HW 3 Solution

3D-1-RRR

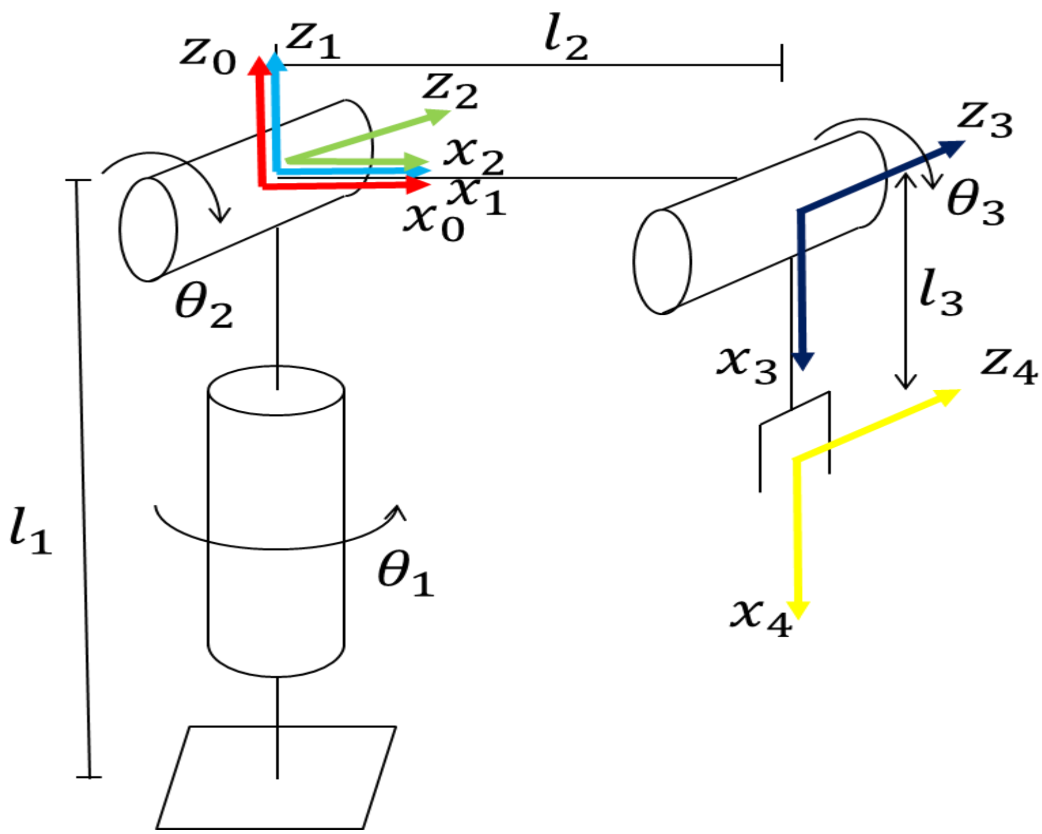

Fig.1 3D-1-RRR

a. Analytical Method

From the lecture notes, the goal transformation matrix could be written as

$$ {}^{0}_{4}T = \begin{bmatrix} r_{11} & r_{12} & r_{13} & x \\ r_{21} & r_{22} & r_{23} & y \\ r_{31} & r_{32} & r_{33} & z \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1} $$

From HW 2, we have derived the goal transformation matrix for 3D-1-RRR

$$ {}^{0}_{4}T = \begin{bmatrix} c1c23 & -c1s23 & -s1 & l3c1c23 + l2c1c2 \\ s1c23 & -s1s23 & c1 & l3s1c23 + l2s1c2 \\ -s23 & -c23 & 0 & -l3s23 - l2s2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2} $$

From equation (1) and (2)

$$r_{13} = -\mathrm{s}1, r_{23} = \mathrm{c}1 \tag{3}$$

Thus, we could solve for θ_1,

$$\theta_1 = Atan2(-r_{13}, r_{23}) \tag{4}$$

Then, solve for θ_2, θ_3. Let $r = x^2 + y^2 + z^2$

$$\mathrm{c}3 = \frac{x^2+y^2+z^2-{l_2}^2-{l_3}^2}{2l_2l_3} \quad ,\mathrm{s}3 = \sqrt{1-c3^2} \tag{5}$$

Solve for θ_3

$$\theta_3 = Atan2(s3, c3) \tag{6}$$

Let

$$k1 = l2 + l3c3,\ \ k_2 = l3s3 \tag{7}$$

$$\theta_2 = Atan2\left(z, \sqrt{x^2+y^2}\right) - Atan2(k2, k1) \tag{8}$$

b. Geometric Method

When we see from the top, we will get a similar figure with fig.2.

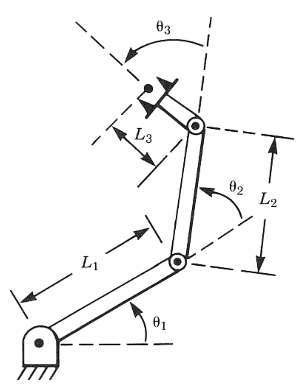

Fig.2 3D-1-RRR

From Fig.2, we could first solve for θ_1,

$$\theta_1 = Atan2(s1, c1) \tag{9}$$

The solve for θ_3, θ_2

$$r = x^2 + y^2 + z^2 \tag{10}$$

$$\mathrm{c}3 = \frac{x^2+y^2+z^2-{l_2}^2-{l_3}^2-1}{2l_2l_3} \quad ,\mathrm{s}3 = \sqrt{1-c3^2} \tag{11}$$

$$\theta_3 = Atan2(s3, c3) \tag{12}$$

$$\theta_2 = \beta \pm \alpha, \quad \beta = Atan2(z, \sqrt{x^2 + y^2}) \tag{13}$$

$$rcos(\alpha) = l1 + l2\cos(\theta_3), rsin(\alpha) = l2\sin(\theta_3) \tag{14}$$

$$\alpha = Atan2(\frac{l2\sin(\theta_2)}{r}, \frac{l1 + l2cos(\theta_2)}{r}) \tag{15}$$

Thus, we could solve for θ_2

$$\theta_2 = \beta - \alpha \tag{16}$$

From the notes, we will have two position for θ_1 and two for θ_2, θ_3

3D-4-RRR

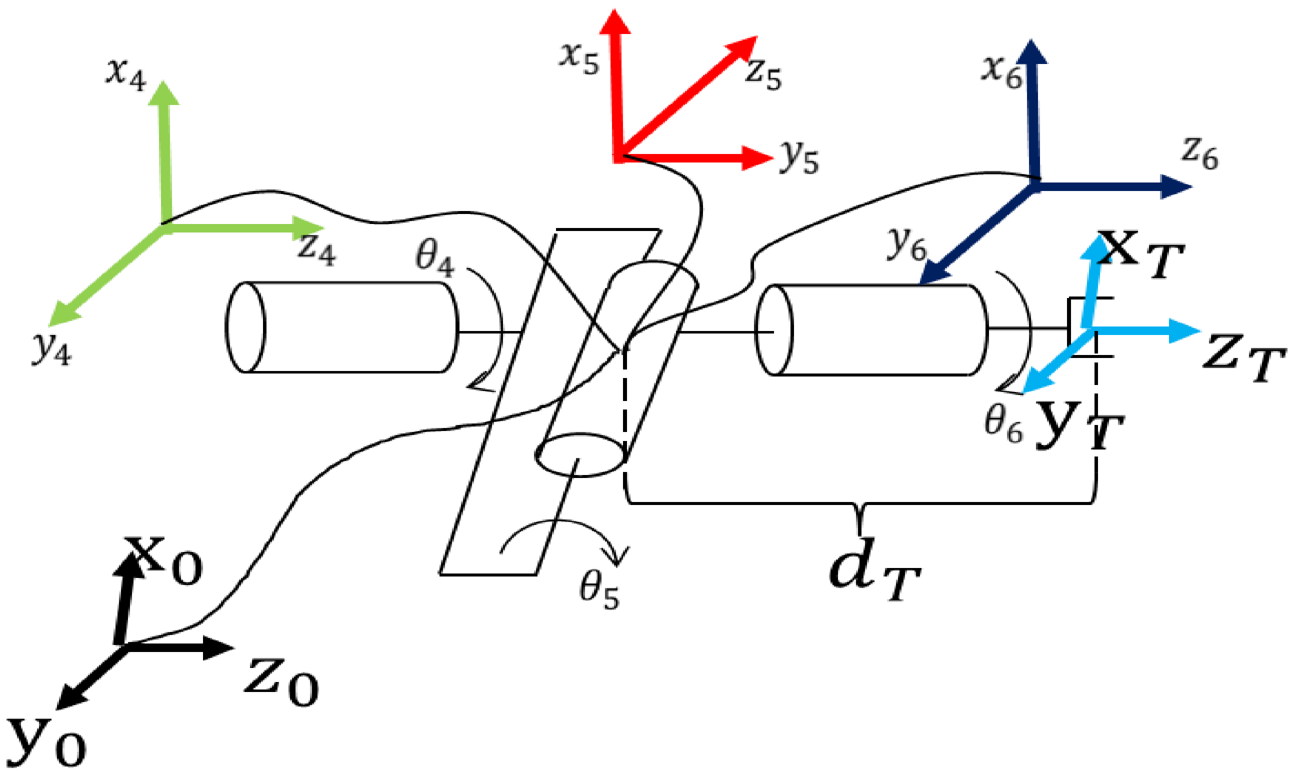

Fig.3 3D-4-RRR

a. Analytical Method

From the lecture notes, the goal transformation matrix could be written as

$$ {}^{3}_{6}T = \begin{bmatrix} r_{11} & r_{12} & r_{13} & x \\ r_{21} & r_{22} & r_{23} & y \\ r_{31} & r_{32} & r_{33} & z \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{17} $$

From HW 2, we have derived the goal transformation matrix for 3D-1-RRR

$$ {}^{3}_{6}T = \begin{bmatrix} c4c5c6 - s4s6 & -c4c5s6 - s4c6 & -c4s5 & 0 \\ s4c5c6 + c4s6 & -s4c5s6 + c4c6 & -s4s5 & 0 \\ s5c6 & -s5s6 & c5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{18} $$

From equation (17) and (18), θ_5 could be solved first

$$ r_{33} = c5, s5 = \sqrt{1 - c5^2} \tag{19} $$

$$ \theta_5 = Atan2(s5, c5) \tag{20} $$

Then solve for θ_6 next

$$r_{31} = \mathrm{c6s5}, \mathrm{c6} = r_{31}/s5 \tag{21}$$

$$r_{32} = -\mathrm{s6s5}, \mathrm{s6} = -r_{32}/s5 \tag{22}$$

$$\theta_6 = Atan2(s6, c6) \tag{23}$$

The solve for θ_4 next

$$r_{13} = -\mathrm{c4s5}, \mathrm{c4} = -r_{13}/s5 \tag{24}$$

$$r_{23} = -\mathrm{s4s5}, \mathrm{s4} = -r_{23}/s5 \tag{25}$$

$$\theta_4 = Atan2(s4, c4) \tag{26}$$

3D-5-RRRP

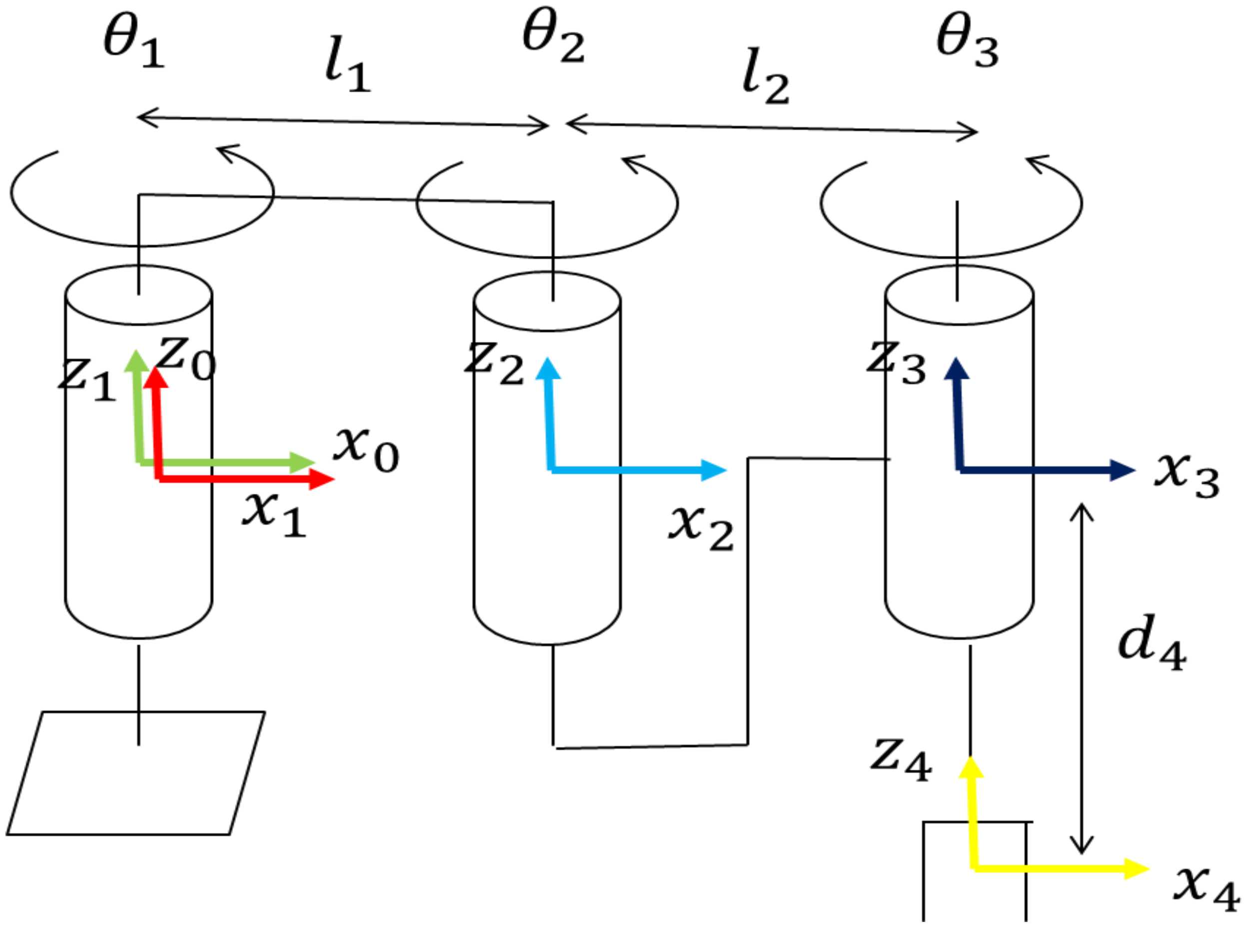

Fig.4 3D-5-RRRP

a. Analytical Method

From the lecture notes, the goal transformation matrix could be written as

$$ {}^0_T T = \begin{bmatrix} r_{11} & r_{12} & r_{13} & x \\ r_{21} & r_{22} & r_{23} & y \\ r_{31} & r_{32} & r_{33} & z \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{27} $$

From HW 2, we have derived the goal transformation matrix for 3D-1-RRR

$$ {}^0_T T = \begin{bmatrix} c123 & -s123 & 0 & l2c12 + l1c1 \\ s123 & c123 & 0 & l2s12 + l1s1 \\ 0 & 0 & 1 & -d4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{28} $$

From equation (27) and (28), d_4 could be solved first,

$$z = -d_4 \quad (29)$$

First solve for θ_2, Let $r = x^2 + y^2$

$$c2 = \frac{x^2+y^2-{l_2}^2-{l_1}^2}{2l_2l_1} \quad ,s2 = \sqrt{1-c2^2} \quad (30)$$

$$\theta_2 = Atan2(s2, c2) \quad (31)$$

Then solve for θ_1, Let

$$k1 = l1 + l2c2,\ \ k_2 = l2s2 \quad (32)$$

$$\theta_1 = Atan2(y, x) - Atan2(k2, k1) \quad (33)$$

The solve for θ_3

$$r_{11} = \mathrm{c123} = \mathrm{c3c12} - \mathrm{s3s12} = \mathrm{c3(c1c2} - \mathrm{s1s2)} - \mathrm{s3(c1s2} + \mathrm{c2s1)} \quad (34)$$

$$\mathrm{c3} = \frac{r_{11}+\mathrm{s3(c1s2+c2s1)}}{\mathrm{(c1c2-s1s2)}} \quad (35)$$

$$r_{21} = \mathrm{s123} = \mathrm{c3s12} + \mathrm{s3c12} = \mathrm{c3(c1s2} + \mathrm{c1s2)} + \mathrm{s3(c1c2} - \mathrm{s2s1)} \quad (36)$$

Insert equation (31) into (32)

$$s3 = \frac{r_{21}-r_{11}\frac{\mathrm{c1s2+c2s1}}{\mathrm{c1c2-s2s1}}}{1+(\frac{\mathrm{c1s2+c2s1}}{\mathrm{c1c2-s1s2}})^2} \quad (37)$$

Insert equation (33) into (31),

$$\mathrm{c3} = \frac{r_{11}+\frac{r_{21}-r_{11}\frac{\mathrm{c1s2+c2s1}}{\mathrm{c1c2-s2s1}}}{1+(\frac{\mathrm{c1s2+c2s1}}{\mathrm{c1c2-s1s2}})^2}*\mathrm{(c1s2+c2s1)}}{\mathrm{c1c2-s1s2}} \quad (38)$$

$$\theta_3 = Atan2(s3, c3) \quad (39)$$

b. Geometric Method

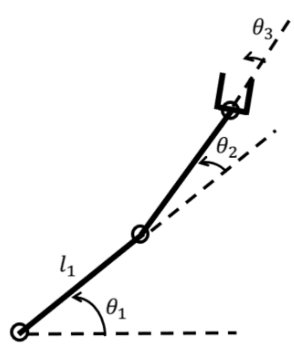

Fig.5 3D-5-RRRP

When we see from the top, we will have a similar picture with figure5 above.

We will use the same method solving θ_1, θ_2. The we will solve for θ_3

$$\theta_3 = \beta - \theta_1 - \theta_2 \quad (40)$$

$$\mathrm{c}\beta = c123 = r_{11} \text{ and } \mathrm{s}\beta = s123 = r_{21} \quad (41)$$

Solve for θ_3,

$$\theta_3 = Atan2(r_{21}, r_{11}) \quad (42)$$