

Robotic Arms Library











2D – 1 – RR (standard DH)





2D – 2 – RRR (modified DH)







2D – 2 – RRR (standard DH)







3D – 1 – RRR (modified DH)







3D – 1 – RRR (standard DH)















3D – 4 – RPP (Cylindrical Robot, standard DH)











3D – 5 – RRRP (modified DH)







3D – 5 – RRRP (standard DH)





3D - 6 - PRRR (modified DH)







3D – 6 – PRRR (standard DH)





3D – 6 – RRP-RRR (Modified DH) – Stanford Arm





3D – 6 – RRP-RRR (Standard DH) – Stanford Arm





3D – 6 – PPP-RRR (Modified DH)





If the link frame have been attached to the links according to our convention, the following definitions of the DH parameters are valid:

Standard form:

- a_i The distance from \hat{Z}_{i-1} to \hat{Z}_i measured along \widehat{X}_i
- α_i The angle between \hat{Z}_{i-1} and \hat{Z}_i measured about \hat{X}_i
- d_i The distance from \hat{X}_{i-1}^{+} to \hat{X}_i^{+} measured along \hat{Z}_{i-1}^{+}
- θ_i The angle between \hat{X}_{i-1} and \hat{X}_i measured about \hat{Z}_{i-1}

Modified form:

 $\begin{array}{l} a_{i-1} \text{ - The distance from } \hat{Z}_{i-1} \text{ to } \hat{Z}_i & \text{measured along } \hat{X}_{i-1} \\ \alpha_{i-1} \text{ - The angle between } \hat{Z}_{i-1} \text{ and } \hat{Z}_i & \text{measured about } \hat{X}_{i-1} \\ d_i & \text{ - The distance from } \hat{X}_{i-1} & \text{to } \hat{X}_i & \text{measured along } \hat{Z}_i \\ \theta_i & \text{ - The angle between } \hat{X}_{i-1} & \text{and } \hat{X}_i & \text{measured about } \hat{Z}_i \end{array}$

Note: $a_i \ge 0$ α_i d_i θ_i are signed quantities



DH Parameters – Standard / Modified Approach





For a given robot manipulator, one can always choose the frames $0, \ldots, n$ in such a way that the above two conditions are satisfied. In certain circumstances, this will require placing the origin o_i of frame i in a location that may not be intuitively satisfying, but typically this will not be the case. In reading the material below, it is important to keep in mind that the choices of the various coordinate frames are not unique, even when constrained by the requirements above. Thus, it is possible that different engineers will derive differing, but equally correct, coordinate frame assignments for the links of the robot. It is very important to note, however, that the end result (i.e.,





the matrix T_n^0 will be the same, regardless of the assignment of intermediate DH frames (assuming that the coordinate frames for link *n* coincide). We will begin by deriving the general procedure. We will then discuss various common special cases for which it is possible to further simplify the homogeneous transformation matrix.

To start, note that the choice of z_i is arbitrary. In particular, from Equation (3.13), we see that by choosing α_i and θ_i appropriately, we can obtain any arbitrary direction for z_i . Thus, for our first step, we assign the axes z_0, \ldots, z_{n-1} in an intuitively pleasing fashion. Specifically, we assign z_i to be the axis of actuation for joint i + 1. Thus, z_0 is the axis of actuation for joint 1, z_1 is the axis of actuation for joint 2, etc. There are two cases to consider: (i) if joint i+1 is revolute, z_i is the axis of revolution of joint i+1; (ii) if joint i+1 is prismatic, z_i is the axis of translation of joint i+1. At first it may seem a bit confusing to associate z_i with joint i+1, but recall that this satisfies the convention that we established above, namely that when joint i is actuated, link i and its attached frame, $o_i x_i y_i z_i$, experience a resulting motion.





Once we have established the z-axes for the links, we establish the base frame. The choice of a base frame is nearly arbitrary. We may choose the origin o_0 of the base frame to be any point on z_0 . We then choose x_0 , y_0 in any convenient manner so long as the resulting frame is right-handed. This sets up frame 0.

Once frame 0 has been established, we begin an iterative process in which we define frame i using frame i-1, beginning with frame 1. Figure 3.4 will be useful for understanding the process that we now describe.





 z_{i-1} , z_i are parallel. Note that in both cases (ii) and (iii) the axes z_{i-1} and z_i are coplanar. This situation is in fact quite common, as we will see in Section 3.2.3. We now consider each of these three cases.

(i) z_{i-1} and z_i are not coplanar: If z_{i-l} and z_i are not coplanar, then there exists a unique shortest line segment from z_{i-1} to z_i , perpendicular to both z_{i-1} to z_i . This line segment defines x_i , and the point where it intersects z_i is the origin o_i . By construction, both conditions (DH1) and (DH2) are satisfied and the vector from o_{i-1} to o_i is a linear combination of z_{i-1} and x_i . The specification of frame *i* is completed by choosing the axis y_i to form a right-handed frame. Since assumptions (DH1) and (DH2) are satisfied, the homogeneous transformation matrix A_i is of the form given in Equation (3.10).





(ii) z_{i-1} is parallel to z_i : If the axes z_{i-1} and z_i are parallel, then there are infinitely many common normals between them and condition (DH1) does not specify x_i completely. In this case we are free to choose the origin o_i anywhere along z_i . One often chooses o_i to simplify the resulting equations. The axis x_i is then chosen either to be directed from o_i toward z_{i-1} , along the common normal, or as the opposite of this vector. A common method for choosing o_i is to choose the normal that passes through o_{i-1} as the x_i axis; o_i is then the point at which this normal intersects z_i . In this case, d_i would be equal to zero. Once x_i is fixed, y_i is determined, as usual by the right hand rule. Since the axes z_{i-1} and z_i are parallel, α_i will be zero in this case.



(iii) z_{i-1} intersects z_i : In this case x_i is chosen normal to the plane formed by z_i and z_{i-1} . The positive direction of x_i is arbitrary. The most natural choice for the origin o_i in this case is at the point of intersection of z_i and z_{i-1} . However, any convenient point along the axis z_i suffices. Note that in this case the parameter a_i will be zero.

This constructive procedure works for frames $0, \ldots, n-1$ in an *n*-link robot. To complete the construction it is necessary to specify frame *n*. The final coordinate system $o_n x_n y_n z_n$ is commonly referred to as the **end effector** or **tool frame** (see Figure 3.5). The origin o_n is most often placed symmetrically between the fingers of the gripper. The unit vectors along the x_n, y_n , and z_n axes are labeled as n, s, and a, respectively. The terminology arises from the fact that the direction a is the **approach** direction, in the sense that the gripper typically approaches an object along the a direction. Similarly the s direction is the **sliding** direction, the direction along which



the fingers of the gripper slide to open and close, and n is the direction **normal** to the plane formed by a and s.



Figure 3.5: Tool frame assignment.

In most contemporary robots the final joint motion is a rotation of the end effector by θ_n and the final two joint axes, z_{n-1} and z_n , coincide. In this case, the transformation between the final two coordinate frames is a translation along z_{n-1} by a distance d_n followed (or preceded) by a rotation of θ_n about z_{n-1} . This is an important observation that will simplify the computation of the inverse kinematics in the next section.

Finally, note the following important fact. In all cases, whether the joint in question is revolute or prismatic, the quantities a_i and α_i are always constant for all *i* and are characteristic of the manipulator. If joint *i* is prismatic, then θ_i is also a constant, while d_i is the *i*th joint variable. Similarly, if joint *i* is revolute, then d_i is constant and θ_i is the *i*th joint variable.