Manipulator Dynamics 3

Iterative Newton-Euler Equations - Solution Procedure

- **Step 1 -** Calculate the link velocities and accelerations iteratively from the robot's base to the end effector

- **Step 2 -** Write the Newton and Euler equations for each link.

Outward iterations: $i : 0 \rightarrow 5$

$$^{i+1}\omega_{i+1} = {}^{i+1}_{i}R\,{}^{i}\omega_i + \dot{\theta}_{i+1}\,{}^{i+1}\hat{Z}_{i+1},$$

$$^{i+1}\dot{\omega}_{i+1} = {}^{i+1}_{i}R\,{}^{i}\dot{\omega}_i + {}^{i+1}_{i}R\,{}^{i}\omega_i \times \dot{\theta}_{i+1}\,{}^{i+1}\hat{Z}_{i+1} + \ddot{\theta}_{i+1}\,{}^{i+1}\hat{Z}_{i+1},$$

$$^{i+1}\dot{v}_{i+1} = {}^{i+1}_{i}R\left({}^{i}\dot{\omega}_i \times {}^{i}P_{i+1} + {}^{i}\omega_i \times \left({}^{i}\omega_i \times {}^{i}P_{i+1}\right) + {}^{i}\dot{v}_i\right),$$

$$^{i+1}\dot{v}_{C_{i+1}} = {}^{i+1}\dot{\omega}_{i+1} \times {}^{i+1}P_{C_{i+1}} + {}^{i+1}\omega_{i+1} \times \left({}^{i+1}\omega_{i+1} \times {}^{i+1}P_{C_{i+1}}\right) + {}^{i+1}\dot{v}_{i+1},$$

$$^{i+1}F_{i+1} = m_{i+1}\,{}^{i+1}\dot{v}_{C_{i+1}},$$

$$^{i+1}N_{i+1} = {}^{C_{i+1}}I_{i+1}\,{}^{i+1}\dot{\omega}_{i+1} + {}^{i+1}\omega_{i+1} \times {}^{C_{i+1}}I_{i+1}\,{}^{i+1}\omega_{i+1}.$$

Iterative Newton-Euler Equations - Solution Procedure

- **Step 3 -** Use the forces and torques generated by interacting with the environment (that is, tools, work stations, parts etc.) in calculating the joint torques from the end effector to the robot's base.

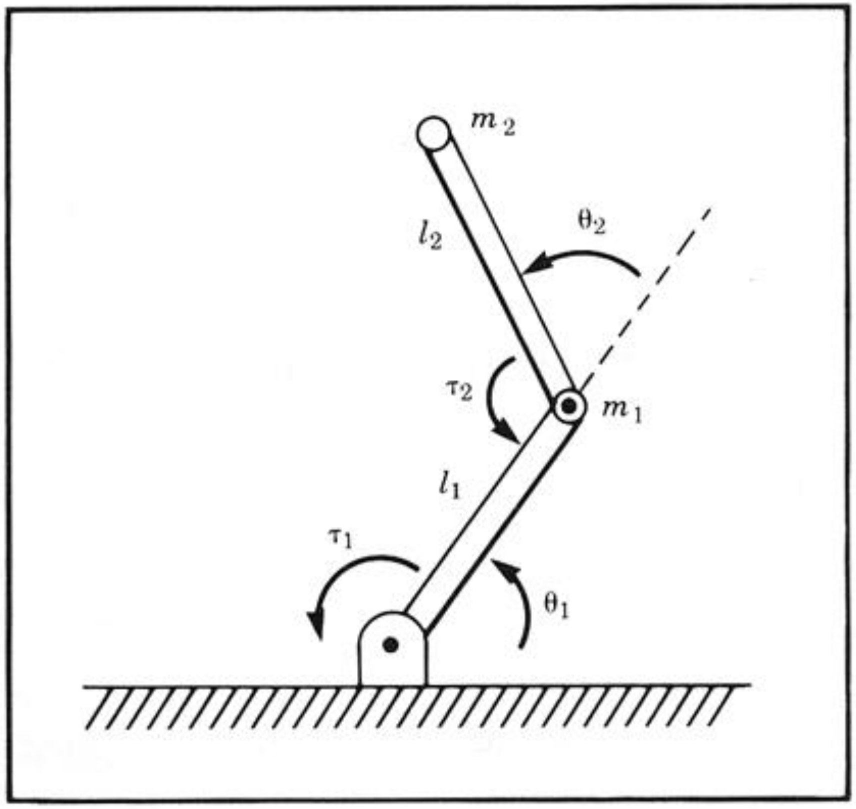
Inward iterations: $i : 6 \rightarrow 1$

$$
\begin{aligned}
{}^{i}f_i &= {}^{i}_{i+1}R\, {}^{i+1}f_{i+1} + {}^{i}F_i, \\
{}^{i}n_i &= {}^{i}N_i + {}^{i}_{i+1}R\, {}^{i+1}n_{i+1} + {}^{i}P_{C_i} \times {}^{i}F_i \\
&\quad + {}^{i}P_{i+1} \times {}^{i}_{i+1}R\, {}^{i+1}f_{i+1}, \\
\tau_i &= {}^{i}n_i^T\, {}^{i}\hat{Z}_i.
\end{aligned}
$$

Iterative Newton-Euler Equations - Solution Procedure

- **Error Checking -** Check the units of each term in the resulting equations
- **Gravity Effect** - The effect of gravity can be included by setting $^{0}\dot{v}_{0} = g$. This is the equivalent to saying that the base of the robot is accelerating upward at 1 g. The result of this accelerating is the same as accelerating all the links individually as gravity does.

Iterative Newton-Euler Equations - 2R Robot Example



Iterative Newton-Euler Equations - 2R Robot Example

- Outward Iteration $i = 0$

$$ {}^{i+1}\omega_{i+1} = {}^{i+1}_{i}R \; {}^{i}\omega_{i} + \dot{\theta}_{i+1} \, {}^{i+1}\hat{Z}_{i+1} $$

Iterative Newton-Euler Equations - 2R Robot Example

Iterative Newton-Euler Equations - 2R Robot Example

- Outward Iteration $i = 0$

$$ {}^{i+1}\dot{\omega}_{i+1} = {}^{i+1}_{i}R\,{}^{i}\dot{\omega}_{i} + {}^{i+1}_{i}R\,{}^{i}\omega_{i} \times \dot{\theta}_{i+1}\,{}^{i+1}\hat{Z}_{i+1} + \ddot{\theta}_{i+1}\,{}^{i+1}\hat{Z}_{i+1} $$

Iterative Newton-Euler Equations - 2R Robot Example

Iterative Newton-Euler Equations - 2R Robot Example

- Outward Iteration $i = 0$

$$ {}^{i+1}\dot{v}_{i+1} = {}^{i+1}_{i}R\left({}^{i}\dot{\omega}_{i} \times {}^{i}P_{i+1} + {}^{i}\omega_{i} \times \left({}^{i}\omega_{i} \times {}^{i}P_{i+1}\right) + {}^{i}\dot{v}_{i}\right) $$

Iterative Newton-Euler Equations - 2R Robot Example

Iterative Newton-Euler Equations - 2R Robot Example

- Outward Iteration $i = 0$

$$^{i+1}\dot{v}_{C_{i+1}} = {}^{i+1}\dot{\omega}_{i+1} \times {}^{i+1}P_{C_{i+1}} + {}^{i+1}\omega_{i+1} \times \left({}^{i+1}\omega_{i+1} \times {}^{i+1}P_{C_{i+1}} \right) + {}^{i+1}\dot{v}_{i+1}$$

Iterative Newton-Euler Equations - 2R Robot Example

Iterative Newton-Euler Equations - 2R Robot Example

- Outward Iteration $i = 0$

$$^{i+1}F_{i+1} = m_{i+1} \, ^{i+1}\dot{v}_{C_{i+1}}$$

Iterative Newton-Euler Equations - 2R Robot Example

- Outward Iteration $i = 0$

$$^{i+1}N_{i+1} = {}^{C_{i+1}}I_{i+1}\,{}^{i+1}\dot{\omega}_{i+1} + {}^{i+1}\omega_{i+1} \times {}^{C_{i+1}}I_{i+1}\,{}^{i+1}\omega_{i+1}$$

Iterative Newton-Euler Equations - 2R Robot Example

Iterative Newton-Euler Equations - 2R Robot Example

- Outward Iteration $i = 1$

$$^{i+1}\omega_{i+1} = {}^{i+1}_{i}R\,{}^{i}\omega_i + \dot{\theta}_{i+1}\,{}^{i+1}\hat{Z}_{i+1}$$

Iterative Newton-Euler Equations - 2R Robot Example

Iterative Newton-Euler Equations - 2R Robot Example

- Outward Iteration $i = 1$

$$ {}^{i+1}\dot{\omega}_{i+1} = {}^{i+1}_{i}R\,{}^{i}\dot{\omega}_{i} + {}^{i+1}_{i}R\,{}^{i}\omega_{i} \times \dot{\theta}_{i+1}\,{}^{i+1}\hat{Z}_{i+1} + \ddot{\theta}_{i+1}\,{}^{i+1}\hat{Z}_{i+1} $$

Iterative Newton-Euler Equations - 2R Robot Example

Iterative Newton-Euler Equations - 2R Robot Example

- Outward Iteration $i=1$

$${}^{i+1}\dot{v}_{i+1} = {}^{i+1}_{i}R\left({}^{i}\dot{\omega}_i \times {}^{i}P_{i+1} + {}^{i}\omega_i \times \left({}^{i}\omega_i \times {}^{i}P_{i+1}\right) + {}^{i}\dot{v}_i\right)$$

Iterative Newton-Euler Equations - 2R Robot Example

Iterative Newton-Euler Equations - 2R Robot Example

- Outward Iteration $i=1$

$$^{i+1}\dot{v}_{C_{i+1}} = {}^{i+1}\dot{\omega}_{i+1} \times {}^{i+1}P_{C_{i+1}} + {}^{i+1}\omega_{i+1} \times \left({}^{i+1}\omega_{i+1} \times {}^{i+1}P_{C_{i+1}} \right) + {}^{i+1}\dot{v}_{i+1}$$

Iterative Newton-Euler Equations - 2R Robot Example

Iterative Newton-Euler Equations - 2R Robot Example

- Outward Iteration $i=1$

$$^{i+1}F_{i+1} = m_{i+1}\,{}^{i+1}\dot{v}_{C_{i+1}}$$

Iterative Newton-Euler Equations - 2R Robot Example

Iterative Newton-Euler Equations - 2R Robot Example

- Outward Iteration $i = 1$

$$^{i+1}N_{i+1} = {}^{C_{i+1}}I_{i+1}\,{}^{i+1}\dot{\omega}_{i+1} + {}^{i+1}\omega_{i+1} \times {}^{C_{i+1}}I_{i+1}\,{}^{i+1}\omega_{i+1}$$

Iterative Newton-Euler Equations - 2R Robot Example

Iterative Newton-Euler Equations - 2R Robot Example

- Inward iteration $i = 2$

$${}^{i}f_{i} = {}^{i}_{i+1}R\; {}^{i+1}f_{i+1} + {}^{i}F_{i}$$

Iterative Newton-Euler Equations - 2R Robot Example

Iterative Newton-Euler Equations - 2R Robot Example

- Inward iteration $i = 2$

$$ {}^{i}n_i = {}^{i}N_i + {}^{i}_{i+1}R \; {}^{i+1}n_{i+1} + {}^{i}P_{C_i} \times {}^{i}F_i + {}^{i}P_{i+1} \times {}^{i}_{i+1}R \; {}^{i+1}f_{i+1} $$

Iterative Newton-Euler Equations - 2R Robot Example

Iterative Newton-Euler Equations - 2R Robot Example

- Inward iteration $i = 1$

$$ {}^{i}f_{i} = {}^{i}_{i+1}R \; {}^{i+1}f_{i+1} + {}^{i}F_{i} $$

Iterative Newton-Euler Equations - 2R Robot Example

Iterative Newton-Euler Equations - 2R Robot Example

- Inward iteration $i = 1$

$$ {}^{i}n_{i} = {}^{i}N_{i} + {}^{i}_{i+1}R\ {}^{i+1}n_{i+1} + {}^{i}P_{C_i} \times {}^{i}F_{i} + {}^{i}P_{i+1} \times {}^{i}_{i+1}R\ {}^{i+1}f_{i+1} $$

Iterative Newton-Euler Equations - 2R Robot Example

Iterative Newton-Euler Equations - 2R Robot Example

- Inward iteration $i=1 \quad i=2$

$$\tau_i = {}^i n_i^T \; {}^i\hat{Z}_i$$

Iterative Newton-Euler Equations - 2R Robot Example

Iterative Newton-Euler Equations - 2R Robot Example

Iterative Newton-Euler Equations - 2R Robot Example

Iterative Newton-Euler Equations - 2R Robot Example

Equation of Motion – Non Rigid Body Effects

$$\tau = M(\theta)\ddot{\theta} + V(\theta,\dot{\theta}) + G(\theta) + F(\theta,\dot{\theta})$$

- Viscous Friction $\tau_{friction} = v\dot{\theta}$
- Coulomb Friction $\tau_{friction} = c\,\mathrm{sgn}(\dot{\theta})$
- Model of Friction $\tau_{friction} = v\dot{\theta} + c\,\mathrm{sgn}(\dot{\theta}) = f(\theta,\dot{\theta})$

Velocity / Force Transformation - Wrist / Sensor / Tool

Velocity / Force Transformation - Wrist / Sensor / Tool

Velocity / Force Transformation - Wrist / Sensor / Tool

Velocity / Force Transformation - Wrist / Sensor / Tool

Velocity / Force Transformation - Wrist / Sensor / Tool