HW 5 Solution

3D-1-RRR

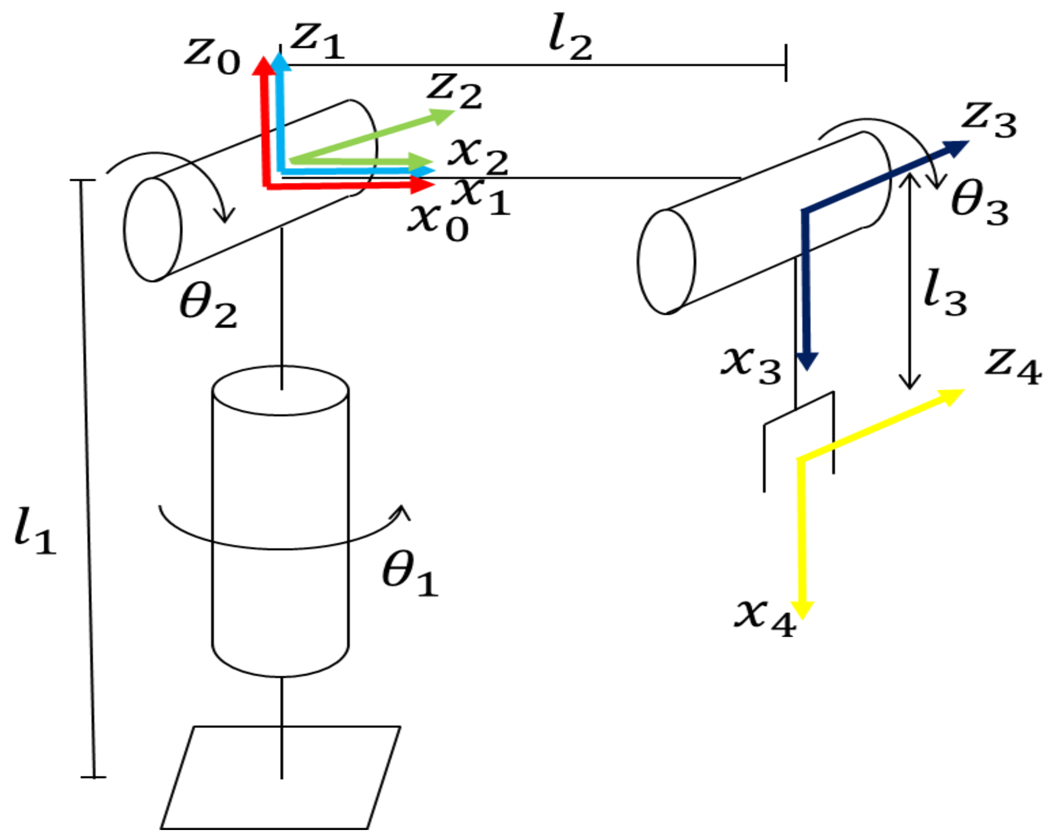

Fig.1 3D-1-RRR

a. Direct Differential

$$
{}^0_4T = \begin{bmatrix} c_1c_{23} & -c_1c_{23} & -s_1 & L_3c_1c_{23} + L_2c_1c_2 \\ sc_{23} & -s_1s_{23} & c_1 & L_3sc_{23} + L_2cs_1 \\ -S_{23} & -c_{23} & 0 & -L_3s_{23} - L_2s_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Position:

$$ {}^0P_{T,x} = x = l3c1c23 + l2c1c2 $$

$$ {}^0P_{T,y} = y = l3s1c23 + l2c2s1 $$

$$ {}^0P_{T,z} = z = -l3s23 - l2s2 $$

Differential:

$$\dot{x} = \frac{\partial x}{\partial \theta_1}\dot{\theta}_1 + \frac{\partial x}{\partial \theta_2}\dot{\theta}_2 + \frac{\partial x}{\partial \theta_3}\dot{\theta}_3$$
$$= -(\mathrm{l3c23} + \mathrm{l2c2})\mathrm{s1}\dot{\theta}_1 + l2c1(-s2)\dot{\theta}_2 - l3c1s23\dot{\theta}_2 - l3c1s23\dot{\theta}_3$$

$$\dot{y} = \frac{\partial y}{\partial \theta_1}\dot{\theta}_1 + \frac{\partial y}{\partial \theta_2}\dot{\theta}_2 + \frac{\partial y}{\partial \theta_3}\dot{\theta}_3$$
$$= -L_3S_1S_{23}\left(\dot{\theta}_2 + \dot{\theta}_3\right) + L_3C_{23}C_1\dot{\theta}_1 + L_2C_2C_1\dot{\theta}_2 - L_2S_1S_2\dot{\theta}_2$$

$$\dot{z} = \frac{\partial z}{\partial \theta_1}\dot{\theta}_1 + \frac{\partial z}{\partial \theta_2}\dot{\theta}_2 + \frac{\partial z}{\partial \theta_3}\dot{\theta}_3$$
$$= -L_2C_{23}\left(\dot{\theta}_2 + \dot{\theta}_3\right) - C_2S_2\dot{\theta}_2$$

$${}^{0}\mathrm{v}_n = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix}$$
$${}^{0}\mathrm{w}_n = {}^{0}_{1}R\begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix}\rho_1 + {}^{0}_{2}R\begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix}\rho_2 + {}^{0}_{3}R\begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_3 \end{bmatrix}\rho_3$$

$$\begin{bmatrix} {}^{0}v_n \\ {}^{0}w_n \end{bmatrix} = {}^{0}J(\theta)\dot{\theta}$$

So,

$${}^{0}J = \begin{bmatrix} -S_1(L_3C_{23} + L_2C_2) & -C_1(L_3S_{23} + L_2S_2) & -L_3S_{23}C_1 \\ C_1(L_3C_{23} + L_2C_2) & -S_1(L_3S_{23} + L_2S_2) & -L_3S_{23}S_1 \\ 0 & -L_3C_{23} - L_2C_2 & -L_3C_{23} \\ 0 & -S_1 & -S_1 \\ 0 & C_1 & C_1 \\ 1 & 0 & 0 \end{bmatrix}$$

b. Force Propagation

Apply force on the end effector $^{4}f_4 = \begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix}$

For $i = 3$,

$$^{3}f_3 = {}^{3}_{4}R\ {}^{4}f_4 = {}^{4}f_4$$

For $i = 2$,

$$^{2}f_2 = {}^{2}_{3}R\ {}^{3}f_3 = \begin{bmatrix} C_3 f_x - S_3 f_y \\ S_3 f_x + C_3 f_y \\ f_2 \end{bmatrix}$$

For $i = 1$

$$^{1}f_1 = {}^{1}_{2}R\ {}^{2}f_2 = \begin{bmatrix} C_{23} f_x - S_{23} f_y \\ f_2 \\ -S_{23} f_x - C_{23} f_y \end{bmatrix}$$

For $i = 3$,

$$^{3}n_3 = {}^{3}_{4}R\ {}^{4}n_4 + {}^{3}_{4}P \times {}^{3}f_3 = \begin{bmatrix} 0 \\ -L_3 f_z \\ L_3 f_y \end{bmatrix}$$

For $i = 2$,

$$^{2}n_2 = {}^{2}_{3}R\ {}^{3}n_3 + {}^{2}_{3}P \times {}^{2}f_2 = \begin{bmatrix} L_3 f_z S_3 \\ -C_3 L_3 f_z - L_2 f_z \\ L_3 f_y + L_2 S_3 f_x + L_2 C_3 f_y \end{bmatrix}$$

For $i = 1$

$$^{1}n_1 = {}^{1}_{2}R\ {}^{2}n_2 + {}^{1}_{2}P \times {}^{1}f_1 = \begin{bmatrix} L_3 S_{23} f_z + L_2 S_2 f_z \\ L_2 S_3 f_x + L_2 C_3 f_y + L_3 f_y \\ L_3 C_{23} f_z + L_2 C_2 f_z \end{bmatrix}$$

And this gives

$$T = \begin{bmatrix} 0 & 0 & L_2C_2 + L_3C_{23} & -S_{23} & -C_{23} & 0 \\ L_2S_3 & L_2C_3 + L_3 & 0 & 0 & 0 & 1 \\ 0 & L_3 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} f_x \\ f_y \\ f_z \\ n_x \\ n_y \\ n_z \end{bmatrix} \qquad (3-20)$$

And

$$^4J = \begin{bmatrix} 0 & L_2S_3 & 0 \\ 0 & L_3 + L_2C_3 & L_3 \\ L_3C_{23} + L_2C_2 & 0 & 0 \\ -S_{23} & 0 & 0 \\ -C_{23} & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix} \qquad (3-12)$$

3D-4-RRR

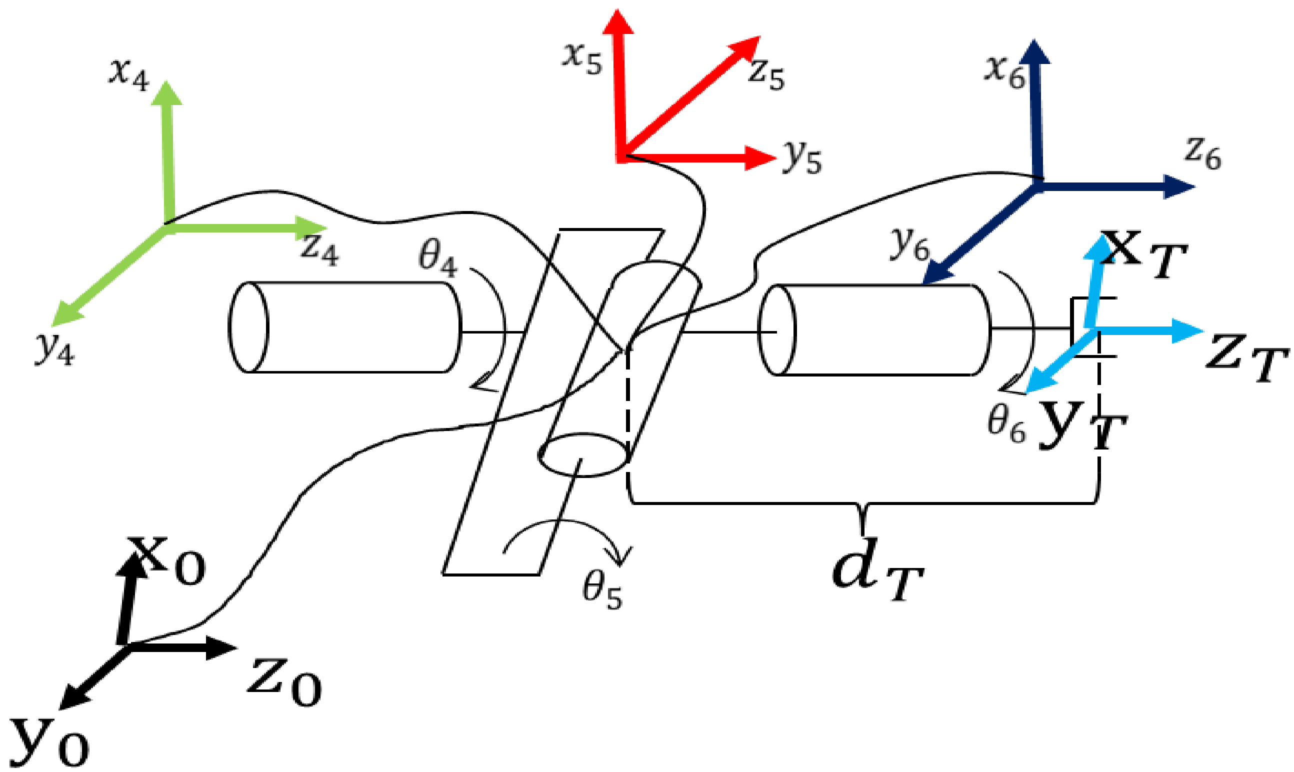

Fig.2 3D-4-RRR

a. Direct Differential

$$ {}^{3}_{6}T = \begin{bmatrix} C_4C_5C_6 - S_4S_6 & -C_6S_4 - C_4C_5 & -C_4S_5 & 0 \\ C_4S_6 + C_5C_6S_4 & C_4C_6 - C_5S_4S_6 & -S_4S_5 & 0 \\ C_6S_5 & -S_5S_4 & C_5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ \dot{x} = d_T S4S5\dot{\theta}_4 - d_T C4C5\dot{\theta}_5 $$
$$ \dot{y} = -d_T C4S5\dot{\theta}_4 - d_T S4C5\dot{\theta}_5 $$
$$ \dot{z} = -d_T S5\dot{\theta}_5 $$

$$
\begin{aligned}
{}^0\theta_x &= S_4\theta_5 - C_4S_5\theta_6 \\
{}^0\theta_y &= -C_4\theta_5 - S_4S_5\theta_6 \\
{}^0\theta_z &= \theta_4 + C_5\theta_6
\end{aligned}
$$

This gives

$$
{}^0J = \begin{bmatrix} d_T S4S5 & -d_T C4C5 & 0 \\ -d_T C4S5 & -d_T S4C5 & 0 \\ 0 & -d_T S5 & 0 \\ 0 & S_4 & -C_4S_5 \\ 0 & -C_4 & -S_4S_5 \\ 1 & 0 & C_5 \end{bmatrix}
$$

b. Force Propagation:

$$
{}^6f_6 = [f_x \quad f_y \quad f_z]^T
$$

$$
{}^5f_5 = {}^5_6R\ {}^6f_6 = \begin{bmatrix} C_6f_x - S_6f_y \\ f_z \\ -S_6f_x - C_6f_y \end{bmatrix}
$$

$$
{}^4f_4 = {}^4_5R\ {}^5f_4 = \begin{bmatrix} C_5C_6f_x - C_5S_6f_y - S_5f_z \\ S_6f_x + C_6f_y \\ C_6S_5f_x - S_5S_6f_y + C_5f_z \end{bmatrix}
$$

and

$$
{}^Tn_T = [n_x \quad n_y \quad n_z]^T
$$

$$
{}^6n_6 = \begin{bmatrix} -d_Tf_y + n_x \\ d_Tf_x + n_y \\ n_z \end{bmatrix}
$$

$$^{5}n_5 = {}^{5}_{6}R\ {}^{6}n_6 + \ {}^{5}P_6 \times \ {}^{5}f_5 = \begin{bmatrix} -d_T S6 f_x - d_T C6 f_y - C_6 n_x - S_6 n_y \\ n_z \\ -d_T C6 f_x + d_T S6 f_y - S_6 n_x - C_6 n_y \end{bmatrix}$$

$$^{4}n_4 = {}^{4}_{5}R\ {}^{5}n_5 + \ {}^{4}P_5 \times \ {}^{4}f_4 = \begin{bmatrix} -d_T S6C5 f_x - d_T C5C6 f_y + C_5 C_6 n_x - C_5 S_6 n_y - S_5 n_z \\ +d_T C6 f_x - d_T S6 f_y + S_6 n_x + C_6 n_y \\ -d_T S6S5 f_x - d_T S5C6 f_y + S_5 S_6 n_x - S_6 S_5 n_y + C_5 n_z \end{bmatrix}$$

This gives

$$T = \begin{bmatrix} -d_T S6S5 & -d_T S5C6 & 0 & S_5 C_6 & -S_5 S_6 & C_5 \\ -d_T C6 & d_T S6 & 0 & -S_6 & -C_6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} f_x \\ f_y \\ f_z \\ n_x \\ n_y \\ n_z \end{bmatrix}$$

3D-5-RRRP

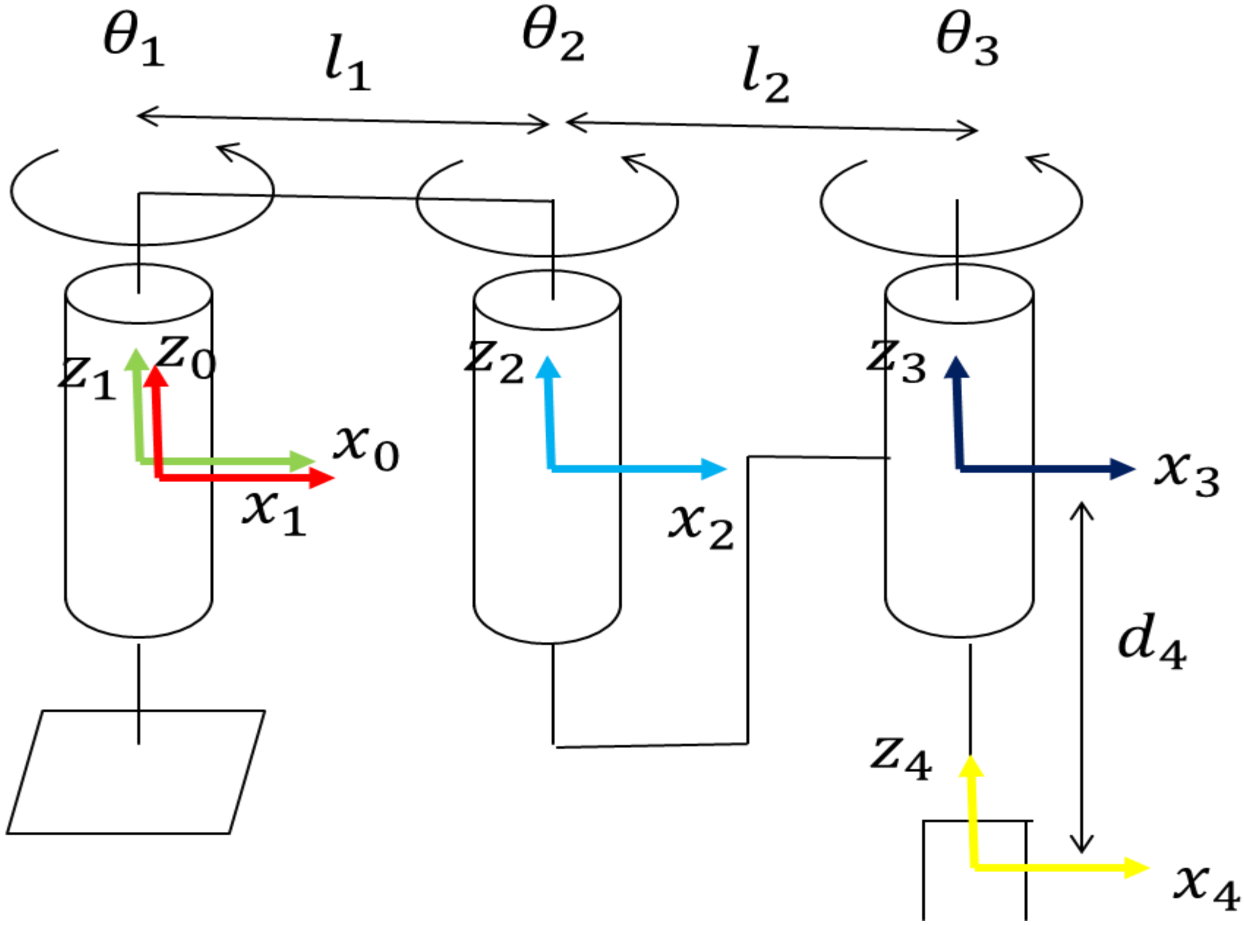

Fig.3 3D-5-RRRP

a. Direct Differentiation

$$\dot{x} = (-L_2S_{12} - L_1S_1)\dot{\theta}_1 - L_2S_{12}\dot{\theta}_2$$
$$\dot{y} = (L_2C_{12} + L_1C_1)\dot{\theta}_1 + L_2C_{12}\dot{\theta}_2$$
$$\dot{z} = -\dot{d}_4$$

$$\dot{\theta}_x = 0$$
$$\dot{\theta}_y = 0$$
$$\dot{\theta}_z = \theta_1 + \theta_2 + \theta_3$$

Then it gives

$$
{}^{0}J = \begin{bmatrix} -L_2S_{12} - L_1S_1 & -L_2S_{12} & 0 & 0 \\ L_2C_{12} + L_1C_1 & L_2C_{12} & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix}
$$

Force Propagation:

$$
{}^{4}f_4 = \begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix}
$$

$$
{}^{4}n_4 = \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix}
$$

$$
{}^{3}f_3 = {}^{3}_{4}R\ {}^{4}f_4 = \begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix}
$$

$$
{}^{2}f_2 = {}^{2}_{3}R\ {}^{3}f_3 = \begin{bmatrix} C_3f_x - S_3f_y \\ S_3f_x + C_3f_y \\ f_z \end{bmatrix}
$$

$$
{}^{1}f_1 = {}^{1}_{2}R\ {}^{2}f_2 = \begin{bmatrix} C_{23}f_x - S_{23}f_y \\ S_{23}f_x + C_{23}f_y \\ f_z \end{bmatrix}
$$

$$
{}^{3}n_3 = {}^{3}_{4}R\ {}^{4}n_4 + {}^{3}_{4}P \times {}^{3}f_3 = \begin{bmatrix} -d_4f_y \\ d_4f_x \\ 0 \end{bmatrix}
$$

$$
{}^{2}n_2 = {}^{2}_{3}R\ {}^{3}n_3 + {}^{2}_{3}P \times {}^{2}f_2 = \begin{bmatrix} -S_3d_4f_x - C_3d_4f_y \\ C_3d_4f_x - S_3d_4f_y \\ L_2S_3f_x + L_2C_3f_y \end{bmatrix}
$$

$$
{}^{1}n_1 = {}^{1}_{2}R\ {}^{2}n_2 + {}^{1}_{2}P \times {}^{1}f_1 = \begin{bmatrix} C_2 & -S_2 & 0 \\ S_2 & C_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} {}^{2}n_2 + \begin{bmatrix} L_1 \\ 0 \\ 0 \end{bmatrix} \times {}^{1}f_1
$$

Then we have ${}^{1}n_{1_2} = L_2S_3f_x + L_2C_3f_y + L_1C_{23}f_y + L_1S_{23}f_x$.

And

$$T = \begin{bmatrix} L_2S_3 + L_1S_{23} & L_2C_3 + L_1C_{23} & 0 & 0 & 0 & 1 \\ L_2S_3 & L_2C_3 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} f_x \\ f_y \\ f_z \\ n_x \\ n_y \\ n_z \end{bmatrix}$$

$${}^{4}J = \begin{bmatrix} L_2S_3 + L_1S_{23} & L_2C_3 + L_1C_{23} & 0 & 0 \\ L_2S_3 & L_2C_3 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$