MAE 263D HW2 Solutions

Refer to HW1 for Forward Kinematics solutions

$$\begin{bmatrix} S_\alpha = \sin\theta_\alpha \\ C_\alpha = \cos\theta_\alpha \end{bmatrix}$$

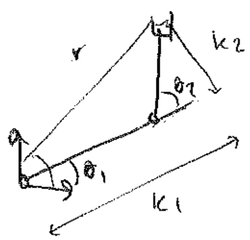
1) 2D-1-RR

$${}^0_3T = \begin{bmatrix} r_{11} & r_{12} & r_{13} & x \\ r_{21} & r_{22} & r_{23} & y \\ r_{31} & r_{32} & r_{33} & z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$x = C_{12}L_2 + C_1L_1$$
$$y = S_{12}L_2 + S_1L_1$$
$$z = 0$$

$$\Rightarrow x^2+y^2 = L_1^2 + L_2^2 + 2L_1L_2C_2$$

$$\Rightarrow C_2 = \frac{x^2+y^2-L_1^2-L_2^2}{2L_1L_2}, \quad S_2 = \pm\sqrt{1-C_2^2} \Rightarrow \underline{\theta_2 = \mathrm{Atan2}(S_2, C_2)}$$

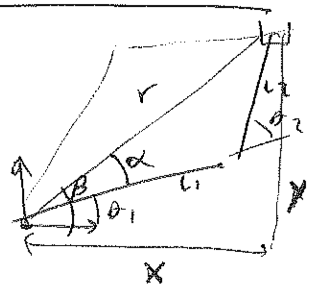


$$\Rightarrow k_1 = L_1 + L_2C_2$$
$$k_2 = L_2S_2$$

$$\Rightarrow \underline{\theta_1 = \mathrm{Atan2}(y,x) - \mathrm{Atan2}(k_2, k_1)}$$

2 Solutions

Geometric:



$$\Rightarrow r^2 = x^2+y^2 = L_1^2 + L_2^2 + 2L_1L_2C_2$$

$$\Rightarrow C_2 = \frac{x^2+y^2-L_1^2-L_2^2}{2L_1L_2}, \quad S_2 = \pm\sqrt{1-C_2^2} \Rightarrow \underline{\theta_2 = \mathrm{Atan2}(S_2, C_2)}$$

$$\theta_1 = \beta \pm \alpha, \quad \beta = \mathrm{Atan2}(y, x)$$

$$r\cos\alpha = L_1 + L_2\cos\theta_2$$
$$r\sin\alpha = L_2\sin\theta_2$$
$$\Rightarrow \underline{\theta_1 = \beta - \alpha}$$

$$\Rightarrow \alpha = \mathrm{Atan2}\left(\frac{L_2\sin\theta_2}{\sqrt{x^2+y^2}}, \frac{L_1+L_2\cos\theta_2}{\sqrt{x^2+y^2}}\right)$$

2 Solutions

2)

2D - 2 - RRR

$\longrightarrow \theta_1, \theta_2$ are identical to problem #1

$${}^0_3T = \begin{bmatrix} r_{11} & r_{12} & r_{13} & x \\ r_{21} & r_{22} & r_{23} & y \\ r_{31} & r_{32} & r_{33} & z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$r_{11} = C_3(c_1c_2 - s_1s_2) - S_3(c_1s_2 + c_2s_1)$$

$$\Rightarrow C_3 = \left[\frac{r_{11} + S_3(c_1s_2 + c_2s_1)}{(c_1c_2 - s_1s_2)}\right] \longrightarrow A$$

$$r_{21} = C_3(c_1s_2 + c_2s_1) + S_3(c_1c_2 - s_1s_2)$$

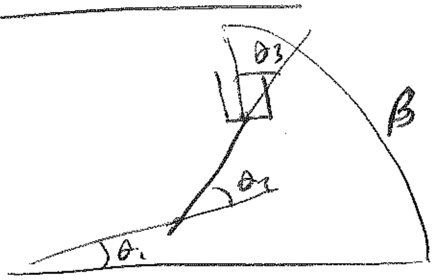
$$\Rightarrow \underline{r_{21} = A\,(c_1s_2 + c_2s_1) + S_3(c_1c_2 - s_1s_2)}$$

$$\Rightarrow S_3 = \left[\frac{r_{21} - r_{11}\dfrac{(c_1s_2 + c_2s_1)}{(c_1c_2 - s_1s_2)}}{1 + \dfrac{(c_1s_2 + c_2s_1)^2}{(c_1c_2 - s_1s_2)}}\right] \longrightarrow B, \quad C_3 = \frac{r_{11} + B\,(c_1s_2 + c_2s_1)}{(c_1c_2 - s_1s_2)}$$

$$\underline{\theta_3 = \text{Atan2}(S_3, C_3)}$$

<u>2 solutions</u>

<u>Geometric:</u>



$$\Rightarrow \theta_3 = \beta - \theta_2 - \theta_1$$

$$c\beta = C_{123} = r_{11}$$

$$s\beta = S_{123} = r_{21}$$

3)

3D - 1 - RRR

$$ {}^{0}_{4}T = \begin{bmatrix} r_{11} & r_{12} & r_{13} & x \\ r_{21} & r_{22} & r_{23} & y \\ r_{31} & r_{32} & r_{33} & z \\ 0 & 0 & 0 & 1 \end{bmatrix} \Rightarrow \begin{matrix} r_{13} = -S_1 \\ r_{23} = C_1 \end{matrix} \Rightarrow \underline{\theta_1 = \text{Atan2}(-r_{13}, r_{23})} $$

$\longrightarrow \theta_2, \theta_3$ Similar to problem #1 only differences: put L_2 for L_1, put L_3 for L_2, $r^2 = x^2+y^2+z^2$

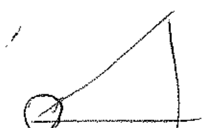
$$ \Rightarrow C_3 = \frac{x^2+y^2-L_2^2-L_3^2}{2L_2L_3}, \quad S_3 = \pm\sqrt{1-C_3^2} \Rightarrow \underline{\theta_3 = \text{Atan2}(S_3, C_3)} $$

$$ \underline{\theta_1 = \text{Atan2}\left(z, \sqrt{x^2+y^2}\right) - \text{Atan2}(k_2, k_1)} $$

$$ k_2 = L_3 S_3, \quad k_1 = L_2 + L_3 C_3 $$

Geometric:

$\theta_1 = \text{Atan2}(S_1, C_1)$



Visible top view y and x

θ_2, θ_3 Similar to problem #1

4 solutions

2 positions θ_1

2 positions for θ_2 and θ_3 together

4)

3D - 2 - RRP (Stanford Arm)

$$ {}^{0}_{4}T = \begin{pmatrix} r_{11} & r_{12} & r_{13} & x \\ r_{21} & r_{22} & r_{23} & y \\ r_{31} & r_{32} & r_{33} & z \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

$\Rightarrow r_{12} = -S_1 , r_{22} = C_1$

$\underline{\theta_1 = \text{Atan2}(-r_{12}, r_{22})}$

$\Rightarrow r_{31} = S_2 , r_{33} = C_2$

$\underline{\theta_2 = \text{Atan2}(r_{31}, r_{33})}$

$z = d_3 C_2 \Rightarrow \underline{d_3 = \frac{z}{C_2}}$

<u>Multiple Solutions</u>

5)

3D-4 - RPP (Cylindrical Robot)

$$ {}^{0}_{3}T = \begin{bmatrix} r_{11} & r_{12} & r_{13} & x \\ r_{21} & r_{22} & r_{23} & y \\ r_{31} & r_{32} & r_{33} & z \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$\Rightarrow r_{13} = c_1 , \ r_{23} = s_1$

$\theta_1 = \mathrm{Atan2}(r_{23}, r_{13})$

$d_2 = z$

$x = d_3 c_1 \Rightarrow d_3 = \dfrac{x}{c_1}$

One solution

(Given $d_{2,3}$ cannot be negative)

Geometric:

$d_2 = z$

$\theta_1 = \mathrm{Atan2}(y, x)$

$d_3 = \sqrt{x^2 + y^2}$

6)

3D - 4 - RRR (Spherical Wrist)

$$
{}^{3}_{6}T = \begin{bmatrix} r_{11} & r_{12} & r_{13} & x \\ r_{21} & r_{22} & r_{23} & y \\ r_{31} & r_{32} & r_{33} & z \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$\Rightarrow r_{33} = C_5$, $S_5 = \pm\sqrt{1 - C_5^2}$

$\theta_5 = \text{Atan2}(S_5, C_5)$

$\Rightarrow r_{31} = C_6 S_5 \Rightarrow C_6 = \dfrac{r_{31}}{S_5}$, $r_{32} = -S_6 S_5 \Rightarrow S_6 = \dfrac{-r_{32}}{S_5}$

$\Rightarrow \theta_6 = \text{Atan2}(S_6, C_6)$

$\Rightarrow r_{13} = -C_4 S_5 \Rightarrow C_4 = \dfrac{-r_{13}}{S_5}$, $r_{23} = -S_4 S_5 \Rightarrow S_4 = \dfrac{-r_{23}}{S_5}$

$\Rightarrow \theta_4 = \text{Atan2}(S_4, C_4)$

Multiple Solutions can exist

7)

3D-5-RRRP

$\Rightarrow z = -d_4 \Rightarrow \underline{d_4 = -z}$

$\rightarrow \theta_1, \theta_2, \theta_3$ identical to problem # 2

8)

3D - 6 -PRRR

$\Rightarrow Z = d_1 + L_1 - L_4 \Rightarrow \underline{d_1 = Z - L_1 + L_4}$

$\rightarrow$ Rest is similar to problem #7