Robotic Arms Library
2D – 1 – RR (modified DH)
2D – 1 – RR (standard DH)
2D – 2 – RRR (modified DH)
2D – 2 – RRR (standard DH)

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3D – 1 – RRR (modified DH)
3D – 1 – RRR (standard DH)
3D – 2 – RRP (modified DH)
3D – 2 – RRP (standard DH)
3D – 4 – RPP (Cylindrical Robot, modified DH)

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3D – 4 – RPP (Cylindrical Robot, standard DH)
3D – 4 – RRR (Spherical Wrist, modified DH)

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3D – 4 – RRR (Spherical Wrist, standard DH)

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3D – 5 – RRRP (modified DH)
3D – 5 – RRRP (standard DH)
3D – 6 – PRRR (modified DH)
3D – 6 – PRRR (standard DH)
3D – 6 – RRP-RRR (Modified DH) – Stanford Arm
Note: The transformation from frame 6 to the tool frame \{T\} and the F/T sensor frame \{S\} doesn't follow the DH parameters. Define the transformation and rotations as needed.
DH Parameters - Summary

If the link frame have been attached to the links according to our convention, the following definitions of the DH parameters are valid:

Standard form:

\( a_i \) - The distance from \( \hat{Z}_{i-1} \) to \( \hat{Z}_i \) measured along \( \hat{X}_i \)

\( \alpha_i \) - The angle between \( \hat{Z}_{i-1} \) and \( \hat{Z}_i \) measured about \( \hat{X}_i \)

\( d_i \) - The distance from \( \hat{X}_{i-1} \) to \( \hat{X}_i \) measured along \( \hat{Z}_{i-1} \)

\( \theta_i \) - The angle between \( \hat{X}_{i-1} \) and \( \hat{X}_i \) measured about \( \hat{Z}_{i-1} \)

Modified form:

\( a_{i-1} \) - The distance from \( \hat{Z}_{i-1} \) to \( \hat{Z}_i \) measured along \( \hat{X}_{i-1} \)

\( \alpha_{i-1} \) - The angle between \( \hat{Z}_{i-1} \) and \( \hat{Z}_i \) measured about \( \hat{X}_{i-1} \)

\( d_i \) - The distance from \( \hat{X}_{i-1} \) to \( \hat{X}_i \) measured along \( \hat{Z}_i \)

\( \theta_i \) - The angle between \( \hat{X}_{i-1} \) and \( \hat{X}_i \) measured about \( \hat{Z}_i \)

**Note:** \( a_i \geq 0 \), \( \alpha_i \), \( d_i \), \( \theta_i \) are signed quantities
DH Parameters – Standard / Modified Approach

(a) Standard form

(b) Modified form
For a given robot manipulator, one can always choose the frames $0, \ldots, n$ in such a way that the above two conditions are satisfied. In certain circumstances, this will require placing the origin $o_i$ of frame $i$ in a location that may not be intuitively satisfying, but typically this will not be the case. In reading the material below, it is important to keep in mind that the choices of the various coordinate frames are not unique, even when constrained by the requirements above. Thus, it is possible that different engineers will derive differing, but equally correct, coordinate frame assignments for the links of the robot. It is very important to note, however, that the end result (i.e.,
DH Parameters – Standard Approach – Assigning Coordinate System

the matrix $T_0^n$) will be the same, regardless of the assignment of intermediate DH frames (assuming that the coordinate frames for link $n$ coincide). We will begin by deriving the general procedure. We will then discuss various common special cases for which it is possible to further simplify the homogeneous transformation matrix.

To start, note that the choice of $z_i$ is arbitrary. In particular, from Equation (3.13), we see that by choosing $a_i$ and $\theta_i$ appropriately, we can obtain any arbitrary direction for $z_i$. Thus, for our first step, we assign the axes $z_0, \ldots, z_{n-1}$ in an intuitively pleasing fashion. Specifically, we assign $z_i$ to be the axis of actuation for joint $i + 1$. Thus, $z_0$ is the axis of actuation for joint 1, $z_1$ is the axis of actuation for joint 2, etc. There are two cases to consider: (i) if joint $i + 1$ is revolute, $z_i$ is the axis of revolution of joint $i + 1$; (ii) if joint $i + 1$ is prismatic, $z_i$ is the axis of translation of joint $i + 1$. At first it may seem a bit confusing to associate $z_i$ with joint $i + 1$, but recall that this satisfies the convention that we established above, namely that when joint $i$ is actuated, link $i$ and its attached frame, $o-x_iy_iz_i$, experience a resulting motion.
Once we have established the z-axes for the links, we establish the base frame. The choice of a base frame is nearly arbitrary. We may choose the origin $o_0$ of the base frame to be any point on $z_0$. We then choose $x_0, y_0$ in any convenient manner so long as the resulting frame is right-handed. This sets up frame 0.

Once frame 0 has been established, we begin an iterative process in which we define frame $i$ using frame $i-1$, beginning with frame 1. Figure 3.4 will be useful for understanding the process that we now describe.

![Figure 3.4: Denavit-Hartenberg frame assignment.](image)

In order to set up frame $i$ it is convenient to consider three cases: (i) the axes $z_{i-1}, z_i$ are not coplanar, (ii) the axes $z_{i-1}, z_i$ intersect, (iii) the axes
$z_{i-1}$, $z_i$ are parallel. Note that in both cases (ii) and (iii) the axes $z_{i-1}$ and $z_i$ are coplanar. This situation is in fact quite common, as we will see in Section 3.2.3. We now consider each of these three cases.

(i) $z_{i-1}$ and $z_i$ are not coplanar: If $z_{i-1}$ and $z_i$ are not coplanar, then there exists a unique shortest line segment from $z_{i-1}$ to $z_i$, perpendicular to both $z_{i-1}$ to $z_i$. This line segment defines $x_i$, and the point where it intersects $z_i$ is the origin $o_i$. By construction, both conditions (DH1) and (DH2) are satisfied and the vector from $o_{i-1}$ to $o_i$ is a linear combination of $z_{i-1}$ and $x_i$. The specification of frame $i$ is completed by choosing the axis $y_i$ to form a right-handed frame. Since assumptions (DH1) and (DH2) are satisfied, the homogeneous transformation matrix $A_i$ is of the form given in Equation (3.10).
(ii) $z_{i-1}$ is parallel to $z_i$: If the axes $z_{i-1}$ and $z_i$ are parallel, then there are infinitely many common normals between them and condition (DH1) does not specify $x_i$ completely. In this case we are free to choose the origin $o_i$ anywhere along $z_i$. One often chooses $o_i$ to simplify the resulting equations. The axis $x_i$ is then chosen either to be directed from $o_i$ toward $z_{i-1}$, along the common normal, or as the opposite of this vector. A common method for choosing $o_i$ is to choose the normal that passes through $o_{i-1}$ as the $x_i$ axis; $o_i$ is then the point at which this normal intersects $z_i$. In this case, $d_i$ would be equal to zero. Once $x_i$ is fixed, $y_i$ is determined, as usual by the right hand rule. Since the axes $z_{i-1}$ and $z_i$ are parallel, $\alpha_i$ will be zero in this case.
(iii) $z_{i-1}$ intersects $z_i$: In this case $x_i$ is chosen normal to the plane formed by $z_i$ and $z_{i-1}$. The positive direction of $x_i$ is arbitrary. The most natural choice for the origin $o_i$ in this case is at the point of intersection of $z_i$ and $z_{i-1}$. However, any convenient point along the axis $z_i$ suffices. Note that in this case the parameter $a_i$ will be zero.

This constructive procedure works for frames $0, \ldots, n - 1$ in an $n$-link robot. To complete the construction it is necessary to specify frame $n$. The final coordinate system $o_n x_n y_n z_n$ is commonly referred to as the **end effector** or **tool frame** (see Figure 3.5). The origin $o_n$ is most often placed symmetrically between the fingers of the gripper. The unit vectors along the $x_n$, $y_n$, and $z_n$ axes are labeled as $n$, $s$, and $a$, respectively. The terminology arises from the fact that the direction $a$ is the **approach** direction, in the sense that the gripper typically approaches an object along the $a$ direction. Similarly the $s$ direction is the **sliding** direction, the direction along which
the fingers of the gripper slide to open and close, and \( n \) is the direction normal to the plane formed by \( a \) and \( s \).

\[
\begin{align*}
    y_n &\equiv s \\
    z_n &\equiv a \\
    x_n &\equiv n
\end{align*}
\]

Figure 3.5: Tool frame assignment.

In most contemporary robots the final joint motion is a rotation of the end effector by \( \theta_n \) and the final two joint axes, \( z_{n-1} \) and \( z_n \), coincide. In this case, the transformation between the final two coordinate frames is a translation along \( z_{n-1} \) by a distance \( d_n \), followed (or preceded) by a rotation of \( \theta_n \) about \( z_{n-1} \). This is an important observation that will simplify the computation of the inverse kinematics in the next section.

Finally, note the following important fact. In all cases, whether the joint in question is revolute or prismatic, the quantities \( \alpha_i \) and \( \alpha_i \) are always constant for all \( i \) and are characteristic of the manipulator. If joint \( i \) is prismatic, then \( \theta_i \) is also a constant, while \( d_i \) is the \( i^{th} \) joint variable. Similarly, if joint \( i \) is revolute, then \( d_i \) is constant and \( \theta_i \) is the \( i^{th} \) joint variable.