HW 4 Solution

3D-1-RRR

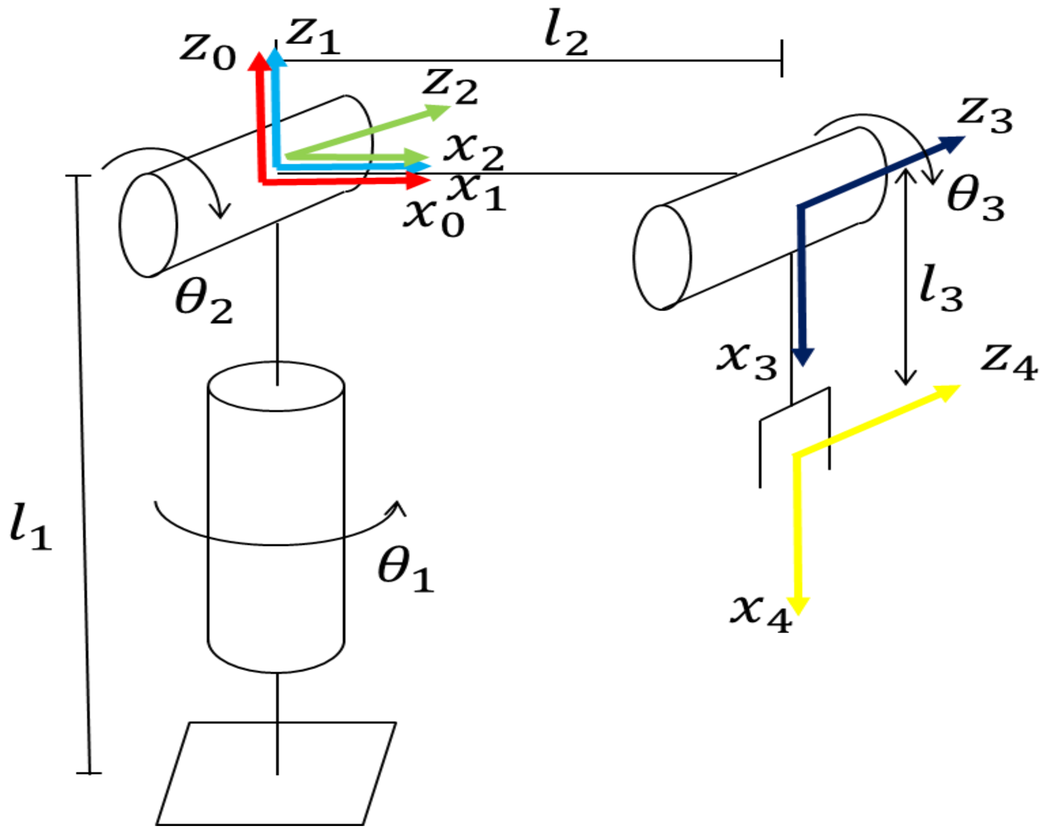

Fig.1 3D-1-RRR

a. Velocity Propagation

Angular Velocity

For $\mathrm{i} = 0$,

$$^1w_1 = {}^1_0R\,{}^0w_0 + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix} = \begin{bmatrix} c1 & s1 & 0 \\ -s1 & c1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix}$$

For $\mathrm{i} = 1$,

$$^2w_2 = {}^2_1R\,{}^1w_1 + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_2 \end{bmatrix} = \begin{bmatrix} c2 & 0 & -s2 \\ s2 & 0 & -c2 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_2 \end{bmatrix} = \begin{bmatrix} -\mathrm{s}2\dot{\theta}_1 \\ -\mathrm{c}2\dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

For i = 2,

$$^{3}w_3 = {}^{3}_{2}R\,{}^{2}w_2 + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_3 \end{bmatrix} = \begin{bmatrix} c3 & -s3 & 0 \\ s3 & c3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -s2\dot{\theta}_1 \\ -c2\dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_3 \end{bmatrix} = \begin{bmatrix} -s23\dot{\theta}_1 \\ -c23\dot{\theta}_1 \\ \dot{\theta}_2 + \dot{\theta}_3 \end{bmatrix}$$

For i = 3,

$$^{4}w_4 = {}^{4}_{3}R\,{}^{3}w_3 + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -s23\dot{\theta}_1 \\ -c23\dot{\theta}_1 \\ \dot{\theta}_2 + \dot{\theta}_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -s23\dot{\theta}_1 \\ -c23\dot{\theta}_1 \\ \dot{\theta}_2 + \dot{\theta}_3 \end{bmatrix}$$

$^{3}w_3 = {}^{4}w_4$

Linear Velocity

For i = 1,

$$^{1}v_1 = {}^{1}_{0}R\{^{0}w_0 \times {}^{0}P_1 + {}^{0}v_0\} = \begin{bmatrix} c1 & s1 & 0 \\ -s1 & c1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

For i = 2,

$$^{2}v_2 = {}^{2}_{1}R\{^{1}w_1 \times {}^{1}P_2 + {}^{1}v_1\} = \begin{bmatrix} c2 & 0 & -s2 \\ s2 & 0 & -c2 \\ 0 & 1 & 0 \end{bmatrix} \left\{ \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

For i = 3,

$$^{3}v_3 = {}^{3}_{2}R\{^{2}w_2 \times {}^{2}P_3 + {}^{2}v_2\} = \begin{bmatrix} c3 & -s3 & 0 \\ s3 & c3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \left\{ \begin{bmatrix} -s2\dot{\theta}_1 \\ -c2\dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} + \begin{bmatrix} l2 \\ 0 \\ 0 \end{bmatrix} \right\} = \begin{bmatrix} l2s3\dot{\theta}_2 \\ l2c3\dot{\theta}_2 \\ l2c2\dot{\theta}_1 \end{bmatrix}$$

b. Jacobian Matrix

$$\begin{bmatrix} ^{4}v_4 \\ ^{4}w_4 \end{bmatrix} = \begin{bmatrix} l2s3\dot{\theta}_2 \\ l3\dot{\theta}_2 + l2c3\dot{\theta}_2 + l3\dot{\theta}_3 \\ l3c23\dot{\theta}_1 + l2c2\dot{\theta}_1 \\ -s23\dot{\theta}_1 \\ -c23\dot{\theta}_1 \\ \dot{\theta}_2 + \dot{\theta}_3 \end{bmatrix} = \begin{bmatrix} 0 & l2s3 & 0 \\ 0 & l3 + l2c3 & l3 \\ l3c23 + l2c2 & 0 & 0 \\ -s23 & 0 & 0 \\ -c23 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix}$$

Thus, we will get the jacobian matrix, that is

$$ {}^{4}J = \begin{bmatrix} 0 & l2s3 & 0 \\ 0 & l3 + l2c3 & l3 \\ l3c23 + l2c2 & 0 & 0 \\ -s23 & 0 & 0 \\ -c23 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix} $$

Singularities:

$\det(J) = 0$ leads to $\begin{vmatrix} 0 & L_2S_3 & 0 \\ 0 & L_2C_3 + L_3 & L_3 \\ L_2C_2 + L_3C_{23} & 0 & 0 \end{vmatrix} = 0$. Solving it gives three singularities which are $\theta_3 = 0°$, $\theta_3 = 180°$, and the location where $L_2C_2 + L_3C_{23} = 0$.

3D-4-RRR

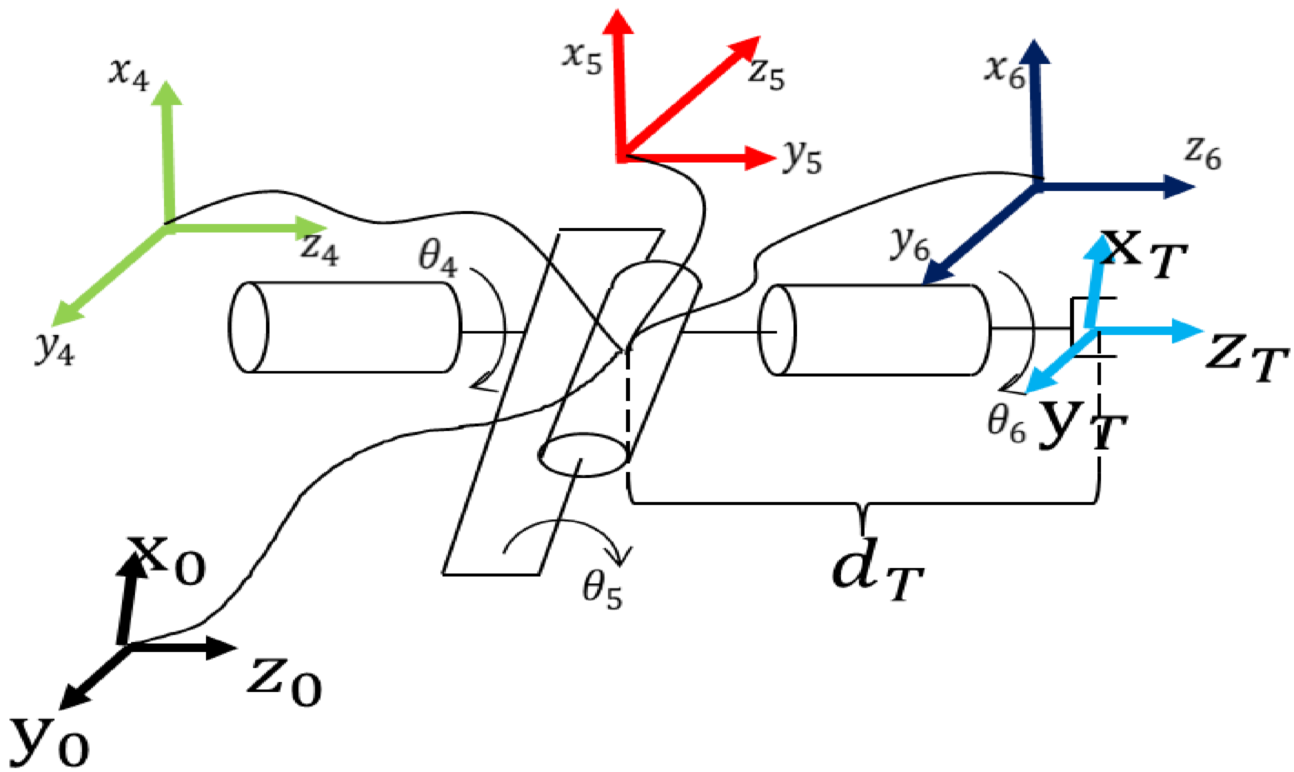

Fig.2 3D-4-RRR

a. Velocity Propagation

Angular Velocity

For $i = 3$,

$$^4\omega_4 = {}^4_3R\ {}^3\omega_3 + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_4 \end{bmatrix} = \begin{bmatrix} C_4 & S_4 & 0 \\ -S_4 & C_4 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_4 \end{bmatrix}$$

For $i = 4$,

$$^5\omega_5 = {}^5_4R\ {}^4\omega_4 + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_5 \end{bmatrix} = \begin{bmatrix} s5\dot{\theta}_4 \\ c5\dot{\theta}_4 \\ \dot{\theta}_5 \end{bmatrix}$$

For $i = 5$,

$$ {}^{6}\omega_6 = {}^{6}_{5}R\ {}^{4}\omega_4 + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_6 \end{bmatrix} = \begin{bmatrix} C_6 S_5 \dot{\theta}_4 - S_6 \dot{\theta}_5 \\ -S_5 S_6 \dot{\theta}_4 - C_6 \dot{\theta}_5 \\ C_5 \dot{\theta}_4 + \dot{\theta}_6 \end{bmatrix} $$

Linear Velocity

From ${}^{i}v_i = {}^{i}_{i-1}R\left({}^{i-1}\omega_{i-1} \times {}^{i-1}P_i + {}^{i-1}v_{i-1}\right)$, we have

$$ {}^{4}v_4 = [0 \quad 0 \quad 0]^T $$
$$ {}^{5}v_5 = [0 \quad 0 \quad 0]^T $$
$$ {}^{6}v_6 = [0 \quad 0 \quad 0]^T $$

$$ {}^{T}v_{T} = \begin{bmatrix} -d_T(\dot{\theta}_5 C6 + S5S6\dot{\theta}_4) \\ d_T(\dot{\theta}_5 S6 - S5C6\dot{\theta}_4) \\ 0 \end{bmatrix} $$

b. Jacobian Matrix

$$ {}^{6}J = \begin{bmatrix} -S5S6d_T & -C6d_T & 0 \\ -S5C6d_T & S6d_T & 0 \\ 0 & 0 & 0 \\ S_5C_6 & -S_6 & 0 \\ -S_5S_6 & -C_6 & 0 \\ C_5 & 0 & 1 \end{bmatrix} $$

3D-5-RRRP

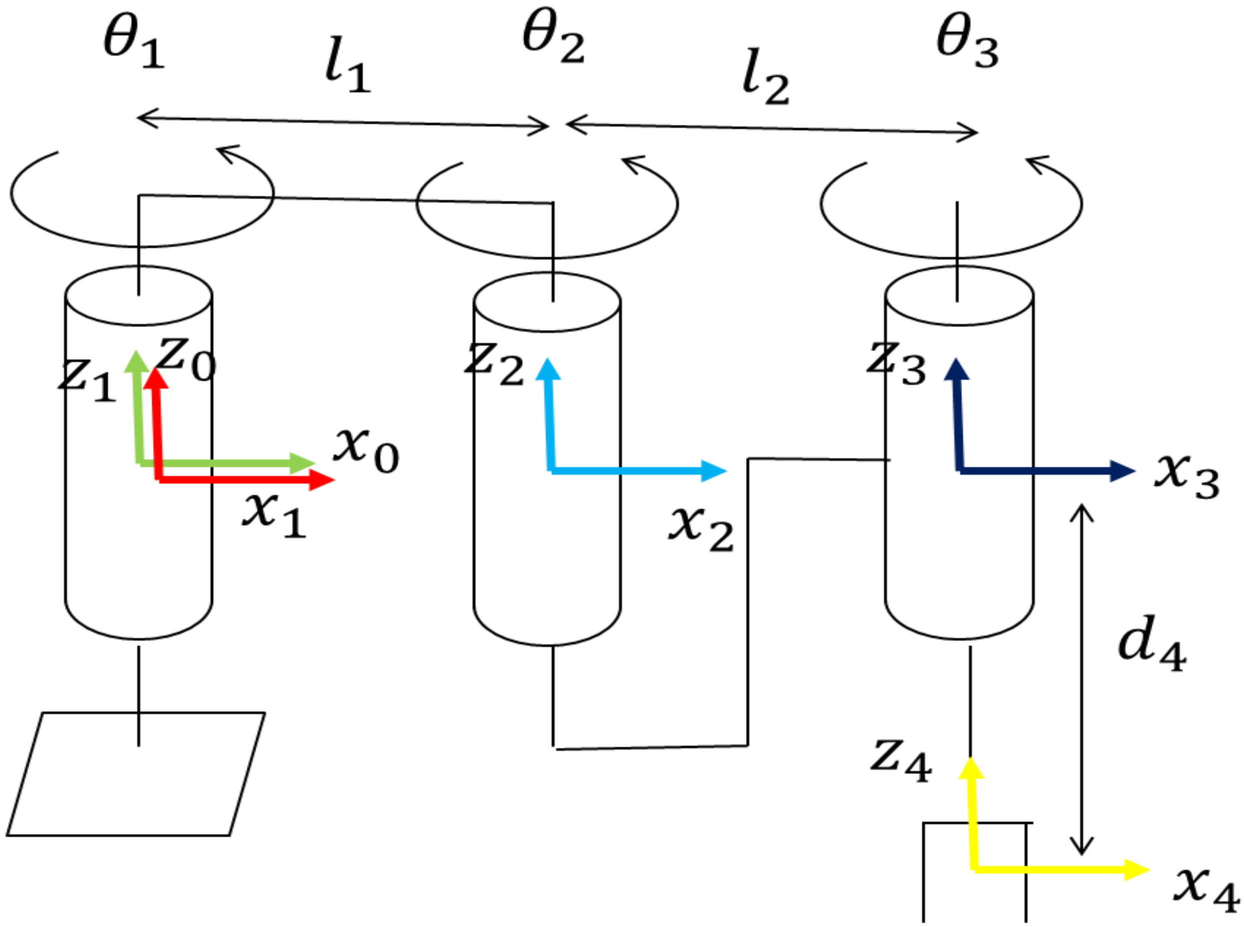

Fig.3 3D-5-RRRP

a. Velocity Propagation
Angular Velocity

For i $= 0$,

$$^{1}\omega_1 = {}^{1}_{0}R\ {}^{0}\omega_0 + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix}$$

For i = 1,

$$^{2}\omega_2 = {}^{2}_{1}R\ {}^{1}\omega_1 + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 + \dot{\theta}_2 \end{bmatrix}$$

For i = 2,

$$^{3}\omega_3 = {}^{3}_{2}R\ {}^{2}\omega_2 + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_3 \end{bmatrix} = {}^{4}\omega_4$$

From

$^{i}v_i = {}^{i}_{i-1}R\left({}^{i-1}\omega_{i-1} \times {}^{i-1}P_i + {}^{i-1}v_{i-1}\right)$

We can get

For i = 1

$$^{1}v_1 = [0 \quad 0 \quad 0]^T$$

For i = 2

$$^{2}v_2 = [L_1 S_2 \dot{\theta}_1 \quad L_1 C_2 \dot{\theta}_1 \quad 0]^T$$

For i = 3

$$^{3}v_3 = \begin{bmatrix} (L_1 S_{23} + L_2 S_3)\dot{\theta}_1 + L_2 S_3 \dot{\theta}_2 \\ (L_1 C_{23} + L_2 C_3)\dot{\theta}_1 + L_2 C_3 \dot{\theta}_2 \\ 0 \end{bmatrix}$$

For i = 4

$$^{4}v_4 = \begin{bmatrix} (L_1 S_{23} + L_2 S_3)\dot{\theta}_1 + L_2 S_3 \dot{\theta}_2 \\ (L_1 C_{23} + L_2 C_3)\dot{\theta}_1 + L_2 C_3 \dot{\theta}_2 \\ -\dot{d}_4 \end{bmatrix}$$

b. Jacobian Matrix

$$ {}^{4}J = \begin{bmatrix} L_1 S_{23} + L_2 S_3 & L_2 S_3 & 0 & 0 \\ L_1 C_{23} + L_2 C_3 & L_2 C_3 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix} $$

Singularities:

$\det({}^{4}J) = 0$ leads to $S_2 = 0$, which means $\theta_2 = 0°$, or $\theta_2 = 180°$.