a. Analytical Method

From the lecture notes, the goal transformation matrix could be written as

$$
\begin{align*}
0_T &= \begin{bmatrix}
    r_{11} & r_{12} & r_{13} & x_1 \\
    r_{21} & r_{22} & r_{23} & y \\
    r_{31} & r_{32} & r_{33} & z \\
    0 & 0 & 0 & 1
\end{bmatrix}
\end{align*}
$$

(1)

From HW 2, we have derived the goal transformation matrix for 3D-1-RRR

$$
\begin{align*}
0_T &= \begin{bmatrix}
    c1c23 & -c1s23 & -s1 & l3c1c23 + l2c1c2 \\
    s1c23 & -s1s23 & c1 & l3s1c23 + l2s1c2 \\
    -s23 & -c23 & 0 & -l3s23 - l2s2 \\
    0 & 0 & 0 & 1
\end{bmatrix}
\end{align*}

(2)

From equation (1) and (2)
\[ r_{13} = -s_1, r_{23} = c_1 \quad (3) \]

Thus, we could solve for \( \theta_1 \),

\[ \theta_1 = \text{Atan2}(-r_{13}, r_{23}) \quad (4) \]

Then, solve for \( \theta_2, \theta_3 \). Let \( r = x^2 + y^2 + z^2 \)

\[ c_3 = \frac{x^2 + y^2 + z^2 - l_2^2 - l_3^2}{2l_2l_3}, \quad s_3 = \sqrt{1 - c_3^2} \quad (5) \]

Solve for \( \theta_3 \)

\[ \theta_3 = \text{Atan2}(s_3, c_3) \quad (6) \]

Let

\[ k_1 = l_2 + l_3c_3, \quad k_2 = l_3s_3 \quad (7) \]

\[ \theta_2 = \text{Atan2}\left(z, \sqrt{x^2 + y^2}\right) - \text{Atan2}(k_2, k_1) \quad (8) \]

b. Geometric Method

When we see from the top, we will get a similar figure with fig.2.

From Fig.2, we could first solve for \( \theta_1 \),

\[ \theta_1 = \text{Atan2}(s_1, c_1) \quad (9) \]

The solve for \( \theta_3, \theta_2 \)

\[ r = x^2 + y^2 + z^2 \quad (10) \]

\[ c_3 = \frac{x^2 + y^2 + z^2 - l_2^2 - l_3^2}{2l_2l_3}, \quad s_3 = \sqrt{1 - c_3^2} \quad (11) \]

\[ \theta_3 = \text{Atan2}(s_3, c_3) \quad (12) \]
\[ \theta_2 = \beta \pm \alpha, \quad \beta = \text{Atan2}(z, \sqrt{x^2 + y^2}) \]  

(13)

\[ r \cos(\alpha) = l_1 + l_2 \cos(\theta_3), \quad r \sin(\alpha) = l_2 \sin(\theta_3) \]

(14)

\[ \alpha = \text{Atan2}\left(\frac{l_2 \sin(\theta_2)}{r}, \frac{l_1 + l_2 \cos(\theta_2)}{r}\right) \]

(15)

Thus, we could solve for \( \theta_2 \)

\[ \theta_2 = \beta - \alpha \]

(16)

From the notes, we will have two position for \( \theta_1 \) and two for \( \theta_2, \theta_3 \)
a. Analytical Method

From the lecture notes, the goal transformation matrix could be written as

$$\begin{bmatrix} r_{11} & r_{12} & r_{13} & x \\ r_{21} & r_{22} & r_{23} & y \\ r_{31} & r_{32} & r_{33} & z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ \hspace{1cm} (17)

From HW 2, we have derived the goal transformation matrix for 3D-1-RRR

$$\begin{bmatrix} c4c5c6 - s4s6 & -c4c5s6 - s4c6 & -c4s5 & 0 \\ s4c5c6 + c4s6 & -s4c5s6 + c4c6 & -s4s5 & 0 \\ s5c6 & -s5s6 & c5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ \hspace{1cm} (18)

From equation (17) and (18), $\theta_5$ could be solved first

$$r_{33} = c5, s5 = \sqrt{1 - c5^2} \hspace{1cm} (19)$$

$$\theta_5 = \text{Atan2}(s5, c5) \hspace{1cm} (20)$$
Then solve for $\theta_6$ next

\[ r_{31} = c6s5, c6 = r_{31}/s5 \]  \hspace{1cm} (21)
\[ r_{32} = -s6s5, s6 = -r_{32}/s5 \]  \hspace{1cm} (22)
\[ \theta_6 = \text{Atan2}(s6, c6) \]  \hspace{1cm} (23)

The solve for $\theta_4$ next

\[ r_{13} = -c4s5, c4 = -r_{13}/s5 \]  \hspace{1cm} (24)
\[ r_{23} = -s4s5, s4 = -r_{23}/s5 \]  \hspace{1cm} (25)
\[ \theta_4 = \text{Atan2}(s4, c4) \]  \hspace{1cm} (26)
a. Analytical Method

From the lecture notes, the goal transformation matrix could be written as

\[ T_0 \rightarrow \begin{bmatrix} r_{11} & r_{12} & r_{13} & x \\ r_{21} & r_{22} & r_{23} & y \\ r_{31} & r_{32} & r_{33} & z \\ 0 & 0 & 0 & 1 \end{bmatrix} \]  \tag{27}

From HW 2, we have derived the goal transformation matrix for 3D-1-RRR

\[ T_0 \rightarrow \begin{bmatrix} c_{123} & -s_{123} & 0 & l_2c_{12} + l_1c_1 \\ s_{123} & c_{123} & 0 & l_2s_{12} + l_1s_1 \\ 0 & 0 & 1 & -d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \]  \tag{28}

From equation (27) and (28), \( d_4 \) could be solved first,
\[ z = -d_4 \]  

First solve for \( \theta_2 \), Let \( r = x^2 + y^2 \)

\[
c_2 = \frac{x^2 + y^2 - l_2^2 - l_1^2}{2l_2l_1}, \quad s_2 = \sqrt{1 - c_2^2} \tag{30}
\]

\[ \theta_2 = \text{Atan}2(s_2, c_2) \tag{31} \]

Then solve for \( \theta_1 \), Let

\[ k_1 = l_1 + l_2c_2, \quad k_2 = l_2s_2 \tag{32} \]

\[ \theta_1 = \text{Atan}2(y, x) - \text{Atan}2(k_2, k_1) \tag{33} \]

The solve for \( \theta_3 \)

\[ r_{11} = c123 = c3c12 - s3s12 = c3(c1c2 - s1s2) - s3(c1s2 + c2s1) \tag{34} \]

\[ c_3 = \frac{r_{11} + s3(c1s2 + c2s1)}{(c1c2 - s1s2)} \tag{35} \]

\[ r_{21} = s123 = c3s12 + s3c12 = c3(c1s2 + c1s2) + s3(c1c2 - s2s1) \tag{36} \]

Insert equation (31) into (32)

\[ s_3 = \frac{r_{21} - r_{11}c1c2 - s2s1}{1 + (c1s2 + c2s1)^2} \tag{37} \]

Insert equation (33) into (31),

\[ c_3 = \frac{c1s2 + c2s1}{1 + (c1c2 - s1s2)^2} \tag{38} \]

\[ \theta_3 = \text{Atan}2(s_3, c_3) \tag{39} \]

b. Geometric Method

![Fig.5 3D-5-RRRP](image-url)
When we see from the top, we will have a similar picture with figure 5 above.

We will use the same method solving $\theta_1, \theta_2$. The we will solve for $\theta_3$

$$\theta_3 = \beta - \theta_1 - \theta_2$$  \hspace{1cm} (40)

$$c\beta = c123 = r_{11} \text{ and } s\beta = s123 = r_{21}$$  \hspace{1cm} (41)

Solve for $\theta_3$,

$$\theta_3 = \text{Atan2}(r_{21}, r_{11})$$  \hspace{1cm} (42)