Velocity Propagation Between Robot Links 3/4
Introduction – Velocity Propagation
In the field of robotics the Jacobian matrix describe the relationship between the joint angle rates $\dot{\theta}_N$ and the translation and rotation velocities of the end effector $\dot{x}$. This relationship is given by:

$$\dot{x} = J(\theta)\dot{\theta}$$

In addition to the velocity relationship, we are also interested in developing a relationship between the robot joint torques $\tau$ and the forces and moments $F$ at the robot end effector (Static Conditions). This relationship is given by:

$$\tau = J(\theta)^T F$$
Jacobian Matrix - Calculation Methods

- Differentiation the Forward Kinematics Eqs.
- Iterative Propagation (Velocities or Forces / Torques)

Jacobian Matrix
Summary – Changing Frame of Representation

• Linear and Rotational Velocity
  – Vector Form
  \[ {^A}_V Q = {^A}_V B O R G + {^A}_B R {^B}_Q + {^A}_B \Omega {^B}_Q \times {^A}_B R {^B}_P Q \]
  – Matrix Form
  \[ {^A}_V Q = {^A}_V B O R G + {^A}_B R {^B}_Q + {^A}_B \hat{\Omega} \left( {^A}_B R {^B}_P Q \right) \]

• Angular Velocity
  – Vector Form
  \[ {^A}_\Omega C = {^A}_\Omega B + {^A}_B R {^B}_\Omega C \]
  – Matrix Form
  \[ {^A}_C \hat{\Omega}_Q = {^A}_B \hat{\Omega}_Q + {^A}_B R {^B}_C \hat{\Omega}_Q {^A}_B R^T \]
Frame - Velocity

- As with any vector, a velocity vector may be described in terms of any frame, and this frame of reference is noted with a leading superscript.

- A velocity vector **computed** in frame \{B\} and **represented** in frame \{A\} would be written

\[
A(B_V) = \frac{d}{dt} B_P^A
\]

Represented (Reference Frame)

Computed (Measured)

\[
A V_{BORG} = 0
\]

\[
A B \ddot{R} = 0
\]
Position Propagation

- The homogeneous transform matrix provides a complete description of the linear and angular position relationship between adjacent links.

- These descriptions may be combined together to describe the position of a link relative to the robot base frame \( \{0\} \).

\[
^0T_i = ^0T_1^1T_2\cdots^i_{i-1}T_i
\]

- A similar description of the linear and angular velocities between adjacent links as well as the base frame would also be useful.
Position Propagation

\[ T^0 = T_1 T_2 T_3 T_4 T_5 T_6 T_7 \]
Motion of the Link of a Robot

In considering the motion of a robot link we will always use link frame \( \{0\} \) as the reference frame (Computed AND Represented). However any frame can be used as the reference (represented) frame including the link’s own frame (\( i \))

Where:

- \( v_i \) is the linear velocity of the origin of link frame (\( i \)) with respect to frame \( \{0\} \) (Computed AND Represented)

- \( \omega_i \) is the angular velocity of the origin of link frame (\( i \)) with respect to frame \( \{0\} \) (Computed AND Represented)

Expressing the velocity of a frame \( \{i\} \) (associated with link \( i \)) relative to the robot base (frame \( \{0\} \)) using our previous notation is defined as follows:

\[
\begin{align*}
    v_i &= \begin{bmatrix} ^0v_i \end{bmatrix} = \begin{bmatrix} ^0V_i \end{bmatrix} \\
    \omega_i &= \begin{bmatrix} ^0\omega_i \end{bmatrix} = \begin{bmatrix} ^0\Omega_i \end{bmatrix}
\end{align*}
\]
The velocities differentiate (computed) relative to the base frame \( \{0\} \) are often represented relative to other frames \( \{k\} \). The following notation is used for this conditions:

\[
\begin{align*}
    k \mathbf{v}_i &\equiv k \begin{bmatrix} 0 \mathbf{v}_i \end{bmatrix} = k R \begin{bmatrix} 0 \mathbf{v}_i \end{bmatrix} = k R \cdot \mathbf{v}_i \\
    k \mathbf{\omega}_i &\equiv k \begin{bmatrix} 0 \mathbf{\Omega}_i \end{bmatrix} = k R \begin{bmatrix} 0 \mathbf{\Omega}_i \end{bmatrix} = k R \cdot \mathbf{\omega}_i
\end{align*}
\]

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**Velocity Propagation**

- **Given:** A manipulator - A chain of rigid bodies each one capable of moving relative to its neighbor

- **Problem:** Calculate the linear and angular velocities of the link of a robot

- **Solution (Concept):** Due to the robot structure (serial mechanism) we can compute the velocities of each link in order starting from the base.

The velocity of link $i+1$ will be that of link $i$, plus whatever new velocity components were added by joint $i+1$
Velocity of Adjacent Links - Angular Velocity 0/5
Velocity of Adjacent Links - Angular Velocity 1/5

- From the relationship developed previously

\[ ^A \Omega_C = ^A \Omega_B + ^A R^B \Omega_C \]

- we can re-assign link names to calculate the velocity of any link \( i \) relative to the base frame \{0\}

\[
\begin{align*}
A & \rightarrow 0 \\
B & \rightarrow i \\
C & \rightarrow i + 1
\end{align*}
\]

\[ ^0 \Omega_{i+1} = ^0 \Omega_i + ^0 R^i \Omega_{i+1} \]

- By pre-multiplying both sides of the equation by \( ^0 R_{i+1} \), we can convert the frame of reference for the base \{0\} to frame \{i+1\}
Velocity of Adjacent Links - Angular Velocity 2/5

\[ i+1 R_0^0 \Omega_{i+1} = i+1 R_0^0 \Omega_i + i+1 R_i^0 R_i^i \Omega_{i+1} \]

- Using the recently defined notation, we have

\[ i+1 \omega_{i+1} = i+1 \omega_i + i+1 R_i^i \Omega_{i+1} \]

* Angular velocity of frame \( \{i+1\} \) measured relative to the robot base, and expressed in frame \( \{i+1\} \) - **Recall the car example**

\[ \begin{bmatrix} \omega_c \\ \omega_c \end{bmatrix} = \begin{bmatrix} w_c \\ -v_c \end{bmatrix} \]

* Angular velocity of frame \( \{i\} \) measured relative to the robot base, and expressed in frame \( \{i+1\} \)

* Angular velocity of frame \( \{i+1\} \) measured relative to frame \( \{i\} \) and expressed in frame \( \{i+1\} \)
Velocity of Adjacent Links - Angular Velocity 3/5

\[ i+1 \omega_{i+1} = i+1 \omega_i + i^R_i \Omega_{i+1} \]

- Angular velocity of frame \{i\} measured relative to the robot base, **expressed in frame \{i+1\}**

\[ i+1 \omega_i = i+1^R_i \omega_i \]
Velocity of Adjacent Links - Angular Velocity 4/5

\[ i+1 \omega_{i+1} = i+1 \omega_i + i R^i \Omega_{i+1} \]

- Angular velocity of frame \( \{i+1\} \) measured (differentiate) in frame \( \{i\} \) and represented (expressed) in frame \( \{i+1\} \)
- Assuming that a joint has only 1 DOF. The joint configuration can be either revolute joint (angular velocity) or prismatic joint (Linear velocity).
- Based on the frame attachment convention in which we assign the \( Z \) axis pointing along the \( i+1 \) joint axis such that the two are coincide (rotations of a link is preformed only along its \( Z \)-axis) we can rewrite this term as follows:

\[
\begin{bmatrix}
0 \\
0 \\
\dot{\theta}_{i+1}
\end{bmatrix}
\]
Velocity of Adjacent Links - Angular Velocity 5/5

• The result is a **recursive equation** that shows the angular velocity of one link in terms of the angular velocity of the previous link plus the relative motion of the two links.

\[ i+1 \omega_{i+1} = i+1 R^i \omega_i + \begin{bmatrix} 0 \\ 0 \\ \hat{\theta}_{i+1} \end{bmatrix} \]

• Since the term \( i+1 \omega_{i+1} \) depends on all previous links through this recursion, the angular velocity is said to propagate from the base to subsequent links.
Velocity of Adjacent Links - Linear Velocity 0/6
Velocity of Adjacent Links - Linear Velocity 1/6

- Simultaneous Linear and Rotational Velocity

- The derivative of a vector in a moving frame (linear and rotation velocities) as seen from a stationary frame

- Vector Form

\[ ^AV_Q = ^AV_{BORG} + _B^ATV_Q + _B^A\Omega_B \times _B^AR_P \]

- Matrix Form

\[ ^AV_Q = ^AV_{BORG} + _B^ATV_Q + _B^AR_P \]
Velocity of Adjacent Links - Linear Velocity 2/6

- From the relationship developed previously (matrix form)

\[ A^V_Q = A^{V_{BORG}} + A^B R^B V_Q + A^B \dot{R}_\Omega \left( A^B R^P Q \right) \]

- We re-assign link frames for adjacent links \((i \text{ and } i+1)\) with the velocity computed relative to the robot base frame \(\{0\}\)

\[
\begin{align*}
A &\rightarrow 0 \\
B &\rightarrow i \\
C &\rightarrow i + 1
\end{align*}
\]

\[ 0^V_{i+1} = 0^V_{i} \dot{R}_\Omega \left( 0^R_{i} P_{i+1} \right) + 0^V_{i} + 0^R_{i} V_{i+1} \]

- By pre-multiplying both sides of the equation by \(i+1^0 R\), we can convert the frame of reference for the left side to frame \(\{i+1\}\)
Velocity of Adjacent Links - Linear Velocity 3/6

\[ i+1 R^0 V_{i+1} = i+1 R^0 \dot{\mathbf{R}}_\Omega \left( R^i P_{i+1} \right) + i+1 R^0 V_i + i+1 R^0 R^i V_{i+1} \]

• Which simplifies to

\[ i+1 R^0 V_{i+1} = i+1 R^0 \dot{\mathbf{R}}_\Omega \left( R^i P_{i+1} \right) + i+1 R^0 V_i + i+1 R^i V_{i+1} \]

• Factoring out \( i+1 R^i \) from the left side of the first two terms

\[ i+1 R^0 V_{i+1} = i+1 R \left( R^0 \dot{\mathbf{R}}_\Omega R^i P_{i+1} + i_0 R^0 V_i \right) + i+1 R^i V_{i+1} \]
Velocity of Adjacent Links - Linear Velocity 4/6

\[ i+1 R^0 V_{i+1} = i+1 R \left( i R^0 \dot{R} \Omega_i R^i P_{i+1} + i R^0 V_i \right) + i R^i V_{i+1} \]

\[ i+1 R^i V_{i+1} \] - Linear velocity of frame \( \{i+1\} \) measured relative to frame \( \{i\} \) and expressed in frame \( \{i+1\} \)

- Assuming that a joint has only 1 DOF. The joint configuration can be either revolute joint (angular velocity) or prismatic joint (Linear velocity).

- Based on the frame attachment convention in which we assign the \( Z \) axis pointing along the \( i+1 \) joint axis such that the two are coincide (translational of a link is performed only along its \( Z \)-axis) we can rewrite this term as follows:

\[
\begin{bmatrix}
0 \\
0 \\
\dot{d}_{i+1}
\end{bmatrix}
\]
Velocity of Adjacent Links - Linear Velocity 5/6

\[ i+10R^0V_{i+1} = i+1R\left( i0R^0\dot{R}_i\Omega_i R^0P_{i+1} + i0R^0V_i \right) + \begin{bmatrix} 0 \\ 0 \\ \dot{d}_{i+1} \end{bmatrix} \]

\[ i0R^0\dot{R}_i\Omega_i R^0 = i0R^0\dot{R}_i\Omega_i R^T = i0R^0\Omega_i = i0R\omega_i = i\omega_i \]

Multiply by Matrix
Definition

\[ i+10R^0V_{i+1} = i+1v_{i+1} \]

Definition

\[ i0R^0V_i = iv_i \]

Definition
The result is a **recursive equation** that shows the linear velocity of one link in terms of the previous link plus the relative motion of the two links.

\[
\dot{v}_{i+1}^{i+1} = \dot{R}^i + \omega_i \times \dot{R}^i P_{i+1}^i + v_i^i + \dot{d}_{i+1}^{i+1}
\]

Since the term \( \dot{v}_{i+1}^{i+1} \) depends on all previous links through this recursion, the angular velocity is said to propagate from the base to subsequent links.
Velocity of Adjacent Links - Summary

- Angular Velocity

\[
i + 1 \omega_{i+1} = i + 1 R^i \omega_i + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_{i+1} \end{bmatrix}
\]

- Linear Velocity

\[
i + 1 v_{i+1} = i + 1 R^i (i \omega \times P_{i+1} + i v_i) + \begin{bmatrix} 0 \\ 0 \\ \dot{d}_{i+1} \end{bmatrix}
\]
For the manipulator shown in the figure, compute the angular and linear velocity of the “tool” frame relative to the base frame expressed in the “tool” frame (that is, calculate $^4\omega$ and $^4v$ ).
Angular and Linear Velocities - 3R Robot - Example

• Frame attachment
Angular and Linear Velocities - 3R Robot - Example

- DH Parameters

<table>
<thead>
<tr>
<th>$i$</th>
<th>$a_{i-1}$</th>
<th>$a_{i}$</th>
<th>$d_{i}$</th>
<th>$\theta_{i}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$\theta_{1}$</td>
</tr>
<tr>
<td>2</td>
<td>90</td>
<td>$L_{1}$</td>
<td>0</td>
<td>$\theta_{2}$</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>$L_{2}$</td>
<td>0</td>
<td>$\theta_{3}$</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>$L_{3}$</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
Angular and Linear Velocities - 3R Robot - Example

- From the DH parameter table, we can specify the homogeneous transform matrix for each adjacent link pair:

\[ i^{-1}_iT = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1}d_i \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1}d \\ 0 & 0 & 0 & 1 \end{bmatrix} \]

\[ 0^1T = \begin{bmatrix} c1 & -s1 & 0 & 0 \\ s1 & c1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ c3 & -s3 & 0 & L2 \end{bmatrix} \]

\[ 1^2T = \begin{bmatrix} c2 & -s2 & 0 & L1 \\ 0 & 0 & -1 & 0 \\ s2 & c2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \]

\[ 2^3T = \begin{bmatrix} 1 & 0 & 0 & L3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \]
Angular and Linear Velocities - 3R Robot - Example

- Compute the angular velocity of the end effector frame relative to the base frame expressed at the end effector frame.

\[ i+1 \omega_{i+1} = i+1 R^i \omega_i + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_{i+1} \end{bmatrix} \]

- For \( i=0 \)

\[ 1 \omega_1 = 1 R^0 \omega_0 + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix} = \begin{bmatrix} c1 & s1 & 0 \\ -s1 & c1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix} \]
Angular and Linear Velocities - 3R Robot - Example

- For $i=1$

\[
\dot{\omega}_2 = \frac{\dot{R}_1}{1} \omega_1 + \begin{bmatrix}
0 & c2 & 0 \\
0 & -s2 & 0 \\
0 & 0 & -1 \\
\end{bmatrix}
\begin{bmatrix}
\dot{\theta}_1 \\
\dot{\theta}_2 \\
\dot{\theta}_3 \\
\end{bmatrix} + \begin{bmatrix}
0 \\
0 \\
1 \\
\end{bmatrix} = \begin{bmatrix}
s2 \dot{\theta}_1 \\
c2 \dot{\theta}_1 \\
0 \\
\end{bmatrix}
\]

- For $i=2$

\[
\dot{\omega}_3 = \frac{\dot{R}_2}{2} \omega_2 + \begin{bmatrix}
0 & c3 & s3 & 0 \\
0 & -s3 & c3 & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\begin{bmatrix}
\dot{\theta}_1 \\
\dot{\theta}_2 \\
\dot{\theta}_3 \\
\end{bmatrix} + \begin{bmatrix}
s23 \dot{\theta}_1 \\
c23 \dot{\theta}_1 \\
0 \\
\end{bmatrix} = \begin{bmatrix}
s23 \dot{\theta}_1 \\
c23 \dot{\theta}_1 \\
\dot{\theta}_2 + \dot{\theta}_3 \\
\end{bmatrix}
\]

- For $i=3$

\[
\dot{\omega}_4 = \frac{\dot{R}_3}{3} \omega_3 + \begin{bmatrix}
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\begin{bmatrix}
\dot{\theta}_1 \\
\dot{\theta}_2 + \dot{\theta}_3 \\
\dot{\theta}_2 + \dot{\theta}_3 \\
\end{bmatrix} + \begin{bmatrix}
s23 \dot{\theta}_1 \\
c23 \dot{\theta}_1 \\
0 \\
\end{bmatrix} = \begin{bmatrix}
s23 \dot{\theta}_1 \\
c23 \dot{\theta}_1 \\
\dot{\theta}_2 + \dot{\theta}_3 \\
\end{bmatrix}
\]

- Note

\[
\dot{\omega}_3 = \frac{\dot{R}_3}{3} \omega_3 + \begin{bmatrix}
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\begin{bmatrix}
\dot{\theta}_1 \\
\dot{\theta}_2 + \dot{\theta}_3 \\
\dot{\theta}_2 + \dot{\theta}_3 \\
\end{bmatrix} + \begin{bmatrix}
s23 \dot{\theta}_1 \\
c23 \dot{\theta}_1 \\
0 \\
\end{bmatrix} = \begin{bmatrix}
s23 \dot{\theta}_1 \\
c23 \dot{\theta}_1 \\
\dot{\theta}_2 + \dot{\theta}_3 \\
\end{bmatrix}
\]

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Angular and Linear Velocities - 3R Robot - Example

- Compute the linear velocity of the end effector frame relative to the base frame expressed at the end effector frame.

- Note that the term involving the prismatic joint has been dropped from the equation (it is equal to zero).

\[
i+1 \mathbf{v}_{i+1} = i+1 R \left( i \mathbf{\omega} \times i P_{i+1} + i v_i \right) + \begin{bmatrix} 0 \\ 0 \\ d_{i+1} \end{bmatrix}
\]
Angular and Linear Velocities - 3R Robot - Example

• For \( i=0 \)

\[
^{1}v_{1}=^{1}R^{0}_{0}\{\omega_{0}\times^{0}P_{1}+^{0}v_{0}\}=
\begin{bmatrix}
c1 & s1 & 0 \\
-s1 & c1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}0 \\
0 \\
0
\end{bmatrix}
+ \begin{bmatrix}0 \\
0 \\
0
\end{bmatrix}
= \begin{bmatrix}0 \\
0 \\
0
\end{bmatrix}
\]

• For \( i=1 \)

\[
^{2}v_{2}=^{2}R^{1}_{1}\{\omega_{1}\times^{1}P_{2}+^{1}v_{1}\}=
\begin{bmatrix}
c2 & 0 & s2 \\
-s2 & 0 & c2 \\
0 & -1 & 0
\end{bmatrix}
\begin{bmatrix}0 \\
0 \\
\dot{\theta}_{1}
\end{bmatrix}
+ \begin{bmatrix}L1 \\
0 \\
0
\end{bmatrix}
= \begin{bmatrix}0 \\
0 \\
-L_{1}\dot{\theta}_{1}
\end{bmatrix}
\]
Angular and Linear Velocities - 3R Robot - Example

- For $i=3$

\[
3v_3 = 3R^{\omega_2 \times 2P_3 + 2v_2} = \begin{bmatrix}
c3 & s3 & 0 \\
-s3 & c30 & 0 \\
0 & 0 & 1 \\
\end{bmatrix}\begin{bmatrix}
s2\dot{\theta}_1 \\
c2\dot{\theta}_1 \\
\dot{\theta}_2 \\
\end{bmatrix} \times \begin{bmatrix}
0 \\
0 \\
0 \\
\end{bmatrix} + \begin{bmatrix}
0 \\
0 \\
-L1\dot{\theta}_1 \\
\end{bmatrix}
\]

\[
= \begin{bmatrix}
c3 & s3 & 0 \\
-s3 & c30 & 0 \\
0 & 0 & 1 \\
\end{bmatrix}\begin{bmatrix}
0 \\
L2\dot{\theta}_1 \\
-L2c2\dot{\theta}_1 - L1\dot{\theta}_1 \\
\end{bmatrix} = \begin{bmatrix}
L2s3\dot{\theta}_2 \\
L2c3\dot{\theta}_2 \\
(-L1 - L2c2)\dot{\theta}_1 \\
\end{bmatrix}
\]
Angular and Linear Velocities - 3R Robot - Example

- For \( i=4 \)

\[
4v_4 = 4R_3^{\omega_3 \times P_4 + 3v_3} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} s23\dot{\theta}_1 \\ c23\dot{\theta}_1 \\ \dot{\theta}_2 + \dot{\theta}_3 \end{bmatrix} \begin{bmatrix} L3 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} L2s3\dot{\theta}_2 \\ L2c3\dot{\theta}_2 \\ -(L1-L2c2)\dot{\theta}_1 \end{bmatrix}
\]

\[
= \begin{bmatrix} L2s3\dot{\theta}_2 \\ (L2c3 + L3)\dot{\theta}_2 + L3\dot{\theta}_3 \\ -(L1-L2c2-L3c23)\dot{\theta}_1 \end{bmatrix}
\]
Angular and Linear Velocities - 3R Robot - Example

• Note that the linear and angular velocities \( (4\omega_4, 4v_4) \) of the end effector where differentiate (measured) in frame \{0\} however represented (expressed) in frame \{4\}.

• In the car example: Observer sitting in the “Car”
  Observer sitting in the “World”

\[
\begin{align*}
  k_v &\equiv [0V_i] = kR [0V_i] = kR \cdot v_i \\
  k\omega &\equiv [0\Omega_i] = kR [0\Omega_i] = kR \cdot \omega_i
\end{align*}
\]

Solve for \( v_4 \) and \( \omega_4 \) by multiply both side of the questions from the left by \( 0R^{-1} \)

\[
\begin{align*}
  4v_4 &= 0R \cdot v_4 \\
  4\omega_4 &= 0R \cdot \omega_4
\end{align*}
\]
Angular and Linear Velocities - 3R Robot - Example

- Multiply both sides of the equation by the inverse transformation matrix, we finally get the linear and angular velocities expressed and measured in the stationary frame \( \{0\} \)

\[
\begin{align*}
\mathbf{v}_4 &= \mathbf{4R}_0^{-1} \cdot \mathbf{v}_4 = \mathbf{4R}_0^T \cdot \mathbf{v}_4 = \mathbf{0R}_4 \cdot \mathbf{v}_4 \\
\mathbf{w}_4 &= \mathbf{4R}_0^{-1} \cdot \mathbf{w}_4 = \mathbf{4R}_0^T \cdot \mathbf{w}_4 = \mathbf{0R}_4 \cdot \mathbf{w}_4 \\
\mathbf{T}_0^4 &= \mathbf{T}_1^2 \mathbf{T}_2^3 \mathbf{T}_4^3 
\end{align*}
\]
Angular and Linear Velocities - 3R Robot - Example

\[
1\omega_1 = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & \dot{\theta}_1 \end{bmatrix}
\]

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Angular and Linear Velocities - 3R Robot - Example

\[ \omega_2 = \begin{bmatrix} s2\dot{\theta}_1 \\ c2\dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} \]
Angular and Linear Velocities - 3R Robot - Example

\[
\begin{bmatrix}
    s23\dot{\theta}_1 \\
    c23\dot{\theta}_1 \\
    \dot{\theta}_2 + \dot{\theta}_3
\end{bmatrix}
\]
Angular and Linear Velocities - 3R Robot - Example

\[ \omega_3 = \omega_4 \]

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$$1v_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$
Angular and Linear Velocities - 3R Robot - Example

\[ \dot{v}_2 = \begin{bmatrix} 0 \\ 0 \\ -L_1 \dot{\theta}_1 \end{bmatrix} \]
Angular and Linear Velocities - 3R Robot - Example

\[ \mathbf{v} = \begin{bmatrix} L2s3\dot{\theta}_2 \\ L2c3\dot{\theta}_2 \\ (-L1 - L2c2)\dot{\theta}_1 \end{bmatrix} \]
Angular and Linear Velocities - 3R Robot - Example

\[
^4v_4 = \begin{bmatrix}
    L2s3\dot{\theta}_2 \\
    (L2c3 + L3)\dot{\theta}_2 + L3\dot{\theta}_3 \\
    -(L1 + L2c2 + L3c23)\dot{\theta}_1
\end{bmatrix}
\]