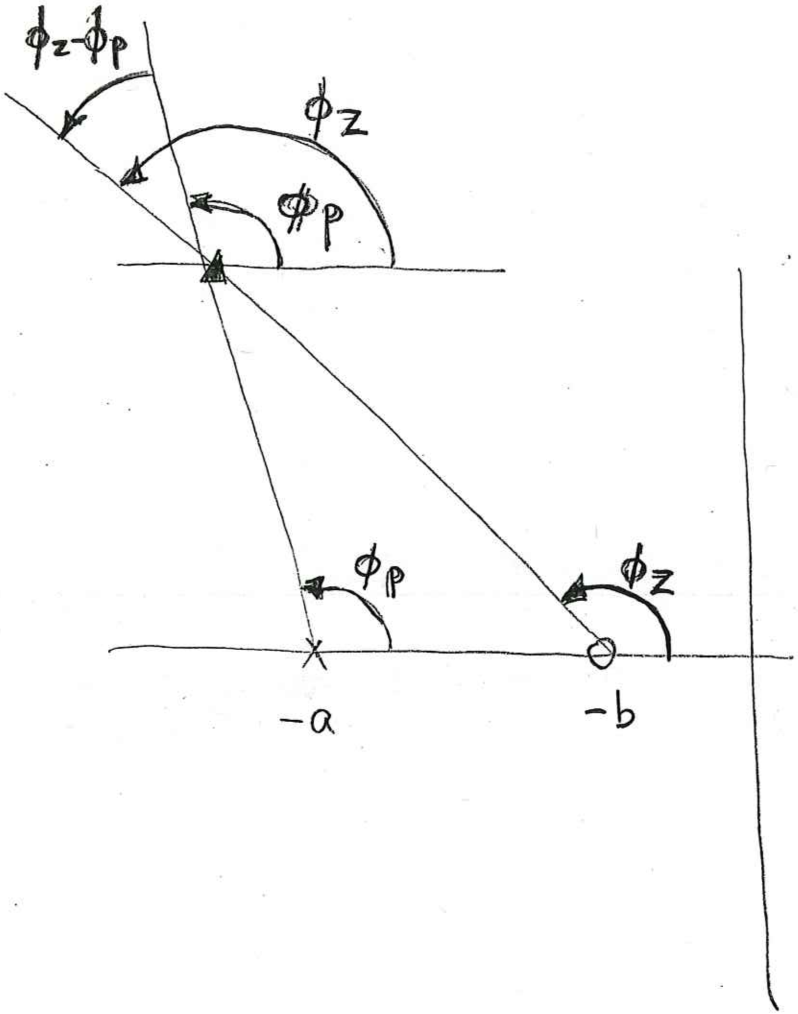
$$-\phi_1 - \phi_2 + \overbrace{\bar{\phi}}^{\text{UNKNOWN}} = -180$$

For example $\quad -135 - 120 + \bar{\phi} = -180$

$$\bar{\phi} = +75$$

- Add $\bar{\phi} = 75°$ to the system to get the RL to pass through ▲

Lead / Lag



- Lead Controler

$$K(s) = K\,\frac{s+b}{s+a}$$

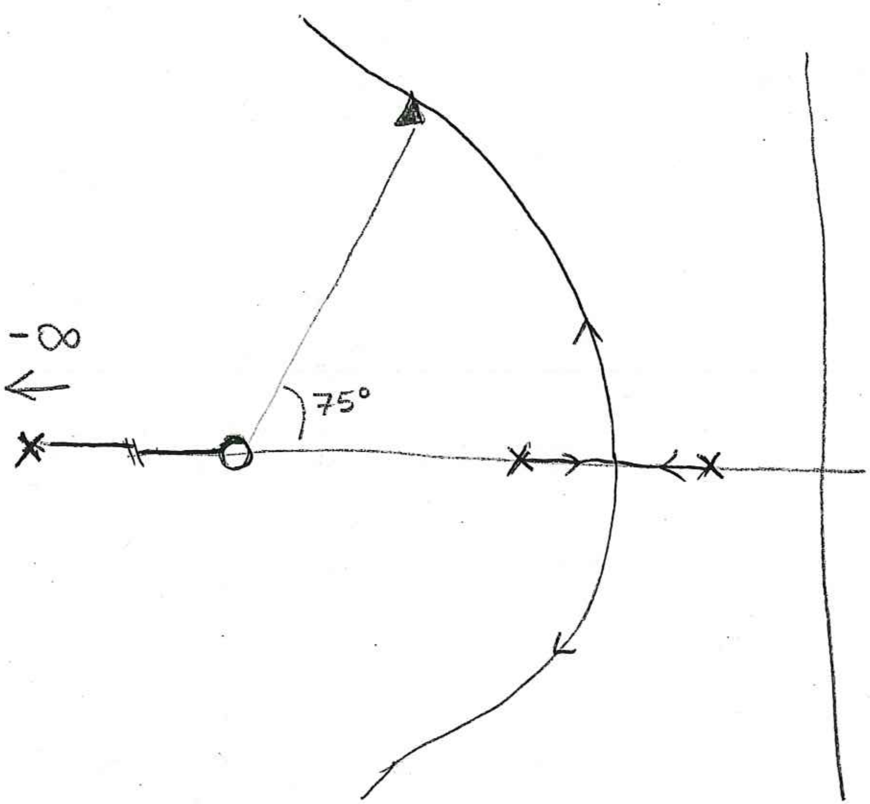
- Add a lead controler with zero (o) and pole (x) such that

$$\phi_z - \phi_p = 75^\circ$$

- The RL will pass through the point ▲ and meet the requirments

→ ⓘ DOF : The differance $\phi_z - \phi_p = 75^\circ$ may be generated by ∞ combinations of poles (x) and zero (o) located along the real axis

Lead / Lag

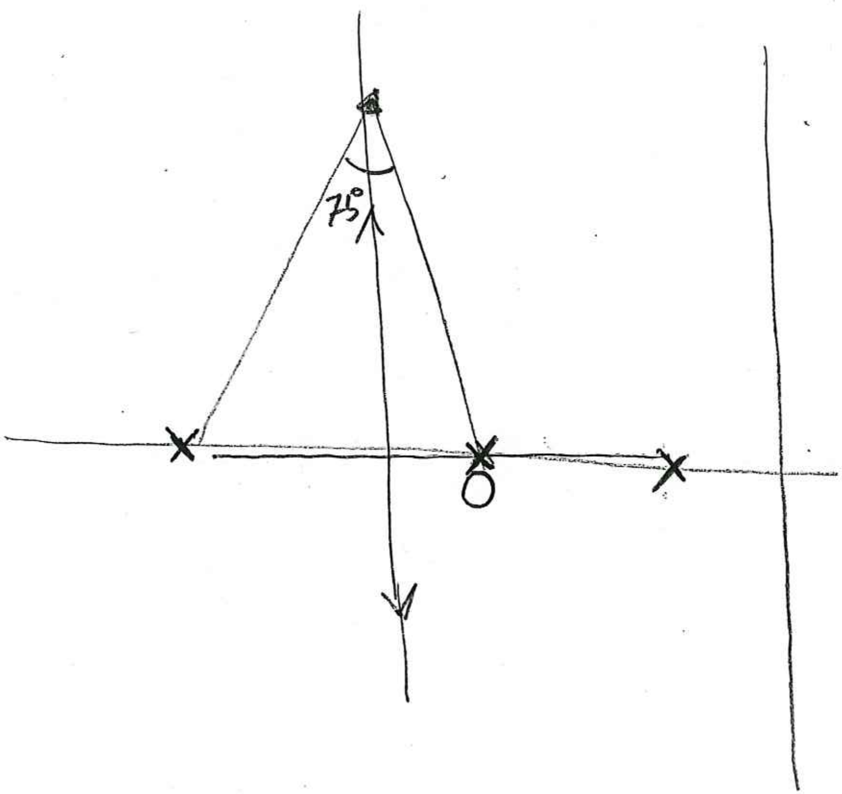


CASE 1 :

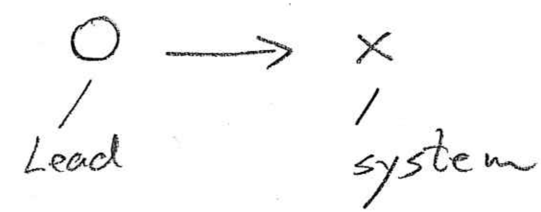
× ⟶ -∞

O ⟶ on the real axis at 75°

Lead / Lag

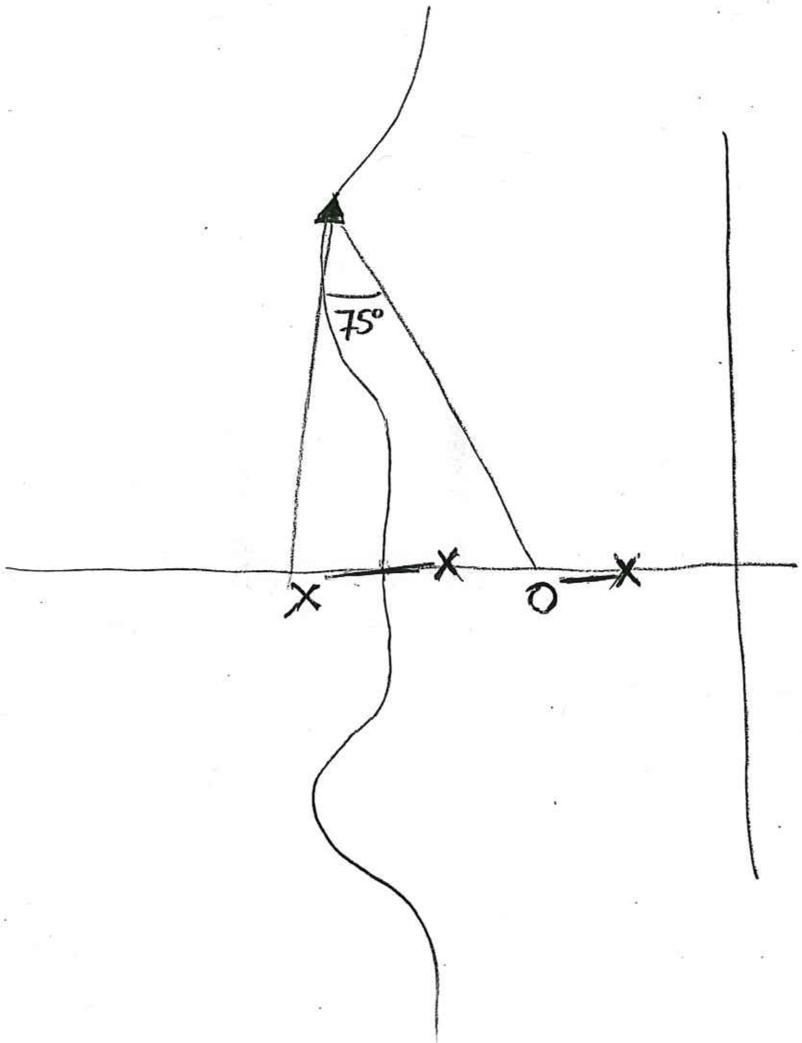


CASE 2



- Put the zero of the Lead ontop of the system pole

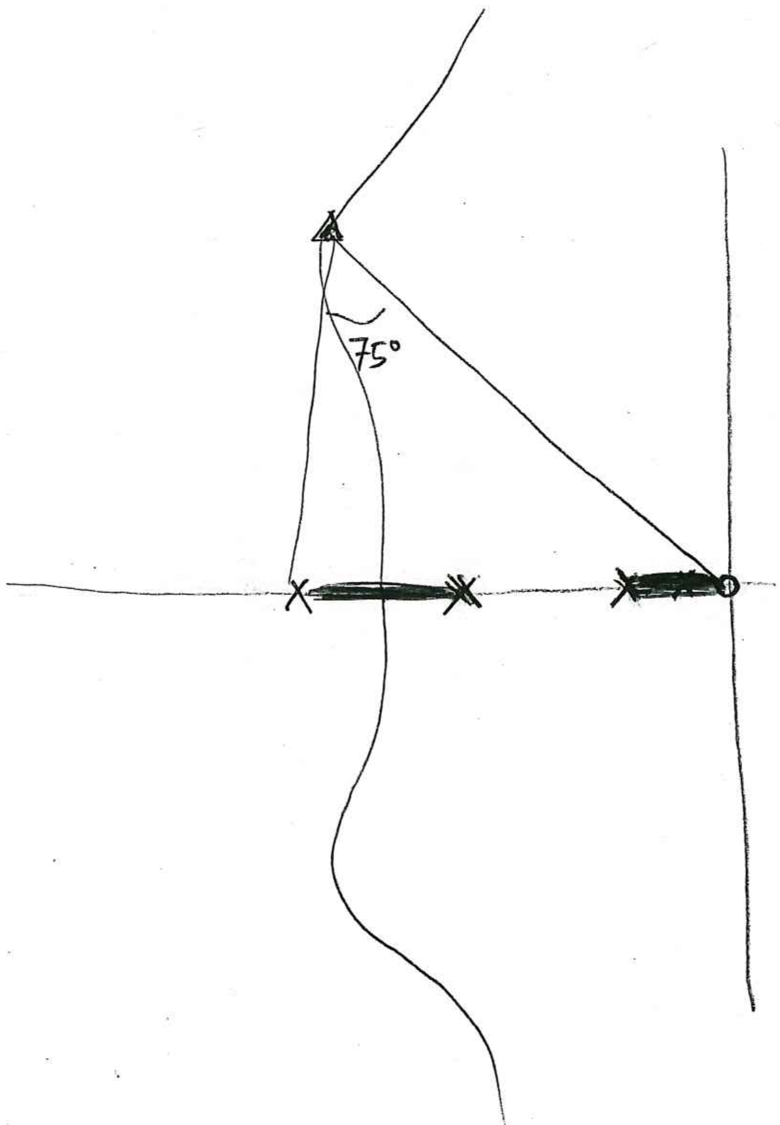
Lead / Lag



CASE 3

- Put the zero of the lead between the two poles of the system

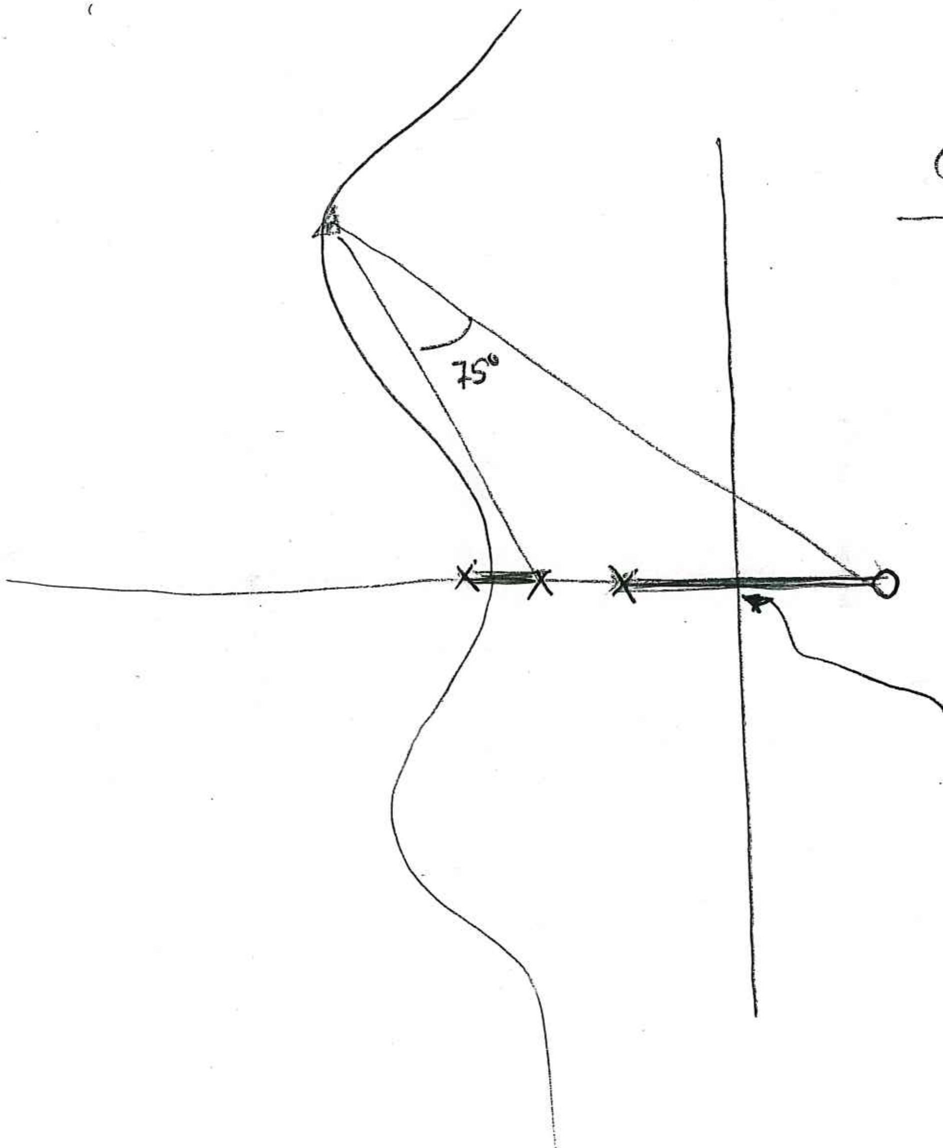
Lead / Lag



CASE 4

- Put the zero of the lead at the origine

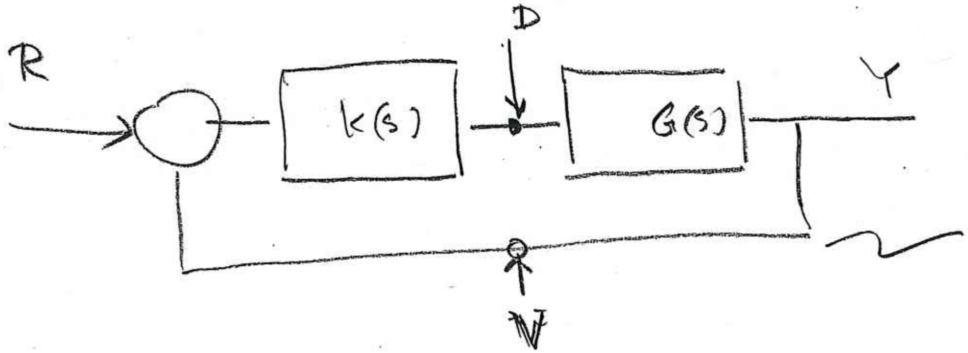
Lead / Lag



CASE 5

- Put the zero of the lead on the right hand side of the RT
- Problem: Do we hit the point ▲ before the system get unstable

Lead / Lag



Perfect sensor with noise

Requirment for KG

small kG

$$\frac{Y}{V} = \frac{kG}{1+kG} \; ; \quad \frac{Y}{V} \simeq 0 \; ; \quad \frac{kG}{1+kG} \to 0 \; ; \quad kG \to 0$$

- small kG
- Noise of the sensor does not affect output

Large kG

$$\frac{Y}{D} = \frac{G}{1+kG} \qquad \frac{Y}{D} \simeq 0 \qquad \frac{G}{1+kG} \to 0 \; ; \quad kG \to \infty$$

- Large KG
- Distarbance noise does not affect the output

$$\frac{Y}{R} = \frac{kG}{1+kG} \; ; \quad \frac{Y}{R} \to 1 \; ; \quad \frac{kG}{1+kG} \to 1 \; ; \quad kG \to \infty$$

- Large kG
- Good tracking respose

LEAD / LAG

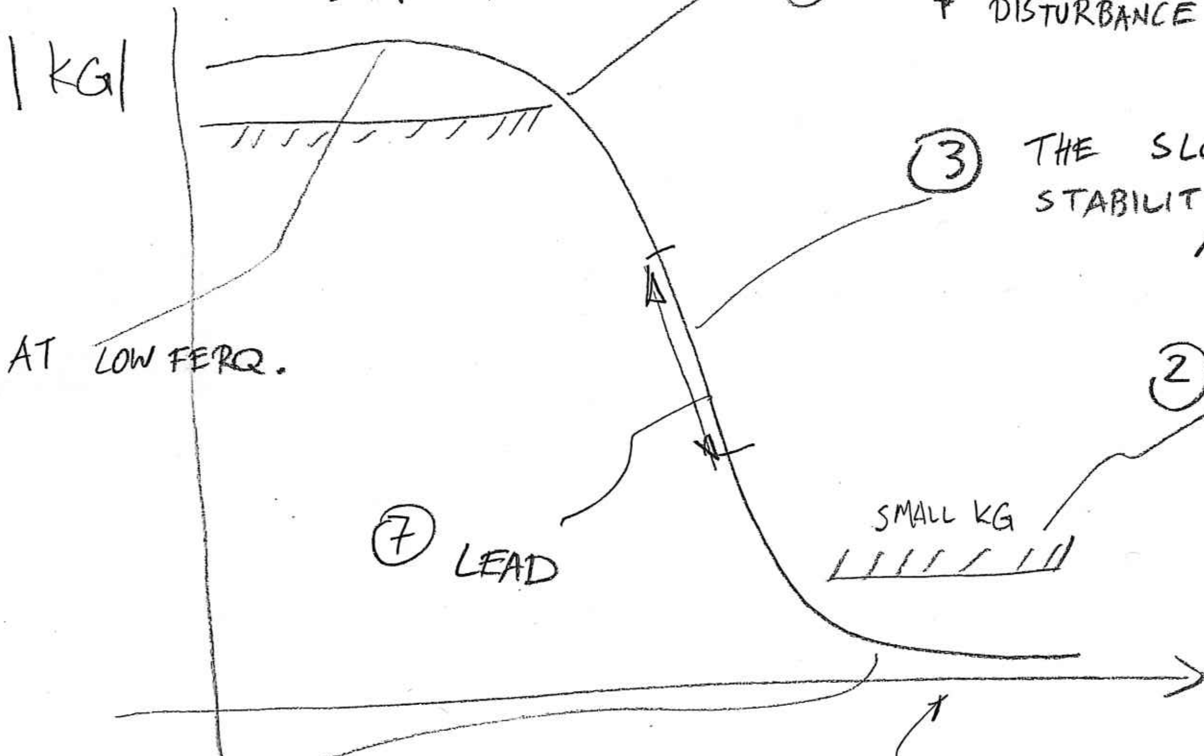
① LOW FREQ.

* TRACKING RESPONSE $\frac{Y}{R} = \frac{KG}{1+KG} \simeq 1$
* DISTURBANCE REJECTION $\frac{Y}{D} = \frac{G}{1+KG} \approx 0$

④ HIGH GAIN AL LOW FREQ

BIG |KG|

③ THE SLOPE DEFINES THE STABILITY – SEE BODE



⑥ LAG AT LOW FERQ.

② HIGH FREQ.

* NOISE REJECTION $\frac{Y}{V} = \frac{KG}{1+KG} \approx 0$
* PLANT UNCERTENTY

⑦ LEAD

⑧ LOW PASS FILTERS

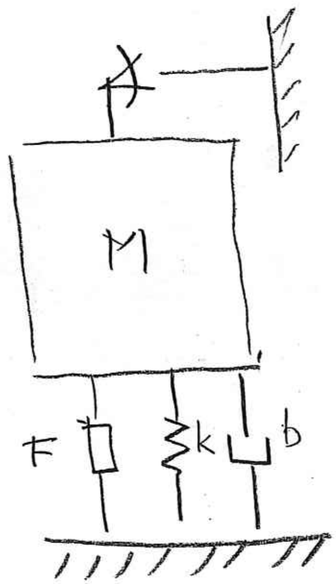
⑤ LOW GAIN AT HIGH FREQ.

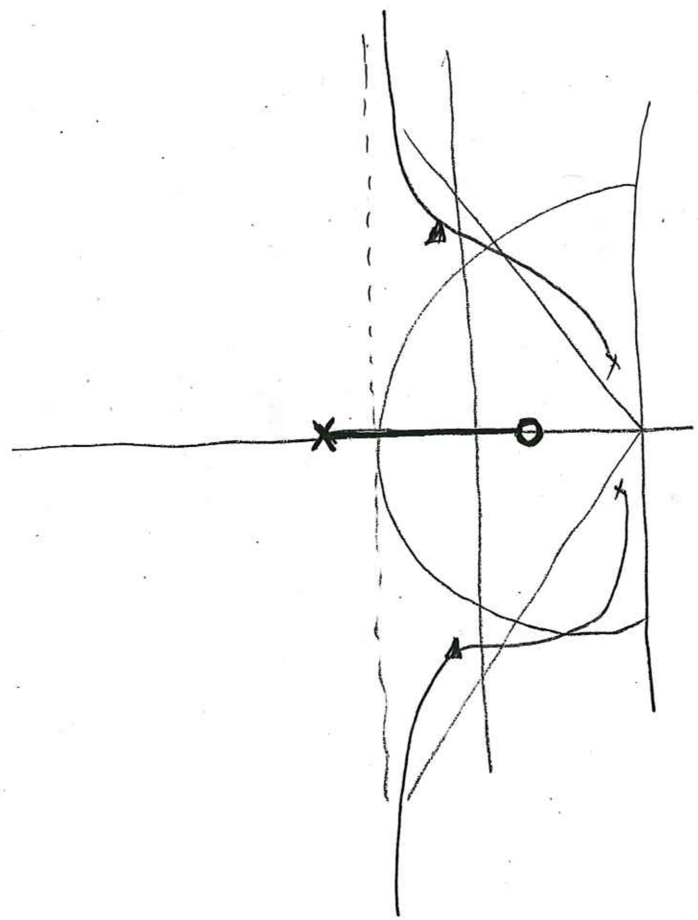
ATTENUATION

SOME TIMES I DO NOT KNOW HOW TO MODEL ALL THE MODES OF THE SYSTEM (ESPECIALY AT HIGH FREQ

Lead / Lag

OPTICAL TABLE EXAMPLE

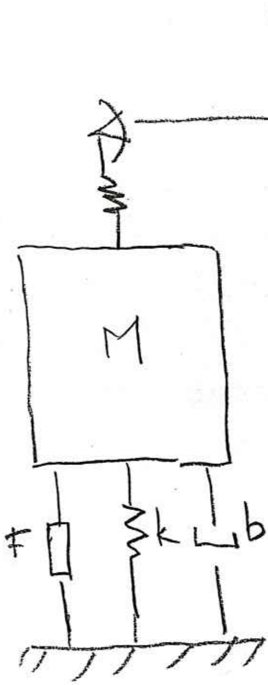


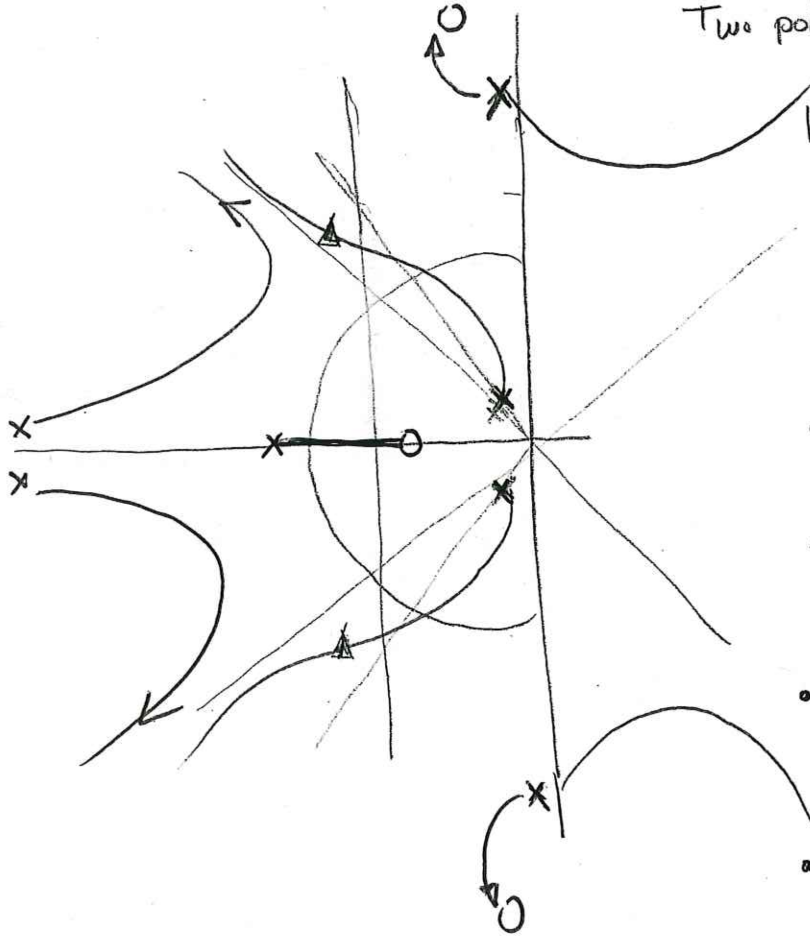


- Add a lead controler

Lead / Lag

OPTICAL TABLE EXAMPLE



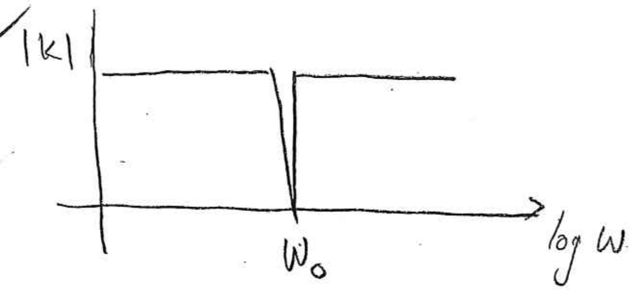


solution – Notch Filter

Two Zero

$$K(s) = \frac{s^2 + 2\zeta\omega_0 s + \omega_0^2}{s^2 + 2\omega_0 s + \omega_0^2}$$

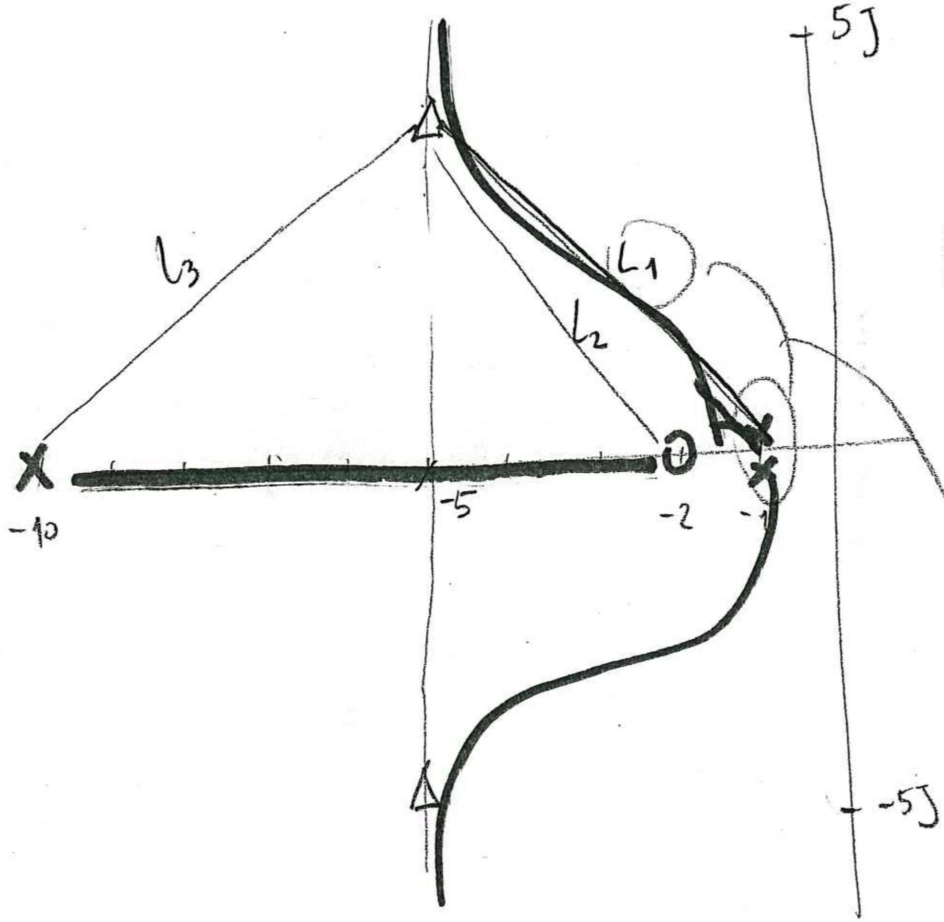
Two poles



- Notch Filter – Band stop
- Alter the departure of the remote poles
- The no. of asymptot will still be the same
- For small k wee still meet the spec. BUT also avoide instability

LEAD/LAG EXAMPLE

$$G(s) = \frac{5(s+2)}{(s+1)^2(s+10)}$$



CENTER OF ASY.

$$\alpha = \frac{-10-2+2}{2} = -5$$

FIND K FOR WHICH $|KG| \to -5 \pm 5J$

$$K \cdot 5 \frac{l_2}{L_1^2 l_3} = 1$$

$\Rightarrow K \simeq 10$

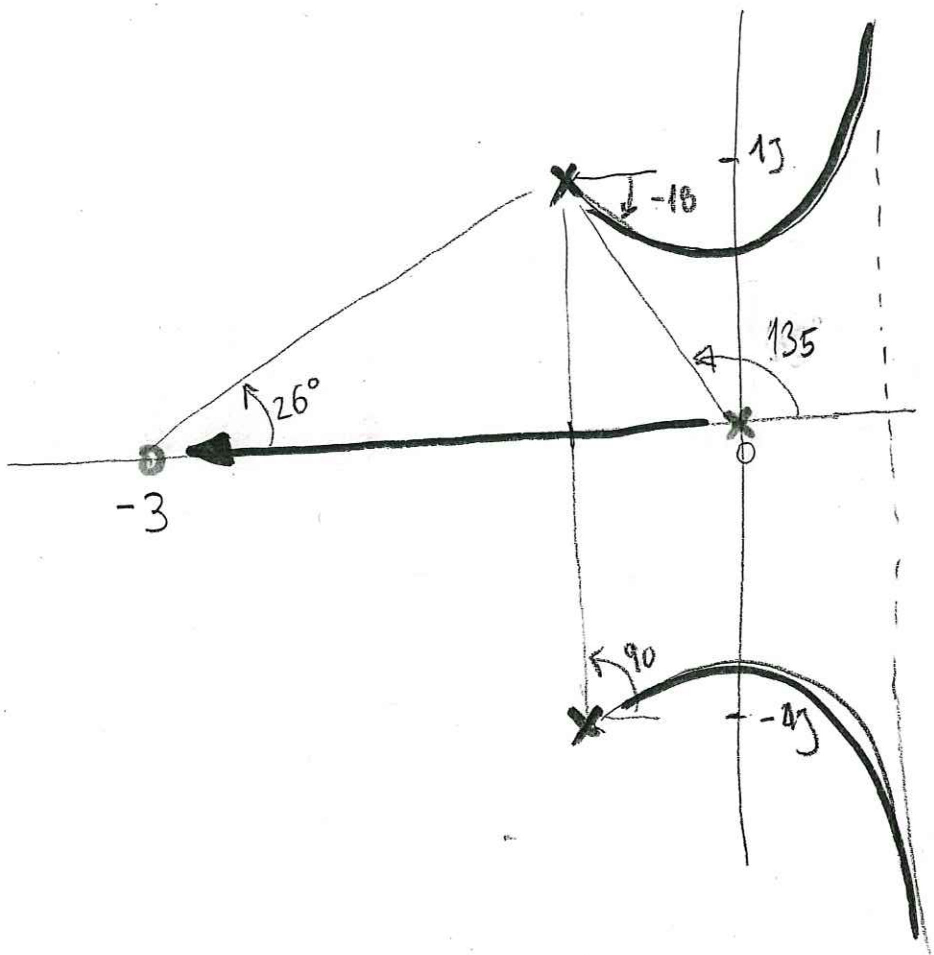
$$L_1 = \sqrt{5^2+4^2} = \sqrt{25+16} = 6.4$$

$$L_2 = \sqrt{5^2+3^2} = \sqrt{25+9} = 5.8$$

$$L_3 = \sqrt{5^2+5^2} = \sqrt{50} = 7.1$$

$$K = \frac{L_1^2 L_3}{5 L_2} = \frac{41 \times 7.1}{5 \times 5.8} \simeq 10$$

LEAD/LAG EXAMPLE



$$G = \frac{1}{s^2 + 2s + 2}$$

$$s_1, s_2 = \left\{ \frac{-2 \pm \sqrt{4-8}}{2} = -1 \pm j \right.$$

$$\left\{ K = \frac{s+3}{s} \right.$$

PI CONTROL (LAG)

$$-\phi_d - 135° - 90 + 26 = -180$$

$$\phi_d = -18°$$

WHAT IS THE VALUE OF K IN WHICH THE ROOT LOCUS CROSSES THE Jω AXIS & BECOME UNSTABLE

LEAD / LAG EXAMPLE

$$D(s) = 1 + kG = 1 + \frac{k(s+3)}{s(s^2+2s+2)} = 0$$

$$s^3 + 2s^2 + s(k+2) + 3k = 0$$

s^3	1	$k+2$	0
s^2	2	$3k$	
s^1	$-\dfrac{\begin{vmatrix}1 & k+2\\ 2 & 3k\end{vmatrix}}{2} = \dfrac{4-k}{2}$	0	
s^0	$-\dfrac{\begin{vmatrix}2 & 3k\\ \frac{4-k}{2} & 0\end{vmatrix}}{\frac{4-k}{2}} = 3k$		

$$-\left(\frac{3k - 2(k+2)}{2}\right) = -\frac{3k-2k-4}{2} =$$

$$= -\frac{k-4}{2} = \frac{4-k}{2}$$

$$-\frac{-3k\left(\frac{\cancel{4-k}}{\cancel{2}}\right)}{\frac{\cancel{4-k}}{\cancel{2}}} = +3k$$

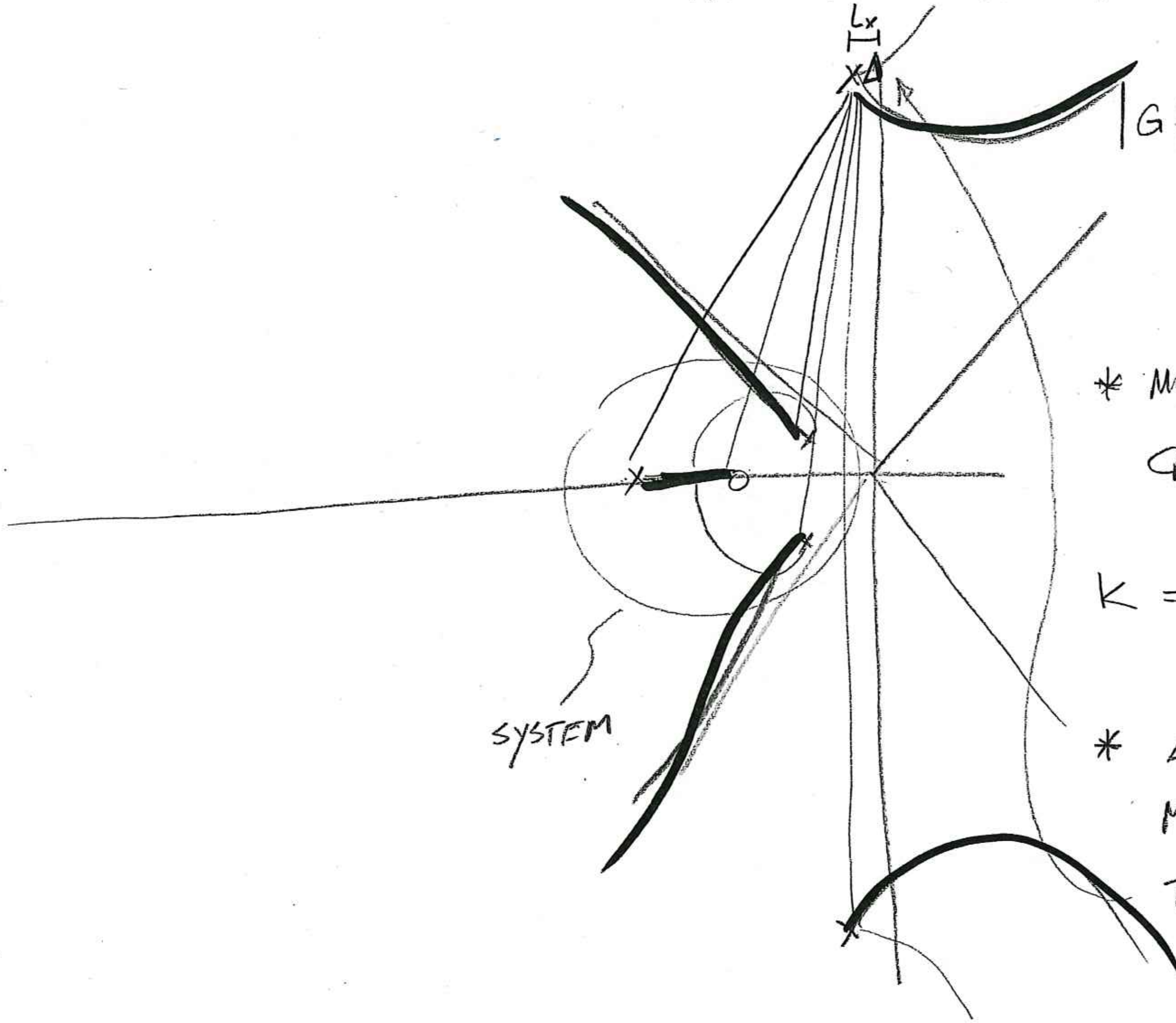
$$\cap \left\{ \begin{array}{lll} 3k > 0 & \Rightarrow & k > 0 \\ \frac{4-k}{2} > 0 & \Rightarrow & k < 4 \end{array} \right\} \cap$$

$$0 < k < 4$$

$k = 4 \Rightarrow$ [illegible]

HOW FAST THE POLE ARE MOVING

MISSING INFO FROM THE ROOT LOCUS



$$|GK| = \underbrace{\frac{\Pi\, l_z}{l_x\, \Pi\, l_p}}_{G} \approx \frac{C}{l_x}$$

* MOVING Δ ALONG THE RL
  G WILL REMAIN THE SAME

$$K = \frac{1}{|G|} \approx l_x$$

* ALL THE OTHER POLES MOVE QUICK COMPARE TO THE POLE