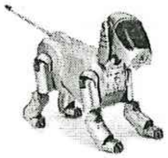
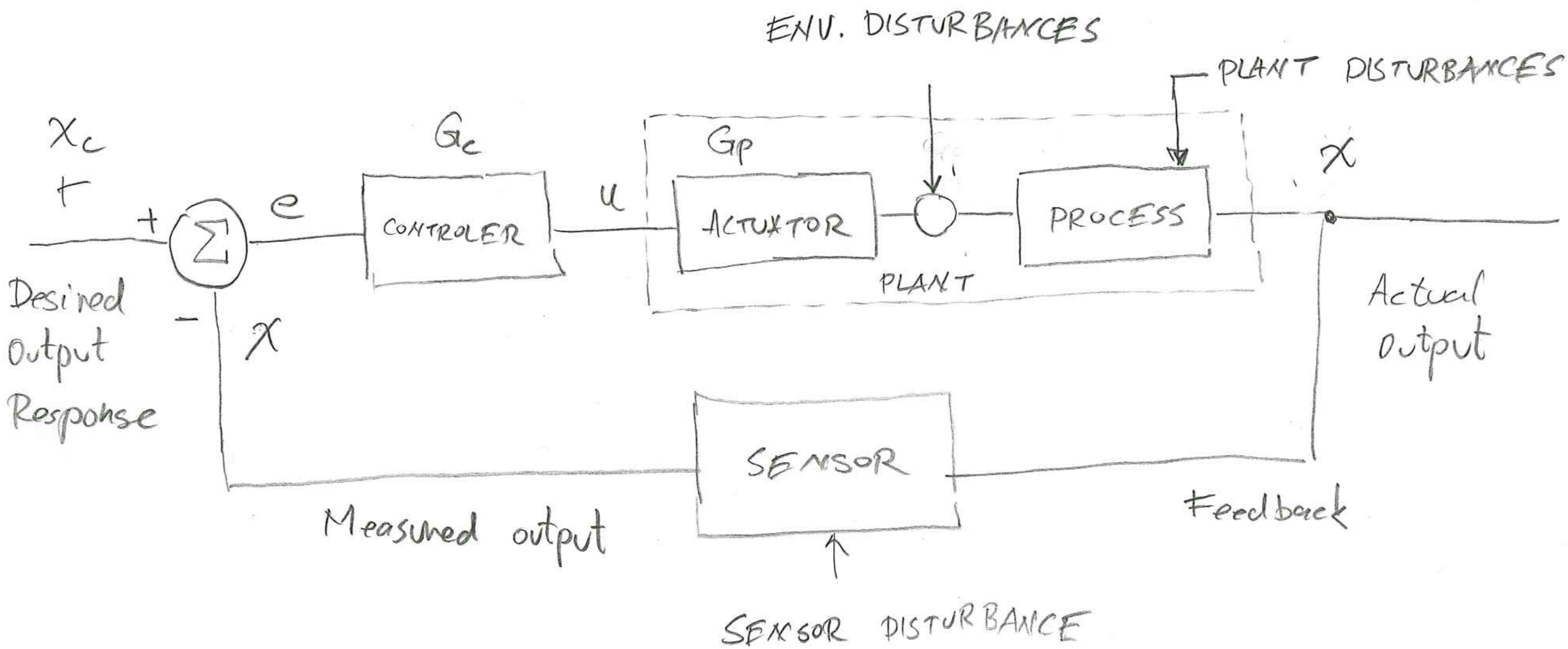
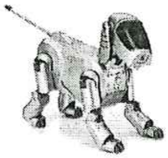
Control - PID



CONTROL - PID

CLOSE LOOP FEEDBACK CONTROL





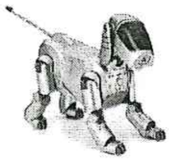
PID · CONTROLER - DEFENITION

PID $\longrightarrow$

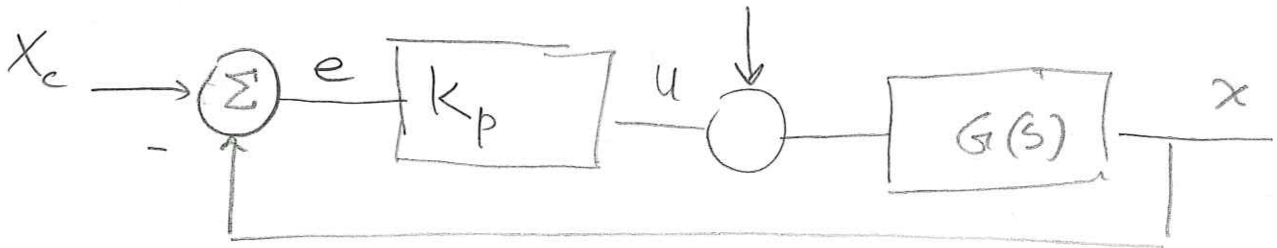
- P - Proportional $\longrightarrow k_p$
- I - Integral $\longrightarrow k_I$
- D - Derivative $\longrightarrow k_d$

$$u(t) = k_p e(t) + k_I \int e(\tau)\, d\tau + k_d \frac{d}{dt} e(t)$$

$$U(s) = E(s)\left[k_p + k_I \frac{1}{s} + k_d s \right] = E(s) = \frac{k_p s + k_I + k_d s^2}{s}$$

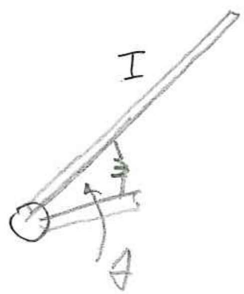


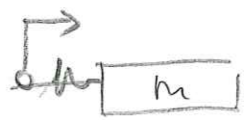
PID - CONTROLER - (P) - PROPORTIONAL CONTROL



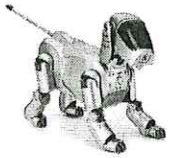
$$u = -k_p(x - x_c) = -k_p e$$

SPRING





$$F = -k\Delta x$$

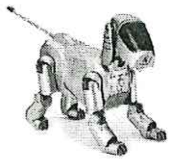


(P) - Problems

1. ENGERGY - k_p (spring) Does not remove energy out of the system

2. RESONANCE - If the input x_c will match the natural frequnancy of the system $\rightarrow$ Resonance

3. LARGE ACTUATORS - To have a very small error we need Larage springs i.e. Large actuators



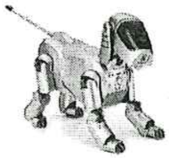
(4) FORCE/ERROR – Force is generated only if a certain amount of error is generated

When error $\rightarrow 0 \Rightarrow F \rightarrow 0$

What if the error $e \rightarrow 0$ (No force)

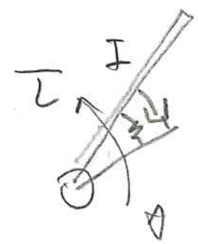
But we can measure velocity

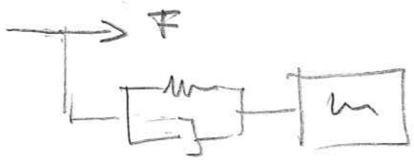
(!) $\Rightarrow$ we can predict the fact that we are about to build up an error



PID - CONTROLER - D - DERIVATIVE CONTROLER

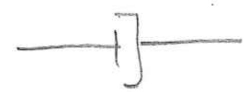
$$u = -k_d(\dot{x} - x_c) = -k_d\,\dot{e}$$





Damper

$$F = -bv = -b\dot{x}$$

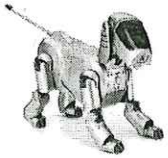


Advantage - Absorb energy → Remove energy out of the system

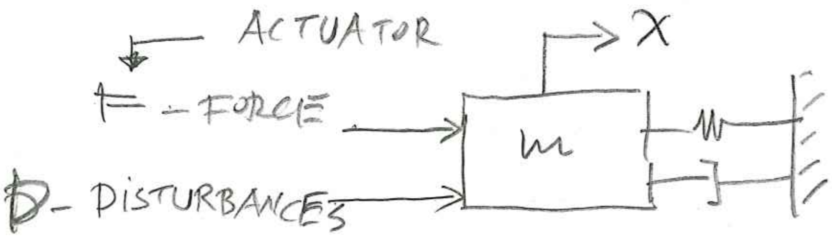
Summary

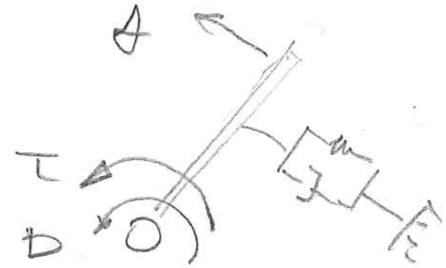
$K_p \rightarrow$ spring $(F = kx)$ - No force is applied until the system moves

$K_d \rightarrow$ Damper $(F = b\dot{x})$ - Anticipate motion



PD CONTROL - EXAMPLE



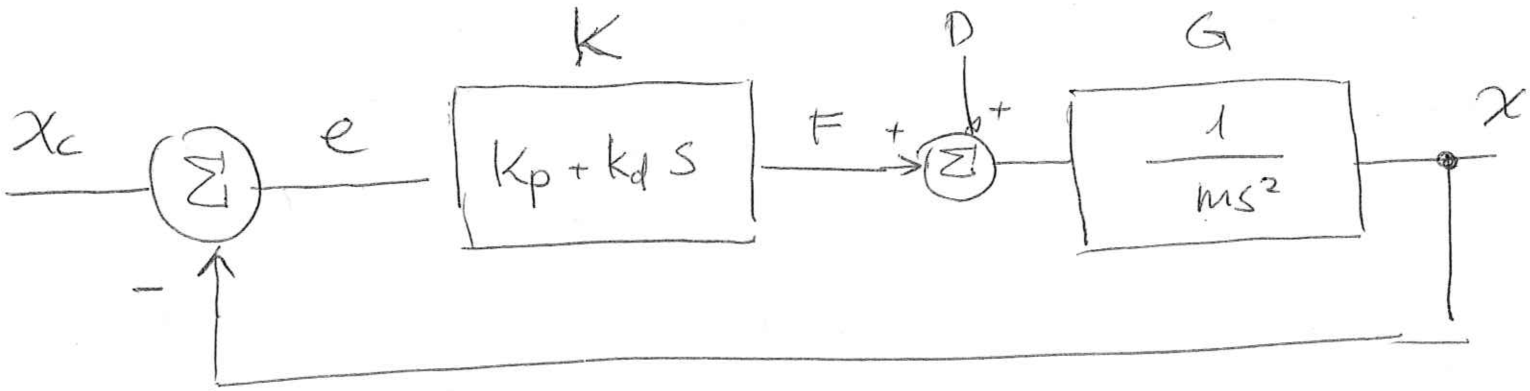


$$+ \rightarrow \quad \sum F = m\dot{x}$$

$$F + D = m\dot{x}$$

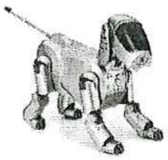
$$F(s) + D(s) = m X s^2$$

$$\frac{X}{F(s) + D(s)} = \frac{1}{m s^2}$$



$$\frac{\text{Output}}{\text{Input}} = \frac{X}{X_c} = \frac{KG}{1+KG} = \frac{(k_p + k_d s)\frac{1}{ms^2}}{1 + (k_p + k_d s)\frac{1}{ms^2}} =$$

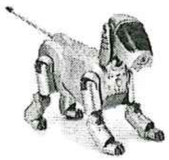
$$= \frac{k_p + k_d s}{ms^2 + k_d s + k_p} = \frac{\frac{k_d}{m} s + \frac{k_p}{m}}{s^2 + \frac{k_d}{m} s + \frac{k_p}{m}}$$



$$\omega_n = \frac{k_p}{m}$$

$$\zeta = \frac{k_d}{2 m \omega_n} = \frac{k_d}{2}\sqrt{\frac{1}{m k_p}}$$

$$\frac{X}{X_c} = \frac{\frac{k_d}{m} s + \frac{k_p}{m}}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$



STEP 1 – { Set $k_d = 0$

→ [Increasing k_p

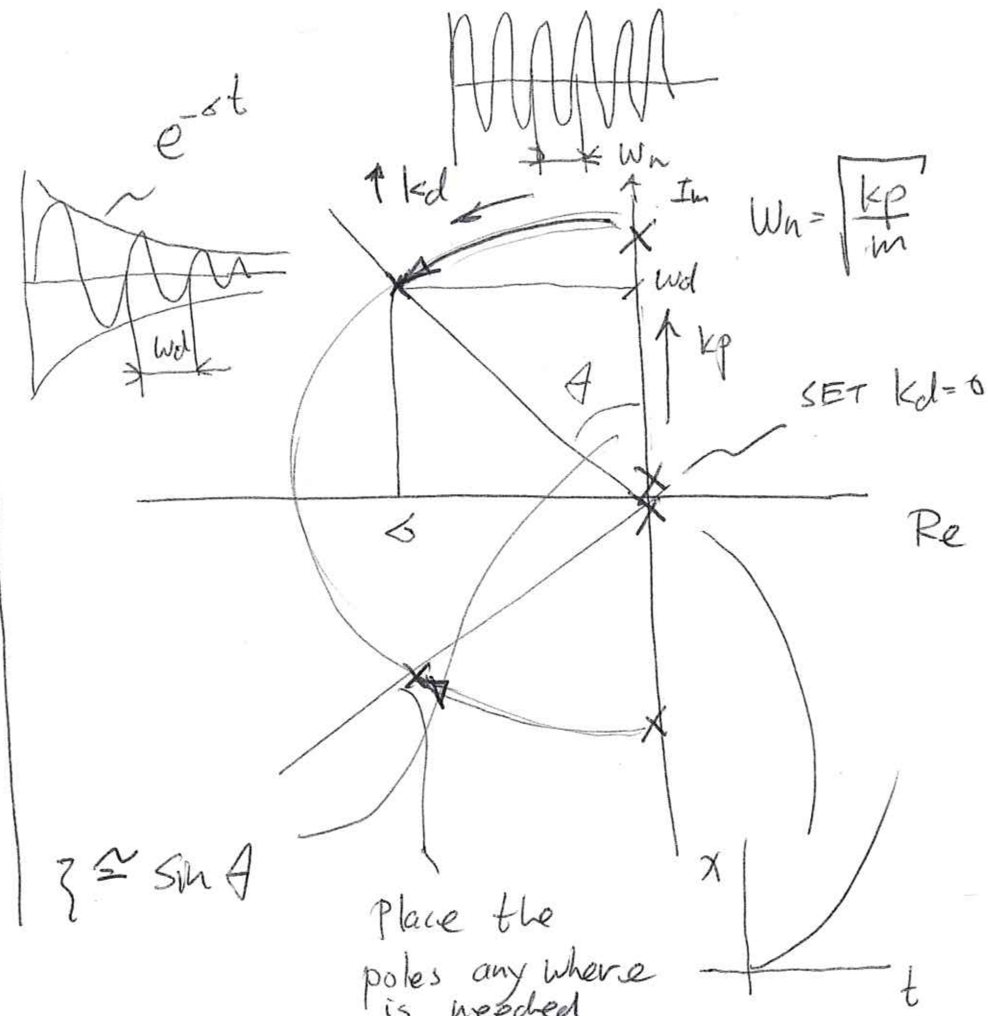
→ [Move the poles along the Imaginary axis to the value of $\omega_n = \sqrt{\frac{k_p}{m}}$

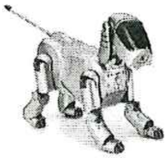
STEP 2 – { set k_p - const

~~Increa~~

[Increase k_d

[The natural Frequency stays the same but the damping increases





PD - ROOT LOCUS

$$KGH = \frac{k_d s + k_p}{m s^2} = k_d \frac{\left(s + \frac{k_p}{k_d}\right)}{m s^2}$$

POLES: $m s^2 = 0$

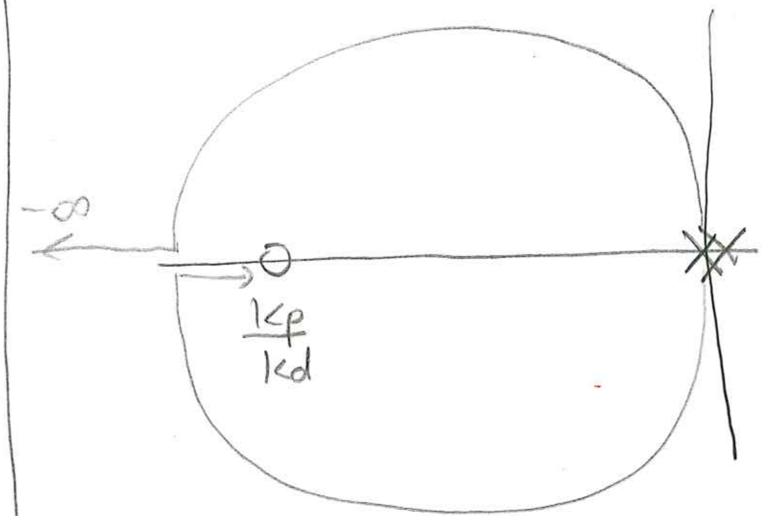
$$\Rightarrow p_1, p_2 = 0$$

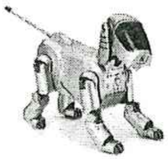
ZEROS $s + \frac{k_p}{k_d} = 0$

$$z_1 = -\frac{k_p}{k_d}$$

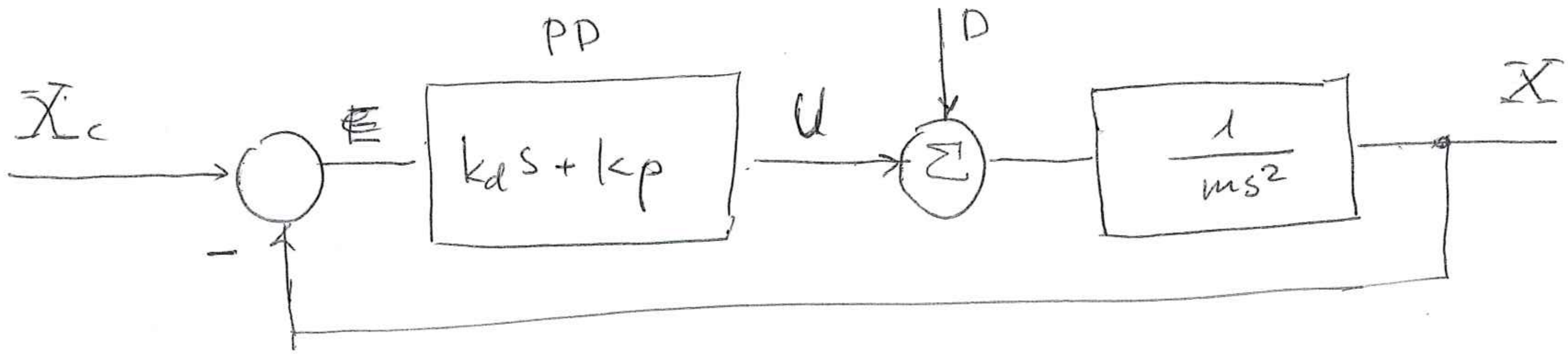
$$k_d \left[\frac{s + \frac{k_p}{k_d}}{m s^2} \right]$$

ROOT LOCUS





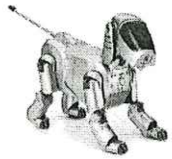
PID CONTROLER - I - Integral



SUPER POSITION

output $\rightsquigarrow$ $$X = \left(\frac{X}{X_c}\right)\Bigg|_{D=0} X_c + \left(\frac{X}{D}\right)_{X_c=0} D$$

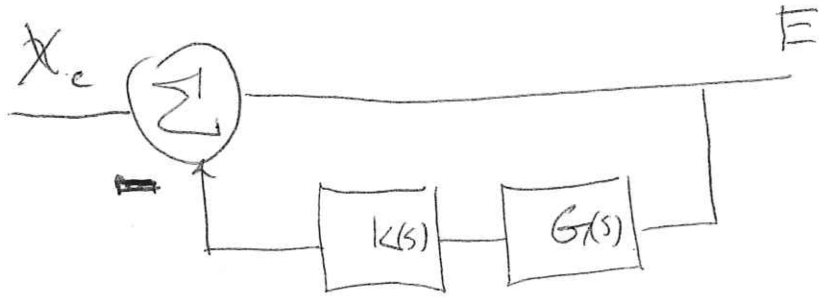
Transfer Function	Transfer Function
Input X_c	Input D
output X	output X
$D = 0$	$X_c = 0$

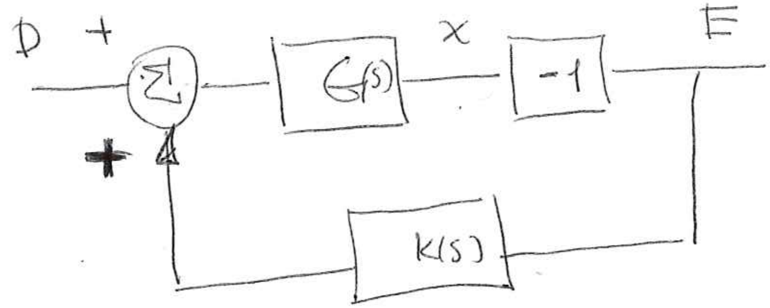


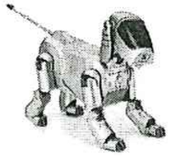
$$E = \left(\frac{E}{X_c}\right)\Bigg|_{D=0} X_c + \left(\frac{E}{D}\right)\Bigg|_{X_c=0} D$$

Input X_c ; Output E

Input $-D$; output E





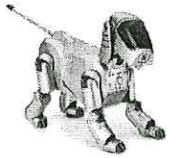


$$\left.\frac{E}{X_c}\right|_{D=0} = \frac{1}{1+kG} = \frac{1}{1+(k_d s+k_p)\cdot\frac{1}{ms^2}} = \frac{ms^2}{ms^2+k_d s+k_p}$$

$$\left.\frac{E}{D}\right|_{X_c=0} = \frac{G}{1-kG} = \frac{-G}{1- -G K_s} = -\frac{G}{1+kG} = \frac{-\frac{1}{ms^2}}{1+(k_d s+k_p)\frac{1}{ms^2}}$$

$$= -\frac{1}{ms^2+k_d s+k_p}$$

Note the + in the first sum



Input

X_c

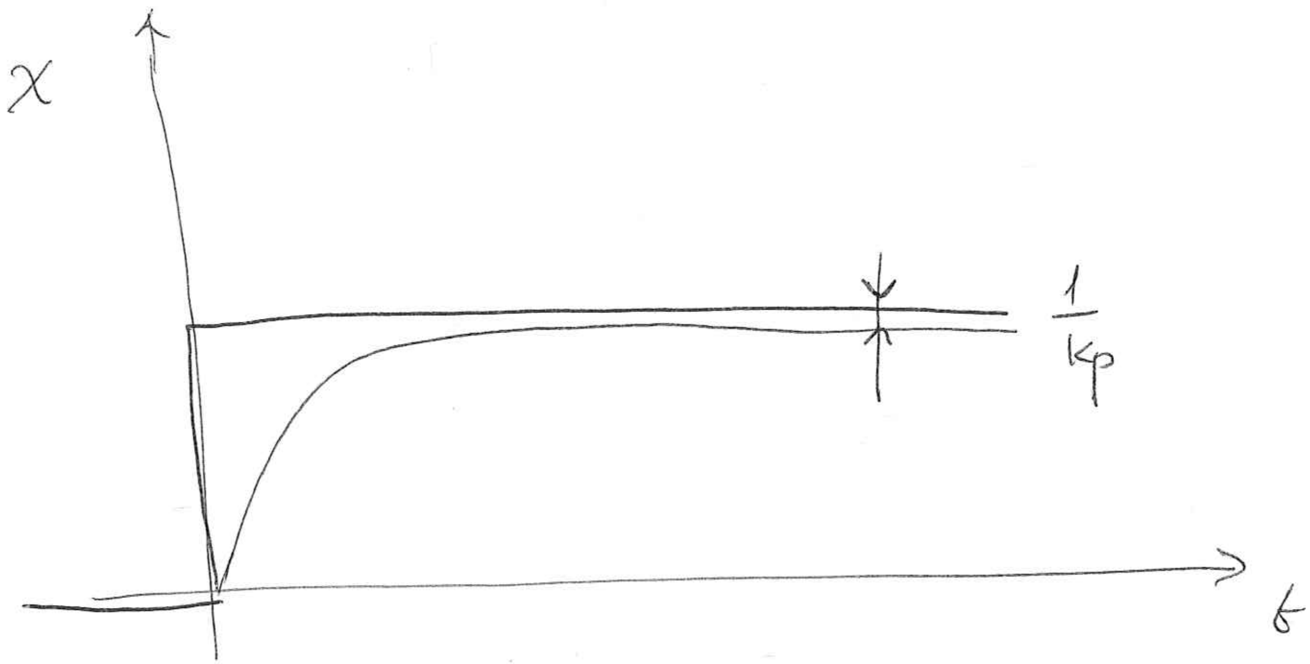
$$e_{ss}\Big|_{\frac{E}{X_c}} = \lim_{s\to 0} \cancel{s}\left[\frac{E}{X_c}\right]\left(\frac{1}{\cancel{s}}\right) = \lim_{s\to 0} \frac{ms^2}{ms^2+k_d s+k_p} = 0$$

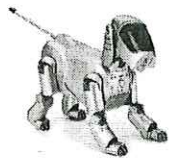
D

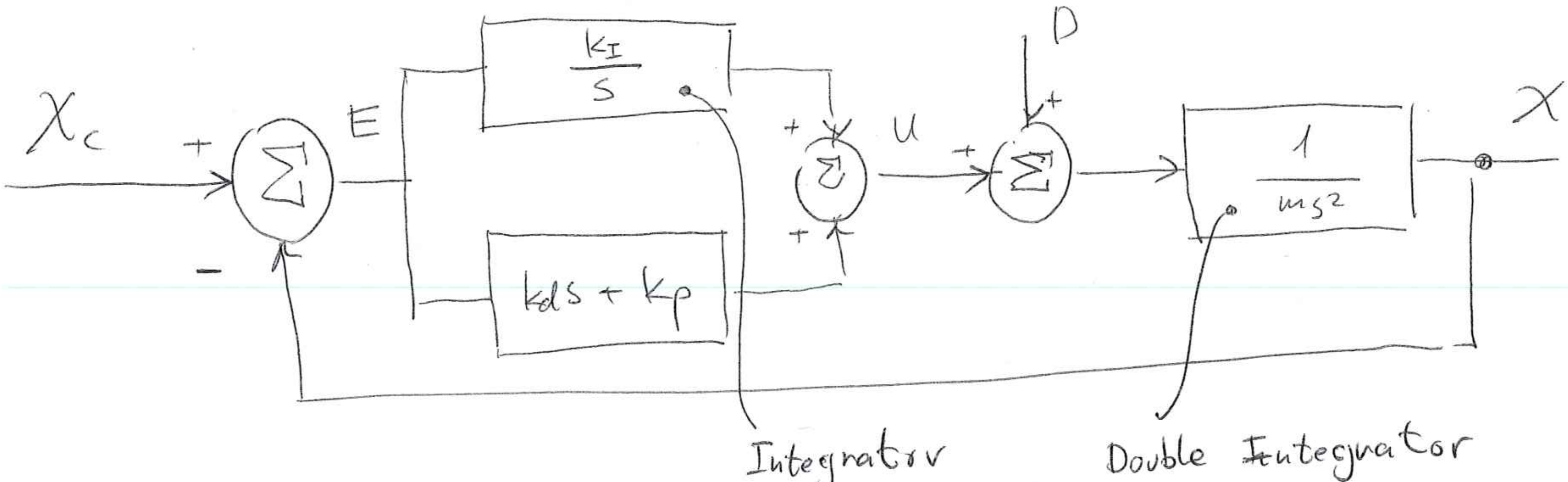
$$e_{ss}\Big|_{\frac{E}{D}} = \lim_{s\to 0} s\left[\frac{E}{D}\right]\left(\frac{1}{s}\right) = \lim_{s\to 0} \frac{1}{ms^2+k_d s+k_p} = \frac{1}{K_p}$$

(*) The disturbance generates a steady state error

$$\longrightarrow \frac{1}{K_p}$$

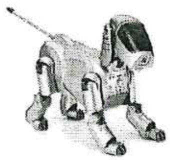

x
1/kp
t



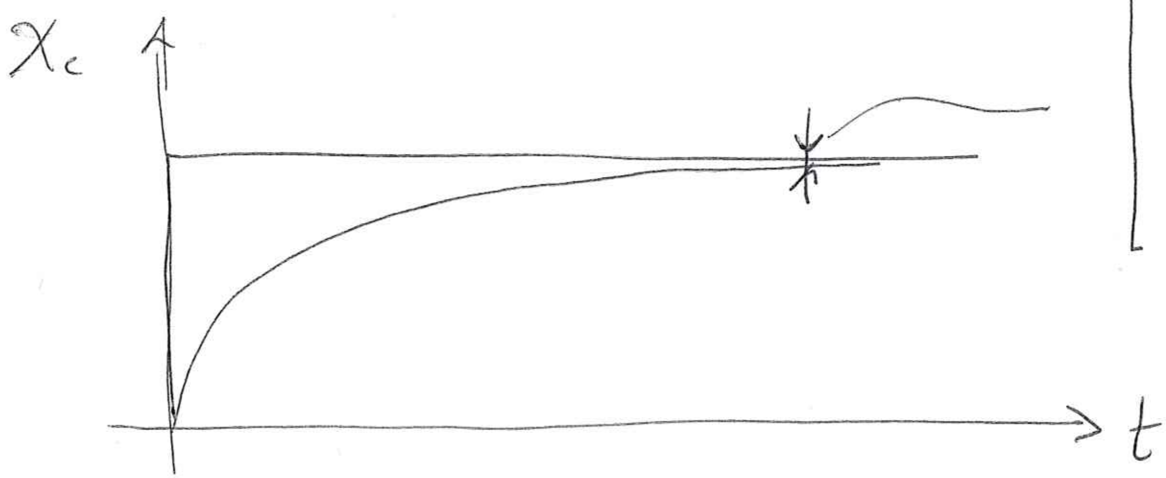


* In Seady - State anything that goes into the integrator must be zero $\int(\sim\sim) = 0$

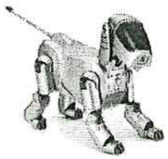
* If somthing gets into the Integrator it will start to accumulate and then we are not in steady State



+ As a result $\rightarrow$ $\begin{cases} U + D = 0 \\ E = 0 \end{cases}$

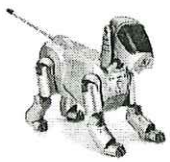


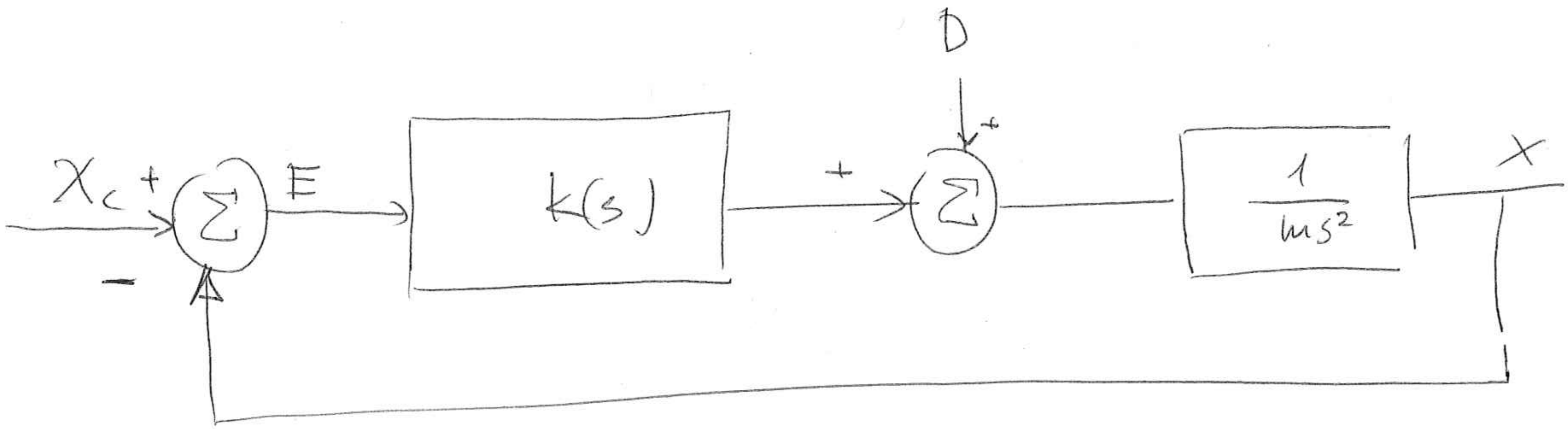
The integrator will reduce the error E to zero



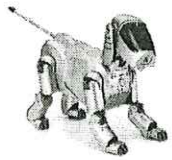
* In general the value of the integral gain K_I should be small $\sim \frac{1}{100}$ of k_d or k_p

* The integral is a pole. Adding a pole to the system can make it unstable





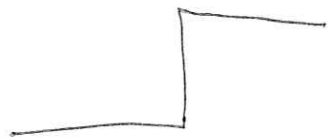
$$PID \rightarrow \quad K(s) = \frac{k_I}{s} + k_d s + k_p = \frac{k_d s^2 + k_p s + k_I}{s}$$



$$\frac{E}{D} = -\frac{G}{1+GH} = -\frac{\frac{1}{ms^2}}{1+\left(\frac{1}{ms^2}\right)\left(\frac{k_d s^2 + k_p s + K_I}{s}\right)} = -\frac{1}{ms^2 + \frac{k_d s^2 + k_p s + K_I}{s}}$$

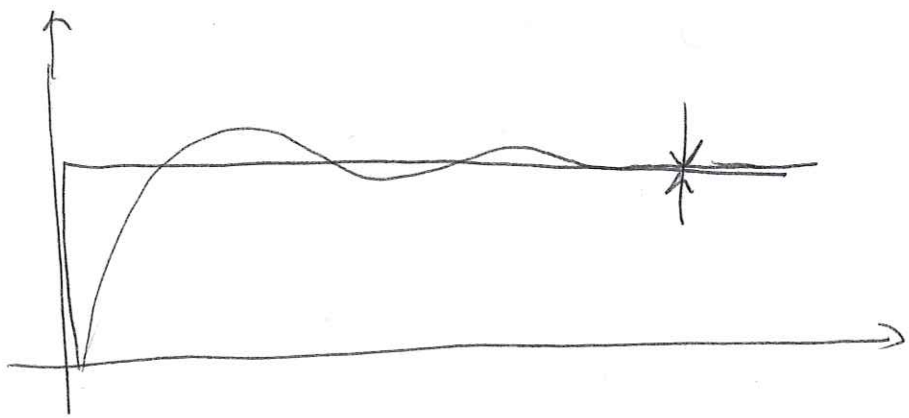
$$= -\frac{s}{ms^3 + k_d s^2 + k_p s + K_I}$$

Input $\frac{1}{s}$

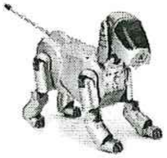


Input

$$e_{ss} = \lim_{s \to 0} \cancel{s} \left[\frac{s}{m s^3 + k_d s^2 + k_p s + k_I} \right] \frac{1}{\cancel{s}} = 0$$



Tracking a step input with a zero steady state error



PID - ROOT LOCUS

$$KGH = \frac{k_p + k_d s + k_I \frac{1}{s}}{m s^2}$$

$$= \frac{k_p s + k_d s^2 + k_I}{m s^3}$$

$$= k_d \left[\frac{s^2 + \frac{k_p}{k_d} s + \frac{k_I}{k_d}}{m s^3} \right]$$

POLES $\quad m s^3 = 0$

$$P_1, P_2, P_3 = 0$$

ZEROS

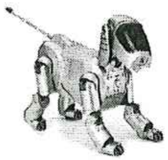
$$s^2 + \frac{k_p}{k_d} s + \frac{k_I}{k_d} = 0$$

$$z_1, z_2 = \frac{-\frac{k_p}{k_d} \pm \sqrt{\left(\frac{k_p}{k_d}\right)^2 - 4\frac{k_I}{k_d}}}{2}$$

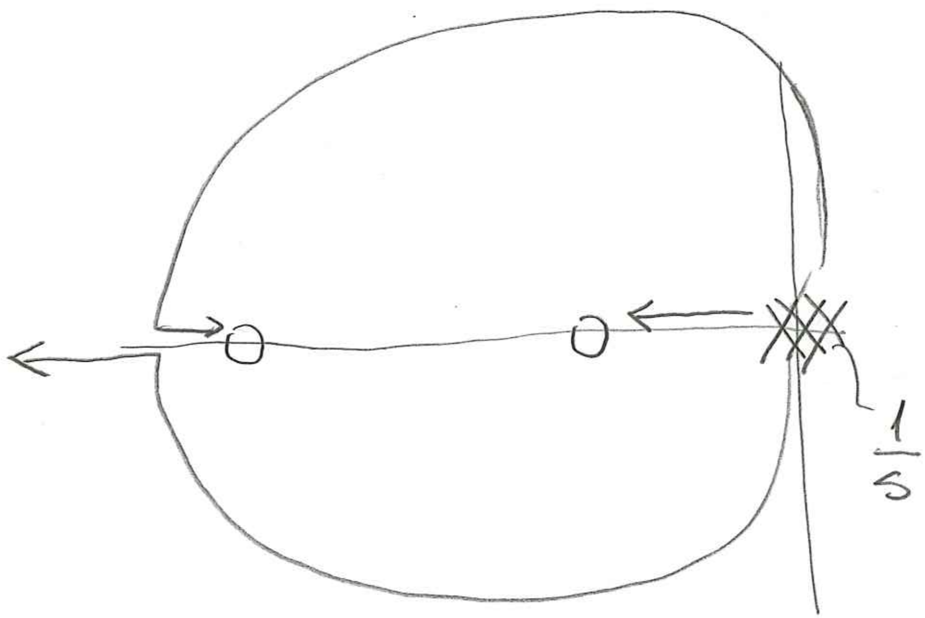
$$\Delta = \left(\frac{k_p}{k_d}\right)^2 - 4\frac{k_I}{k_d}$$

$\Delta > 0 \rightarrow$ 2 Real Zero

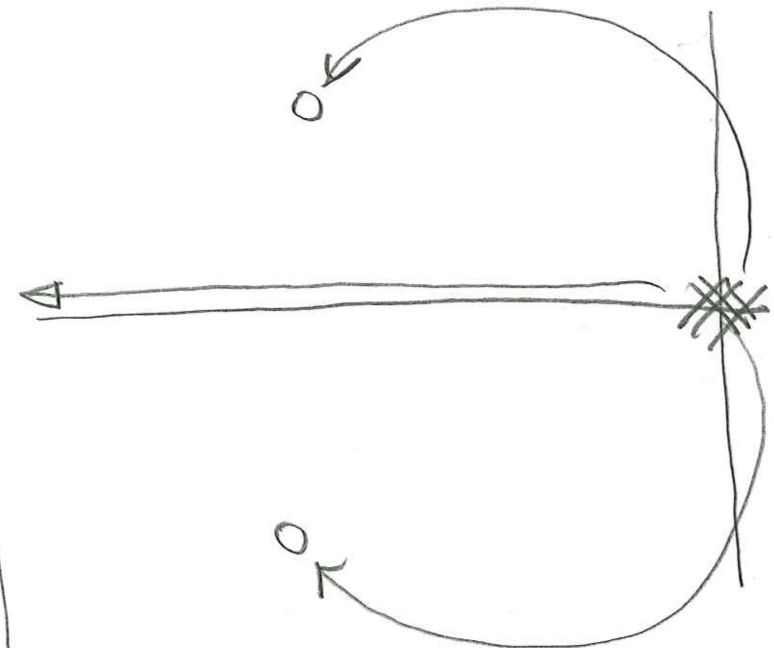
$\Delta < 0 \rightarrow$ 2 Complex Zero



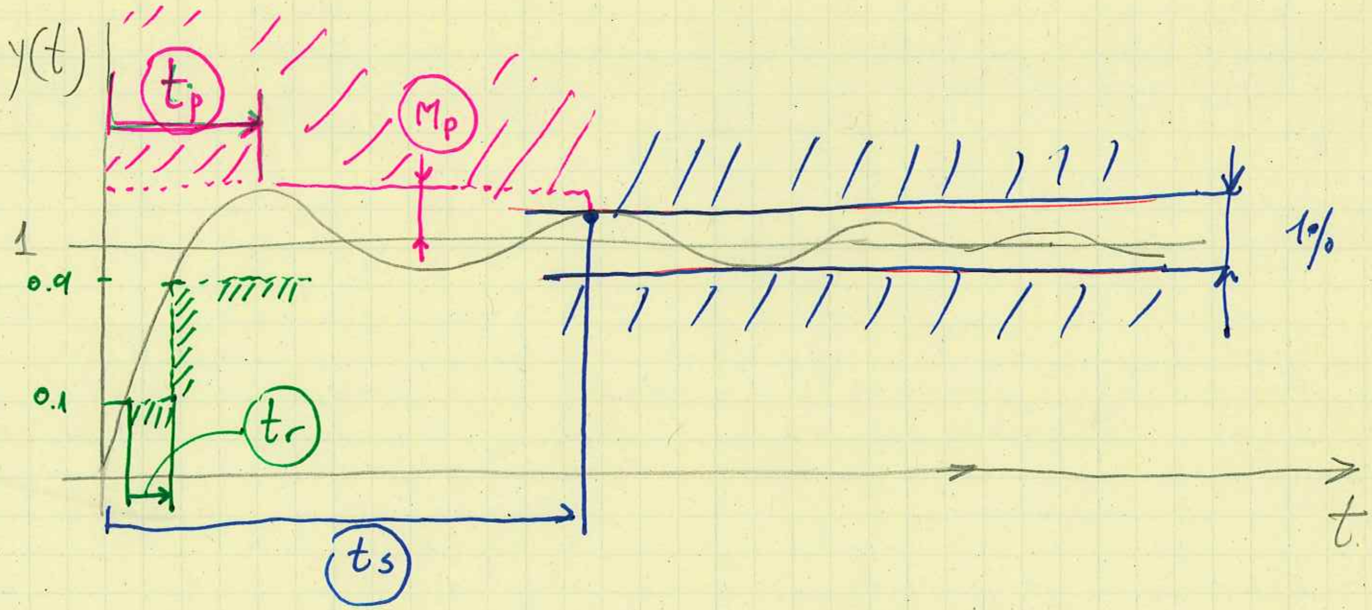
$\Delta > 0 \Rightarrow \left(\frac{k_p}{k_d}\right)^2 > 4\frac{k_I}{k_d}$



$\Delta < 0 \quad \left(\frac{k_p}{k_d}\right)^2 < 4\frac{k_I}{k_d}$



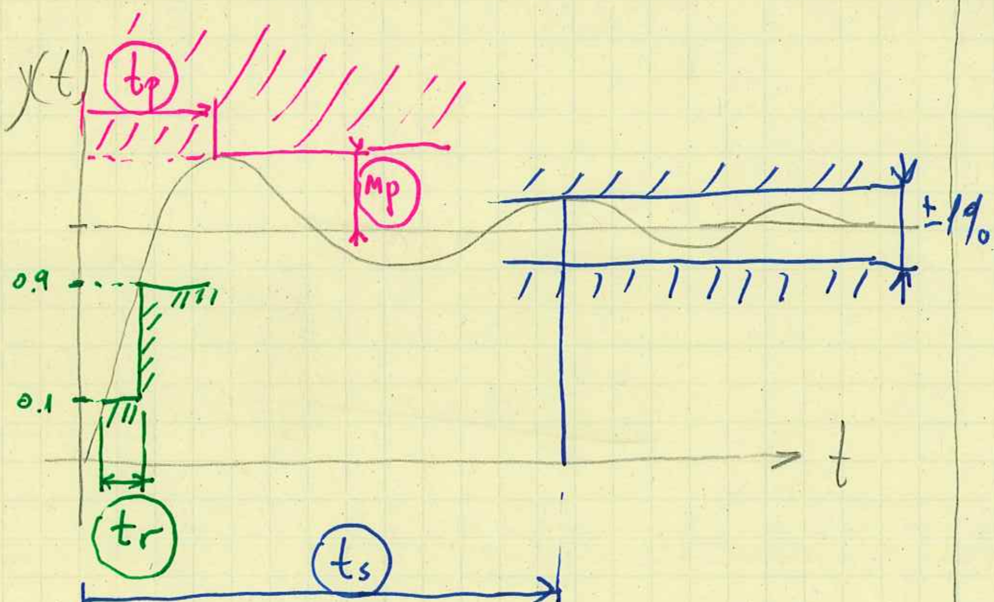
DESIGN - TIME DOMAIN SPECIFICATIONS



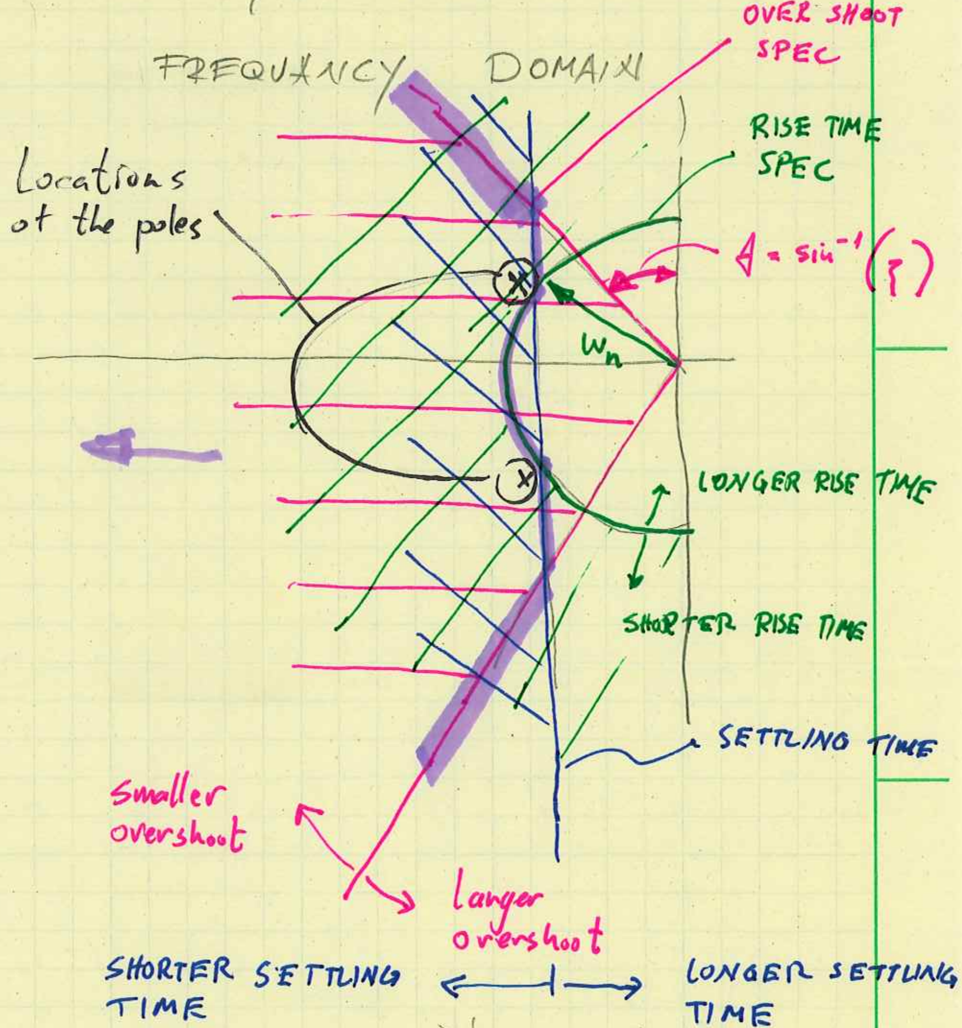
- RISE TIME (t_r) The time it takes the system to reach the vicinity of its new set point
- SETTING TIME (t_s) The time it takes the system transient to decay
- OVERSHOOT (M_p) - The ration between the maximal value and the final value (%)
- Peak time (t_p) - The time it takes the system to reach the maximum overshoot

DESIGN - TIME DOMAIN / FREQUANCY DOMAIN

TIME DOMAIN



FREQUANCY DOMAIN



1. $t_r \rightarrow \omega_n \geq \dfrac{1.8}{t_r}$

2. $M_p \rightarrow \zeta \geq \zeta(M_p)$ Graph

3. $\sigma \rightarrow \sigma \geq \dfrac{4.6}{t_s}$