MAE263B: Dynamics of Robotic Systems
Discussion Section - Week2
: Forward & Inverse Kinematics
(Yasukawa Motoman L-3)

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01.17.2020.

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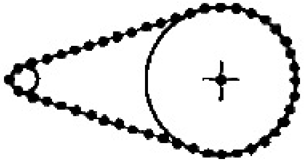
- ❑ Inverse Kinematics
- ❑ Special case

  (**Actuator space $\neq$ Joint space**)

- When the actuator space is not coincident to the joint space, how can we solve it?
- Ex. Chain belt



Forward Kinematics

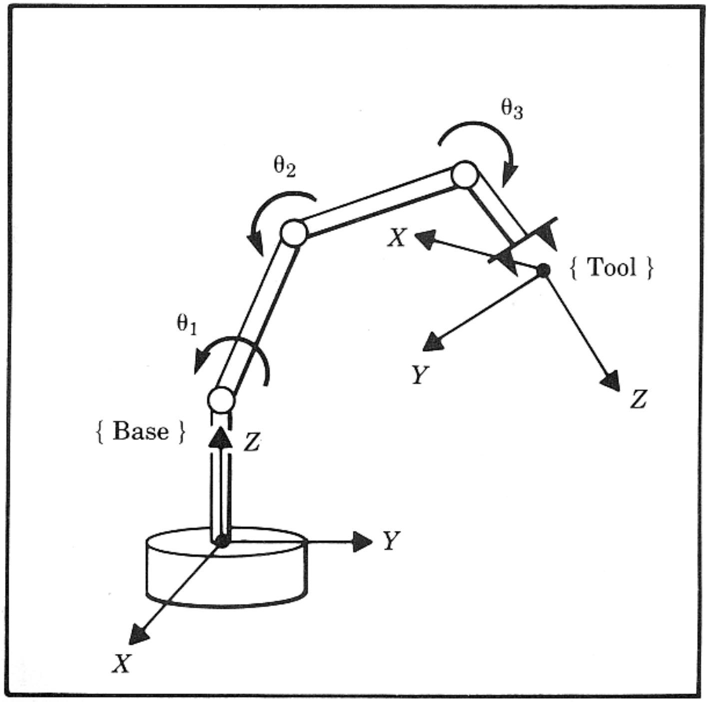
Problem

Given: Joint angles and links geometry

Compute: Position and orientation of the end effector relative to the base frame

Solution

Kinematic Equations - Linear Transformation (4x4 matrix) which is a function of the joint positions (angles & displacements) and specifies the EE configuration in the base frame.



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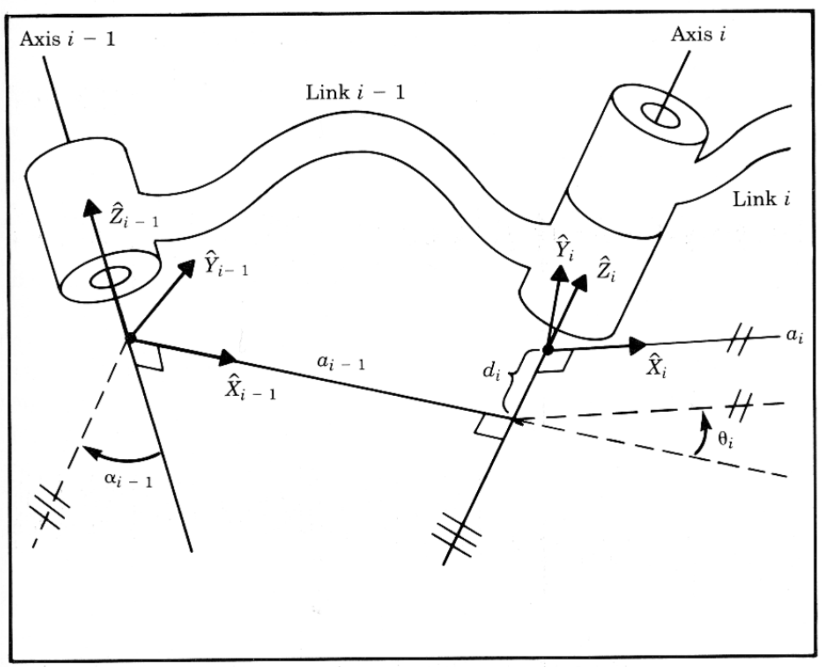
Derivation of link Homogeneous Transform

The homogeneous transform is a 4x4 matrix casting the ***rotation*** and ***translation*** of a general transform into a single matrix.

$$
{}^{i-1}_{\;\;i}T = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1} d_i \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1} d \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$



DH Parameters - Review

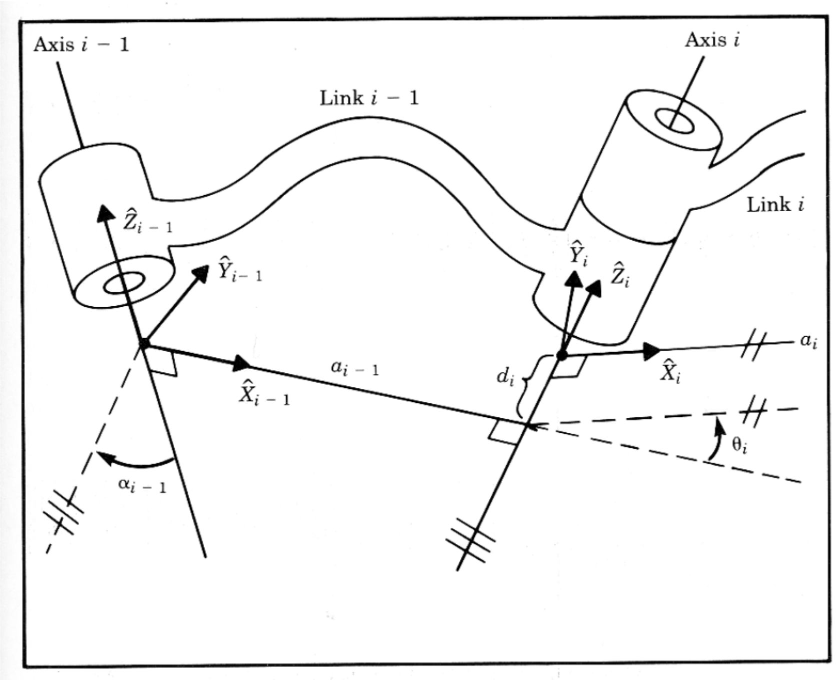
a_{i-1} - **Link Length** - The distance from $\hat{Z}_{i-1}$ to $\hat{Z}_i$ measured along $\hat{X}_{i-1}$

α_{i-1} - **Link Twist** - The angle between $\hat{Z}_{i-1}$ and $\hat{Z}_i$ measured about $\hat{X}_{i-1}$

d_i - **Link Offset** - The distance from $\hat{X}_{i-1}$ to $\hat{X}_i$ measured along $\hat{Z}_i$

θ_i - **Link Angle** - The angle between $\hat{X}_{i-1}$ and $\hat{X}_i$ measured about $\hat{Z}_i$

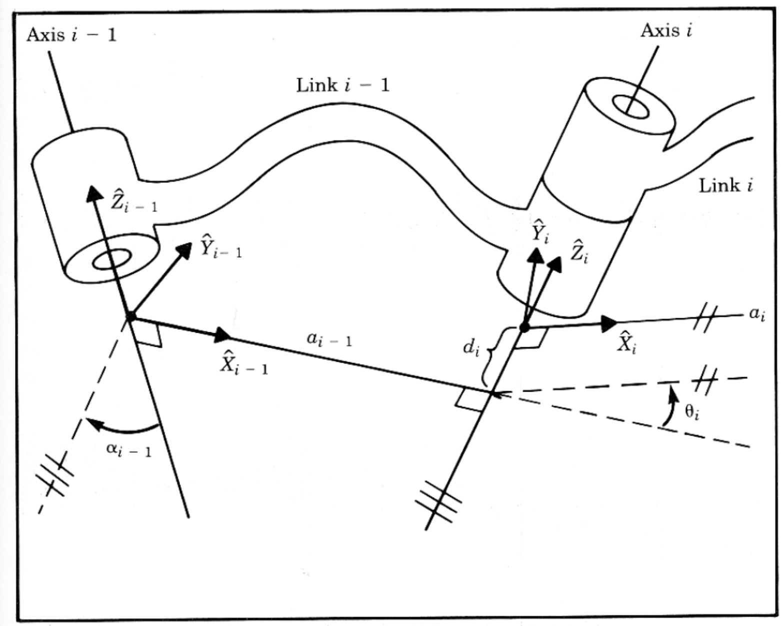
Note: $a_i \geq 0$ α_i d_i θ_i are singed quantities



Derivation of link Homogeneous Transformation

Problem: Determine the transformation which defines frame {***i-1***} relative to the frame {***i***}

$${}^{i-1}_{i}T$$



Note: For any given link of a robot, ${}^{i-1}_{i}T$ will be a function of only one variable out of $a_{i-1}\ \alpha_{i-1}\ d_i\ \theta_i$. The other three parameters being fixed by mechanical design.

Revolute Joint -> θ_i

Prismatic Joint -> d_i

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Forward Kinematics – Yasukawa Motoman L-3

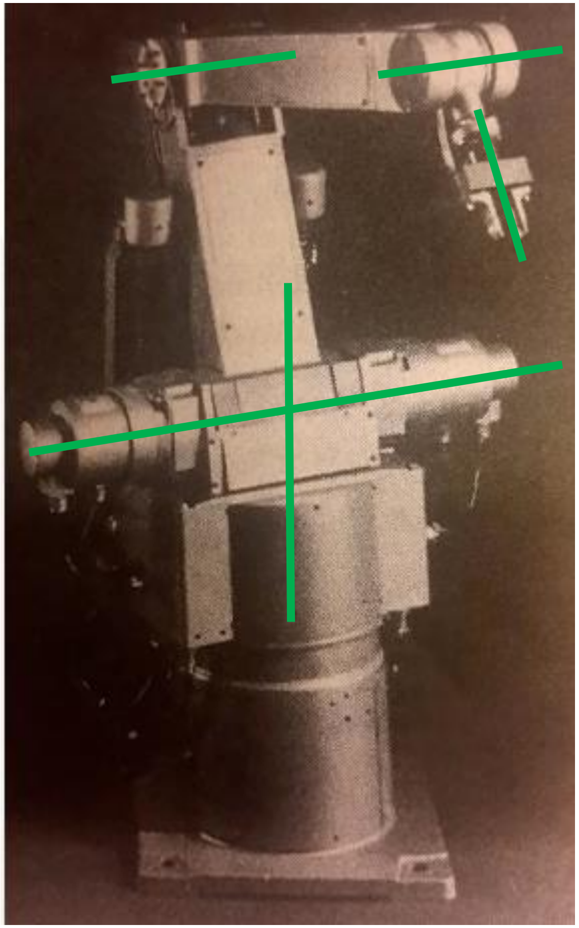
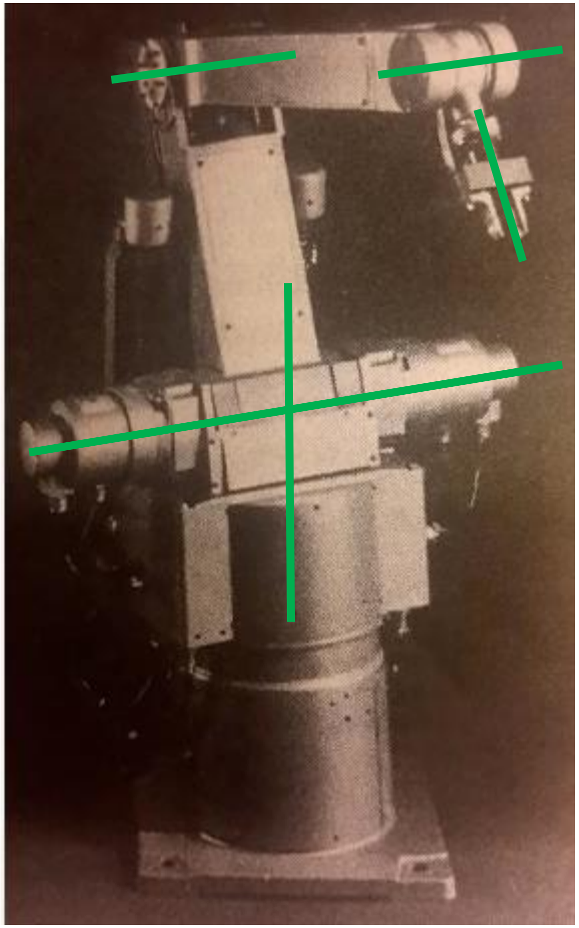


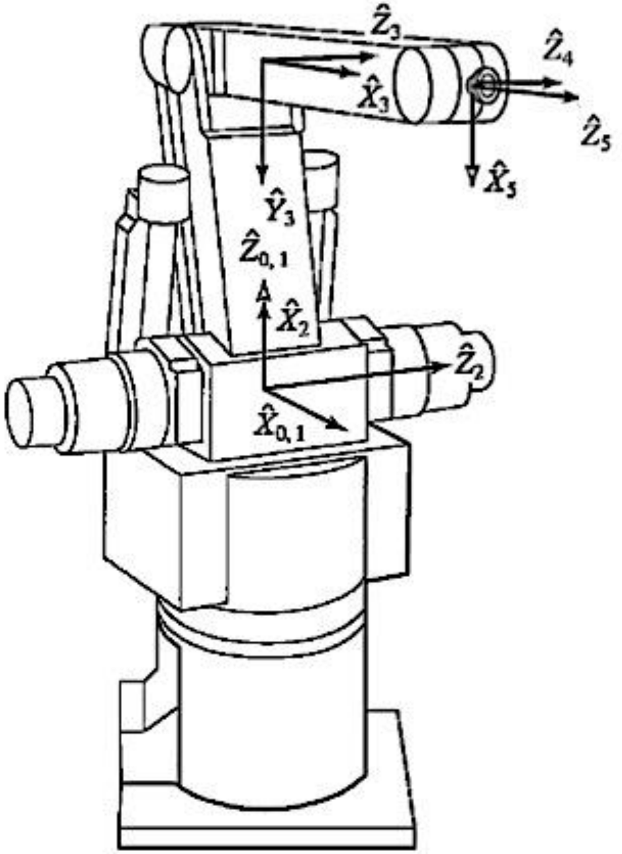
FIGURE 3.22: The Yasukawa Motoman L-3. Courtesy of Yasukawa.

The **Yasukawa Motoman L-3** is an industrial manipulator with **5-axis** (5DOFs).

Overall, the robot behaves as an open kinematic chain.

Forward Kinematics – Yasukawa Motoman L-3

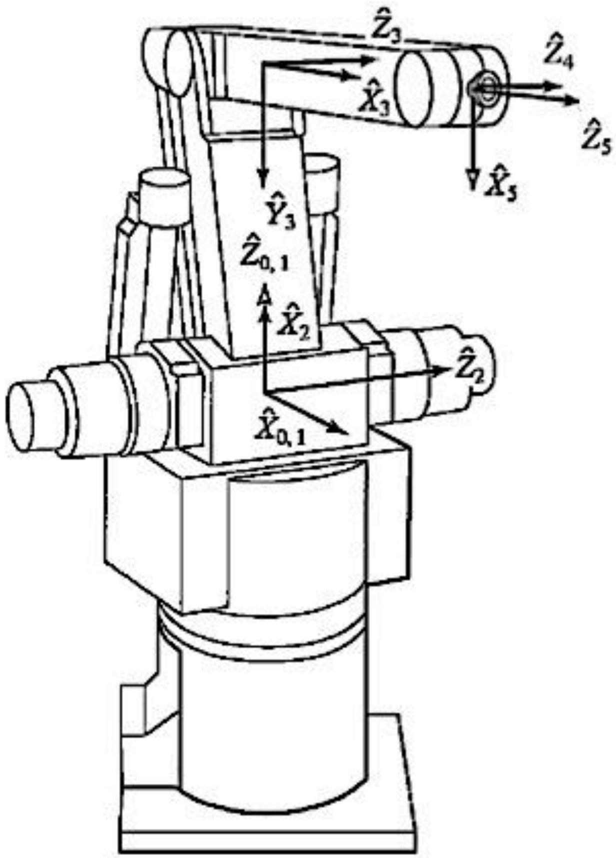




Configuration (0°,-90°,90°,90°,0°)

Forward Kinematics – Yasukawa Motoman L-3

i	$\alpha_i - 1$	$a_i - 1$	d_i	θ_i
1	0	0	0	θ_1
2	$-90°$	0	0	θ_2
3	0	L_2	0	θ_3
4	0	L_3	0	θ_4
5	$90°$	0	0	θ_5



Configuration (0°,-90°,90°,90°,0°)

Forward Kinematics – Yasukawa Motoman L-3

Link parameters of the Yasukawa L-3 manipulator.

i	$\alpha_i - 1$	$a_i - 1$	d_i	θ_i
1	0	0	0	θ_1
2	$-90°$	0	0	θ_2
3	0	L_2	0	θ_3
4	0	L_3	0	θ_4
5	$90°$	0	0	θ_5

$$ {}^{i-1}_{\ \ i}T = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1} d_i \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1} d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}. $$

$$ {}^0_1T = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, $$

$$ {}^1_2T = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s\theta_2 & -c\theta_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, $$

$$ {}^2_3T = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & l_2 \\ s\theta_3 & c\theta_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, $$

$$ {}^3_4T = \begin{bmatrix} c\theta_4 & -s\theta_4 & 0 & l_3 \\ s\theta_4 & c\theta_4 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, $$

$$ {}^4_5T = \begin{bmatrix} c\theta_5 & -s\theta_5 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ s\theta_5 & c\theta_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Forward Kinematics – Yasukawa Motoman L-3

And calculating all the products we obtain

$$
{}^{0}_{5}T = \begin{bmatrix} r_{11} & r_{12} & r_{13} & p_x \\ r_{21} & r_{22} & r_{23} & p_y \\ r_{31} & r_{32} & r_{33} & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
\begin{aligned}
r_{11} &= c_1 c_{234} c_5 - s_1 s_5, \\
r_{21} &= s_1 c_{234} c_5 + c_1 s_5, \\
r_{31} &= -s_{234} c_5,
\end{aligned}
$$

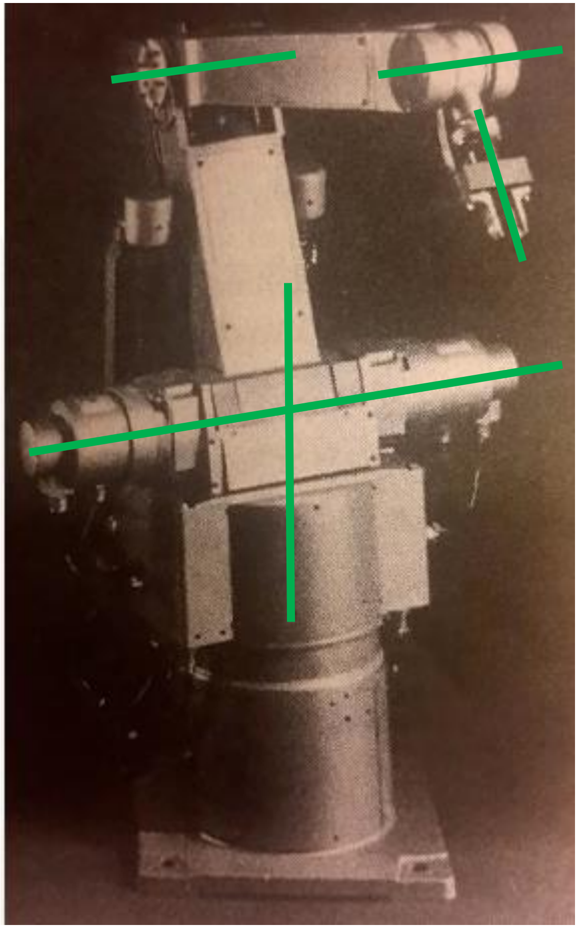
$$
\begin{aligned}
r_{12} &= -c_1 c_{234} s_5 - s_1 c_5, \\
r_{22} &= -s_1 c_{234} s_5 + c_1 c_5, \\
r_{32} &= s_{234} s_5,
\end{aligned}
$$

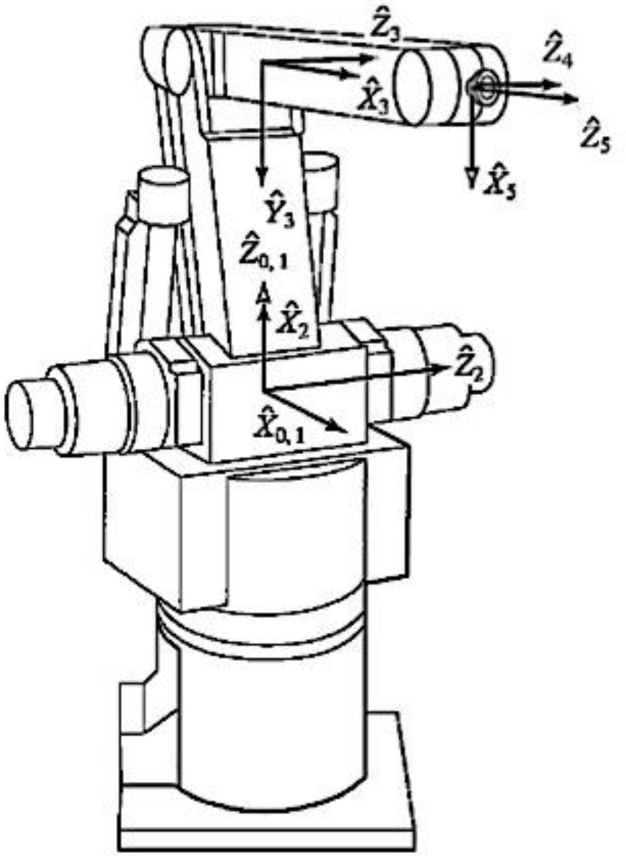
Where $c_{234} = \cos(\theta_2+\theta_3+\theta_4)$.

$$
\begin{aligned}
p_x &= c_1(l_2 c_2 + l_3 c_{23}), \\
p_y &= s_1(l_2 c_2 + l_3 c_{23}), \\
p_z &= -l_2 s_2 - l_3 s_{23}.
\end{aligned}
$$

$$
\begin{aligned}
r_{13} &= c_1 s_{234}, \\
r_{23} &= s_1 s_{234}, \\
r_{33} &= c_{234},
\end{aligned}
$$

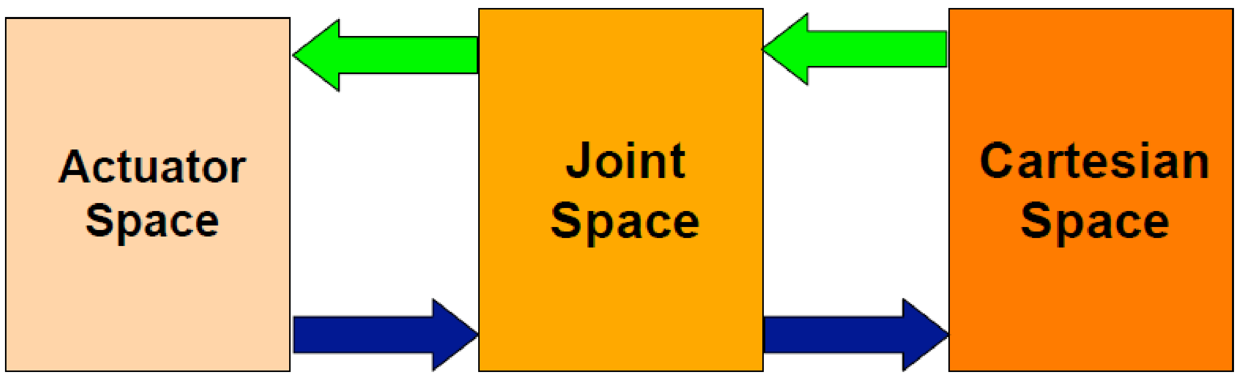
Forward Kinematics – Yasukawa Motoman L-3





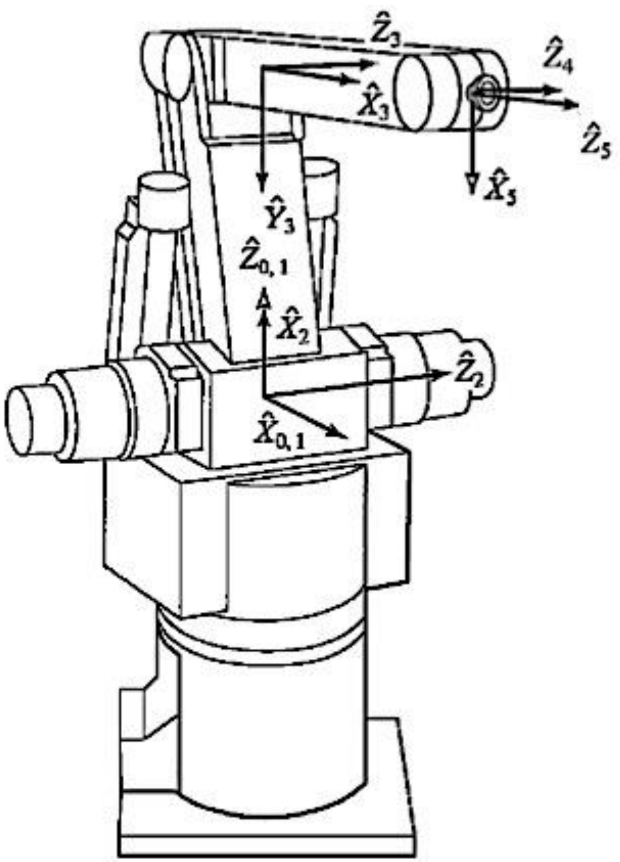
Configuration (0°,-90°,90°,90°,0°)

Actuator Space - Joint Space - Cartesian Space



Task Oriented Space
Operational Space

Forward Kinematics – Yasukawa Motoman L-3

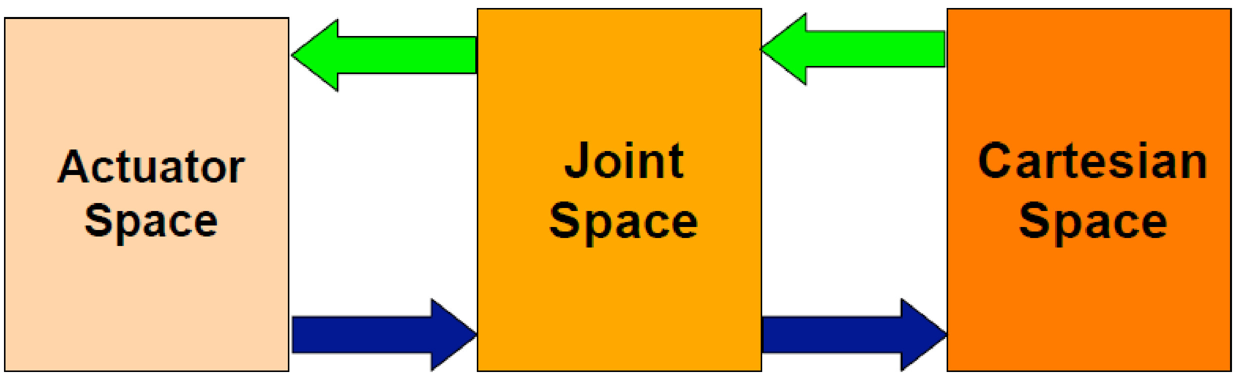


However, if we look at the actuation system (Actuator-Space) we discover that:

- **Joint 2** and **Joint 3** are coupled with a four-linkages mechanism actuated by two linear actuators.

- **Joint 4** and **Joint 5** are actuated by a **chain drive**.

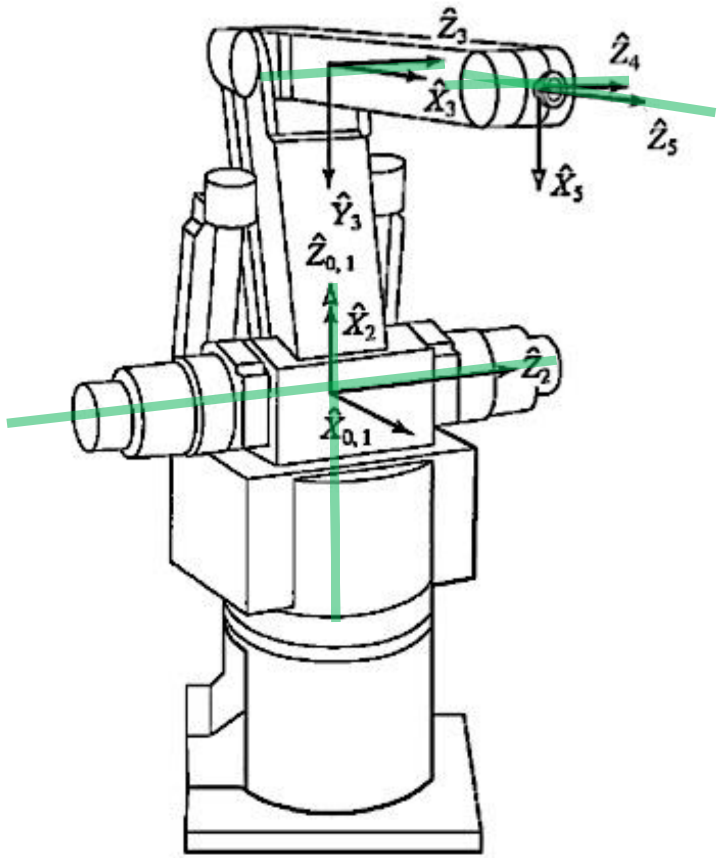
Actuator Space - Joint Space - Cartesian Space



1) We will solve for joint angles from actuator positions
(Actuator-Space →Joint-Space).

Task Oriented Space
Operational Space

Forward Kinematics – Yasukawa Motoman L-3



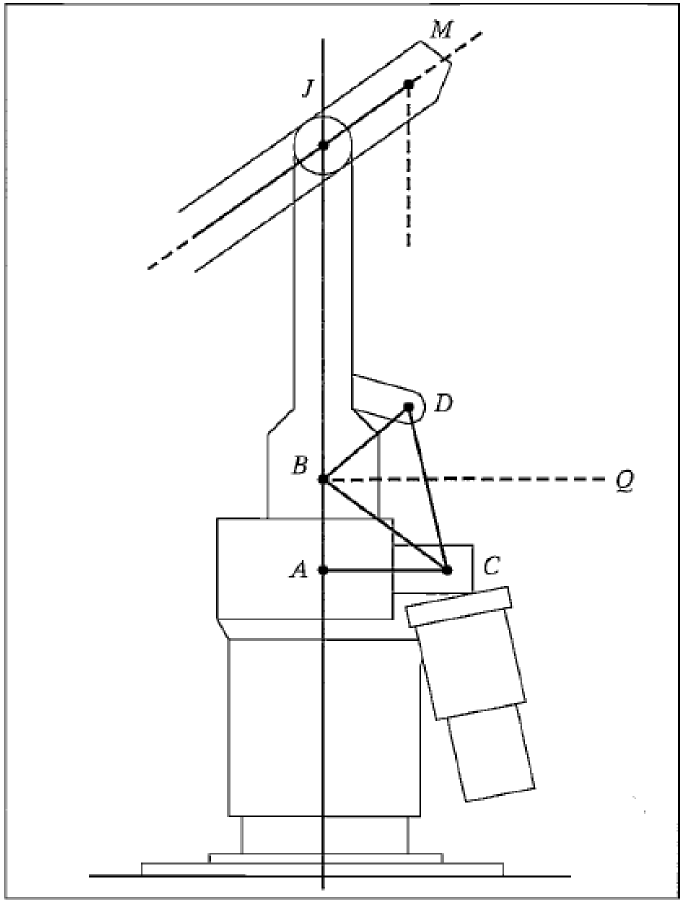
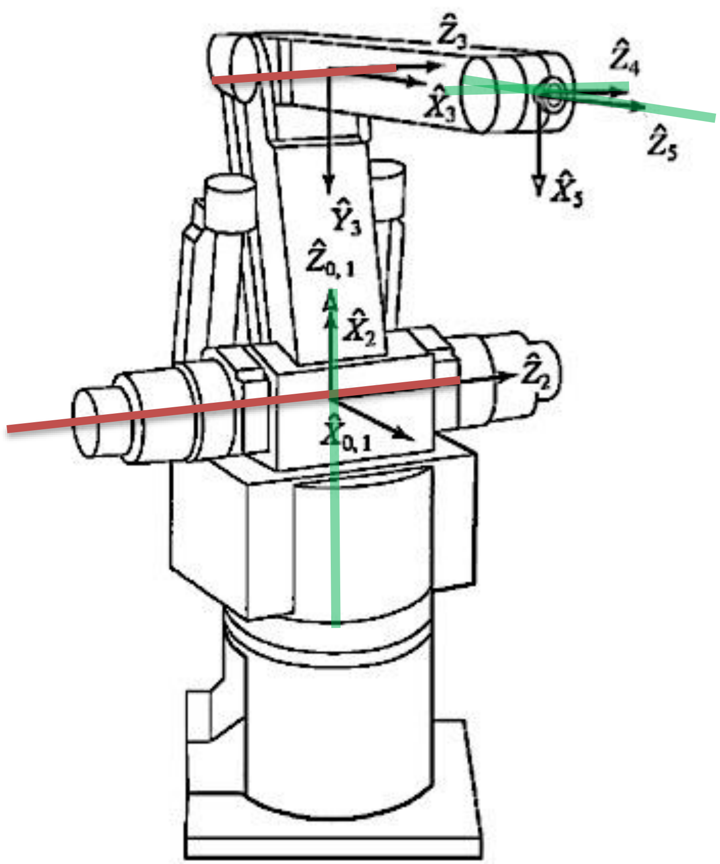
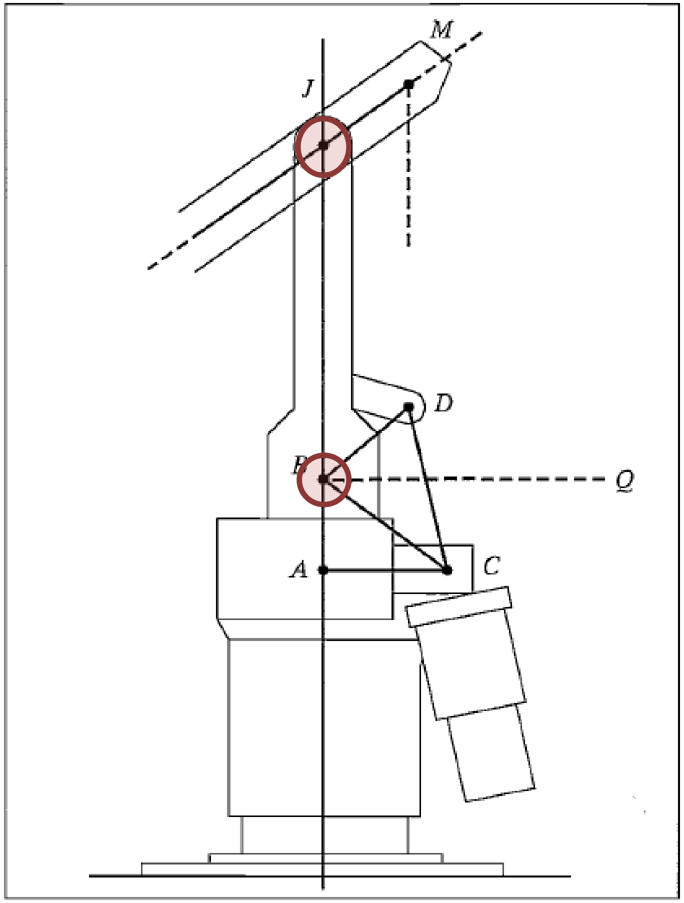


FIGURE 3.23: Kinematic details of the Yasukawa actuator-2 linkage.

Forward Kinematics – Yasukawa Motoman L-3





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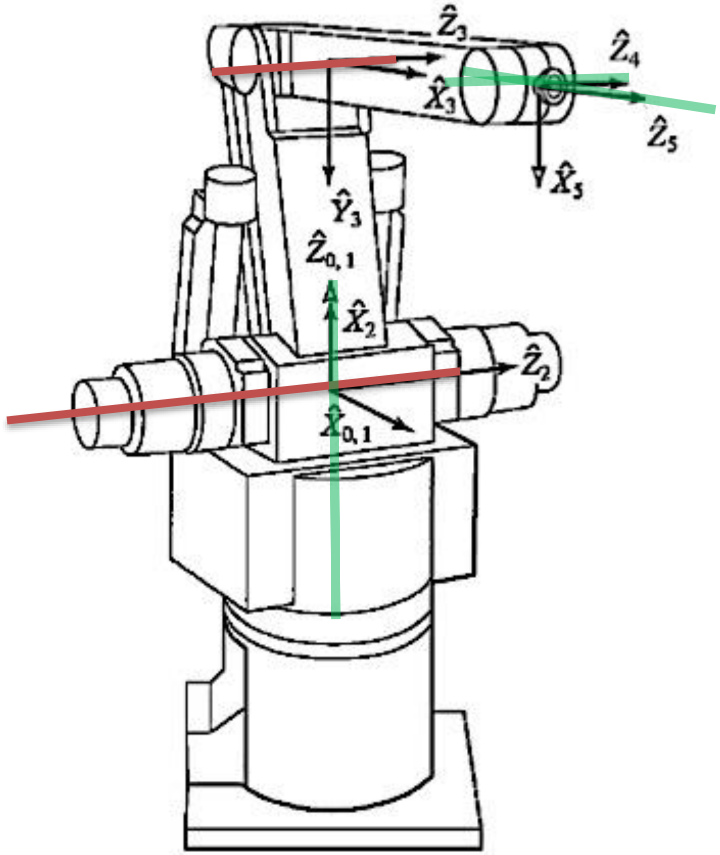
Forward Kinematics – Yasukawa Motoman L-3



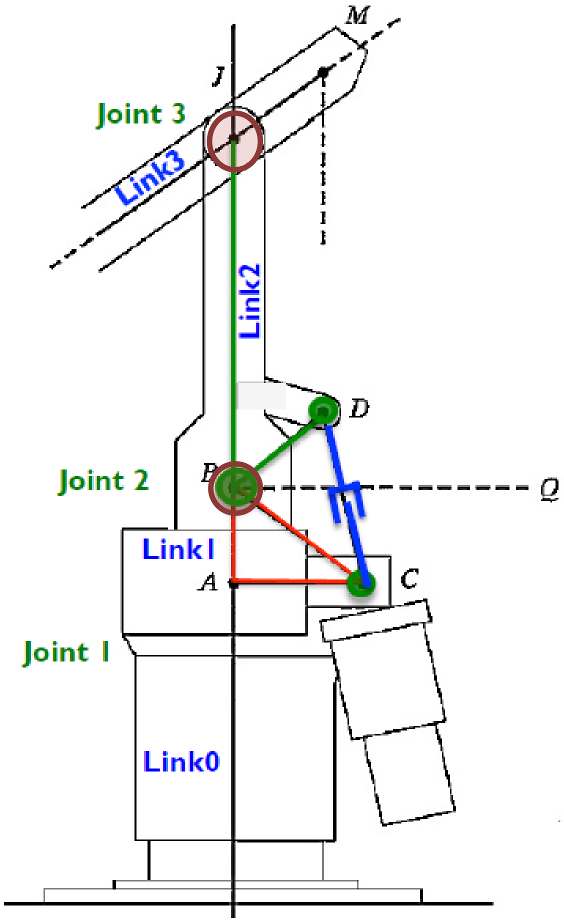


FIGURE 3.23: Kinematic details of the Yasukawa actuator-2 linkage.

Forward Kinematics – Yasukawa Motoman L-3

The linear **actuator 2** effects the **Link 2** and the **Link 3** of the robot:

Constants?

$$\gamma_2 = AB, \phi_2 = AC, \alpha_2 = BC,$$

$$\beta_2 = BD, \Omega_2 = \angle JBD, l_2 = BJ,$$

Variables?

$$\theta_2 = -\angle JBQ, \psi_2 = \angle CBD, g_2 = DC.$$

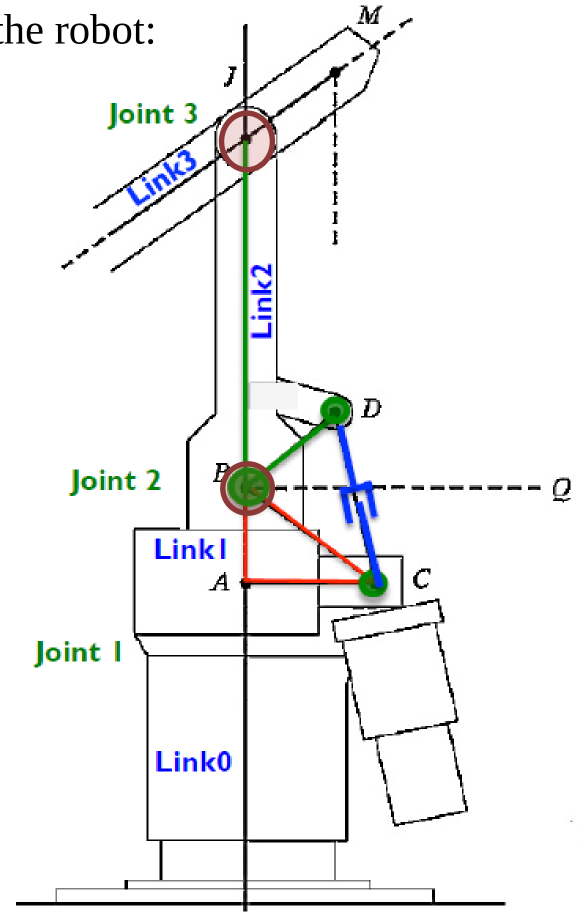
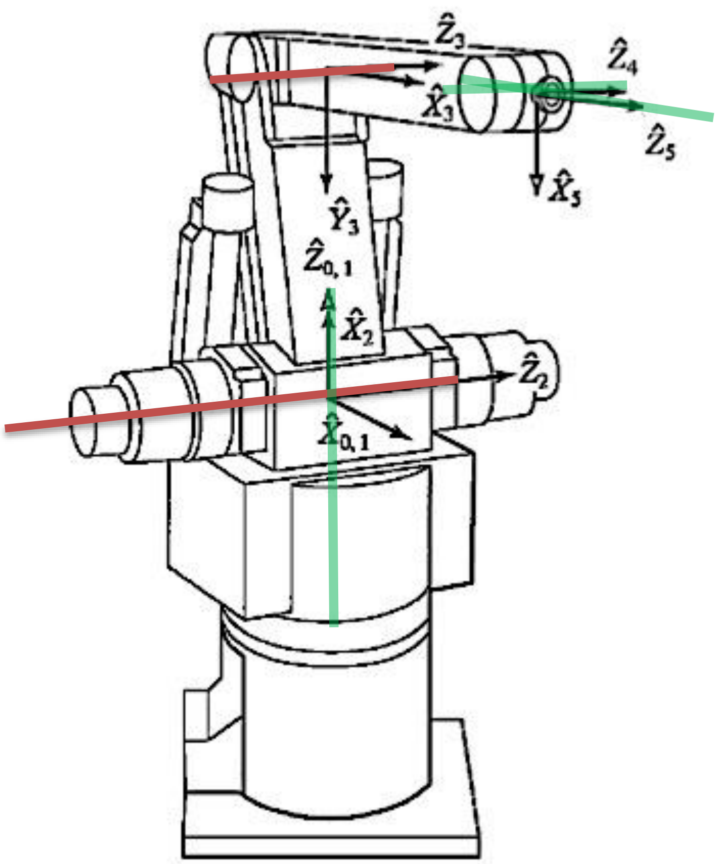
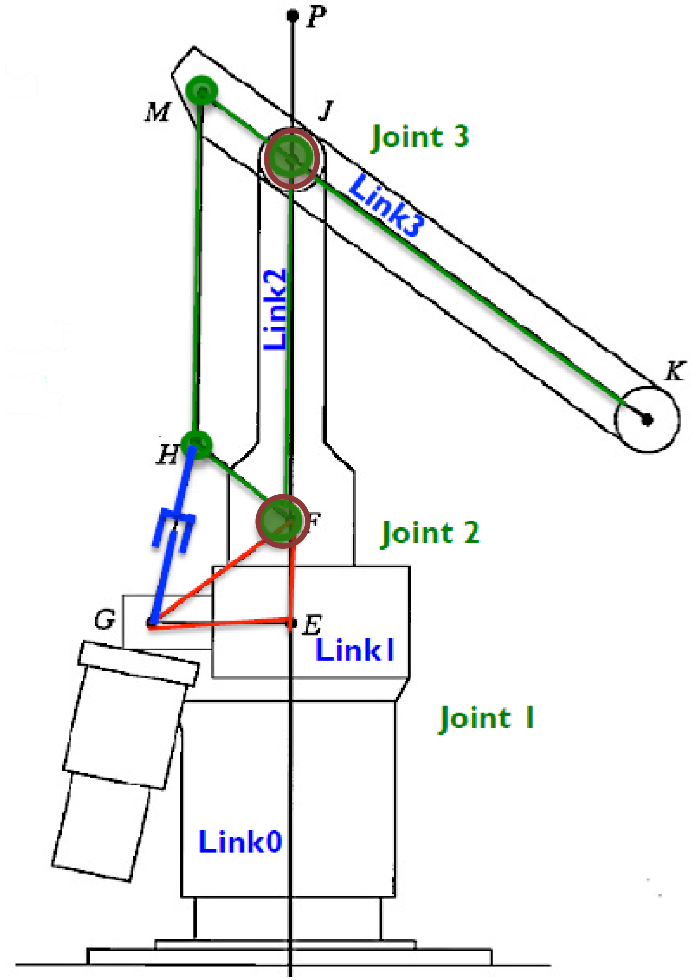


FIGURE 3.23: Kinematic details of the Yasukawa actuator-2 linkage.

Forward Kinematics – Yasukawa Motoman L-3

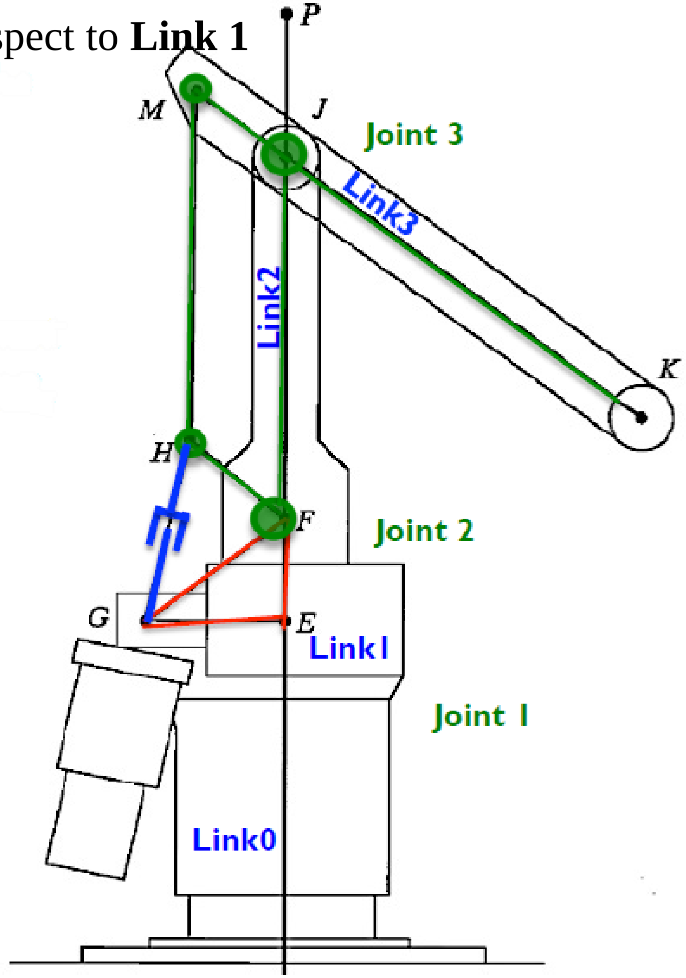




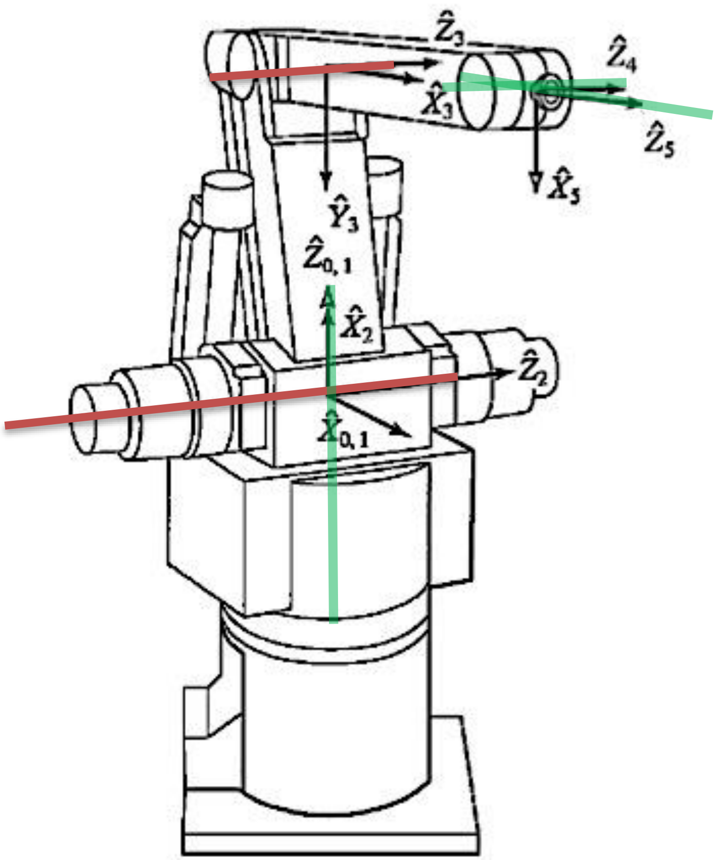
Forward Kinematics – Yasukawa Motoman L-3

- **Actuator 3** changes the orientation of **Link 3** with respect to **Link 1** (rather than relative to **Link 2**).

Obs. When the **actuator 3** keeps its position also the orientation of **Link 3** will be constant (relative to **Link 1**), independently from the position of **Joint 2** (**Actuator 2**).

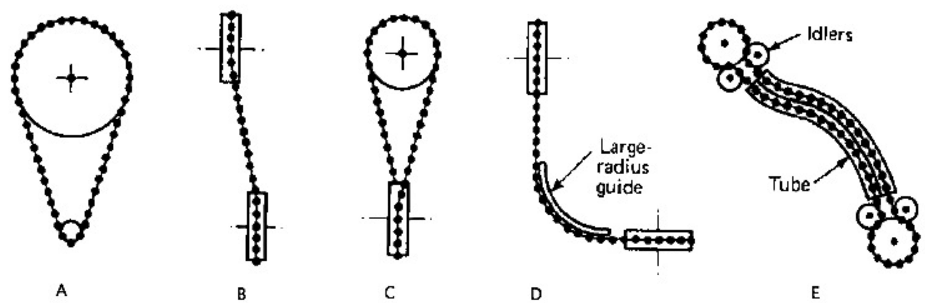


Forward Kinematics – Yasukawa Motoman L-3



The **Actuator 4** and the **Actuator 5**, that move **Joint 4** and **Joint 5** respectively, are attached to **Link 1** and their rotational axes are aligned with the **Joint 2** axis.

The forces are transmitted by chains in a way that:
- **Actuator 4** positions **Joint 4** and orientates **Link 4** relative to **Link 1** (rather than Link 3).
Again it realizes a sort of "absolute" adjustment of the orientation of **Link 4** relative to **Link 1** frame.
- **Actuator 5** positions **Joint 5**, but this time relative to **Link 4**.



Forward Kinematics – Yasukawa Motoman L-3

The equations that map the actuators positions $[\boldsymbol{A_1}\ \boldsymbol{A_2}\ \boldsymbol{A_3}\ \boldsymbol{A_4}\ \boldsymbol{A_5}]^{\boldsymbol{T}}$ to the joint positions $[\boldsymbol{\theta_1}\ \boldsymbol{\theta_2}\ \boldsymbol{\theta_3}\ \boldsymbol{\theta_4}\boldsymbol{\theta_5}]^{\boldsymbol{T}}$ can be calculated by analysing the robot's geometry:

Obs.: for **Joint 1** and **Joint 5** we have a simple linear relationships between the actuator and the joint position, where $\mathbf{k_i}$ is a scaling factor due to the gear system and $\boldsymbol{\lambda_i}$ is representing an offset most probably due to how the position sensor was installed.

Obs.: notice that the position of **Joint 3** depends on $\mathbf{A_3}$ and also $\mathbf{A_2}$

$$\theta_1 = k_1 A_1 + \lambda_1,$$

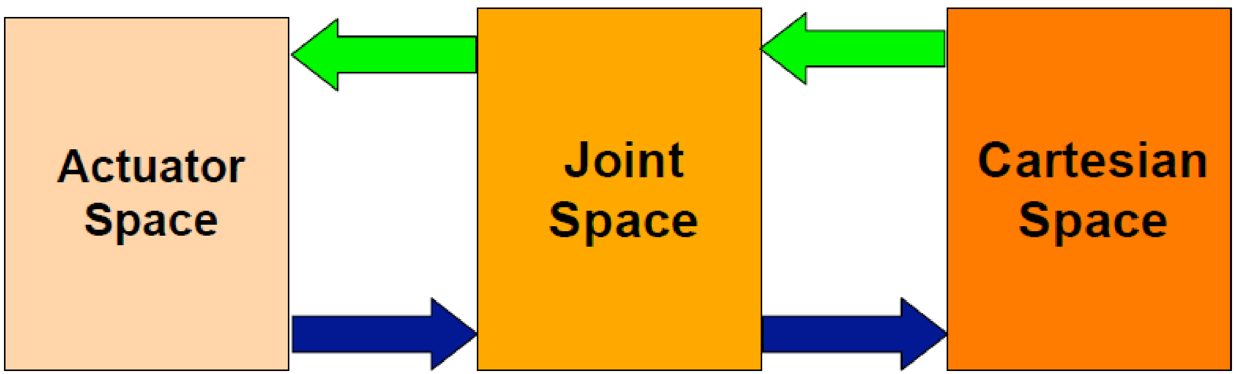
$$\theta_2 = \cos^{-1}\left(\frac{(k_2 A_2 + \lambda_2)^2 - \alpha_2^2 - \beta_2^2}{-2\alpha_2\beta_2}\right) + \tan^{-1}\left(\frac{\phi_2}{\gamma_2}\right) + \Omega_2 - 270°,$$

$$\theta_3 = \cos^{-1}\left(\frac{(k_3 A_3 + \lambda_3)^2 - \alpha_3^2 - \beta_3^2}{-2\alpha_3\beta_3}\right) - \theta_2 + \tan^{-1}\left(\frac{\phi_3}{\gamma_3}\right) - 90°,$$

$$\theta_4 = -k_4 A_4 - \theta_2 - \theta_3 + \lambda_4 + 180°,$$

$$\theta_5 = -k_5 A_5 + \lambda_5.$$

Actuator Space - Joint Space - Cartesian Space

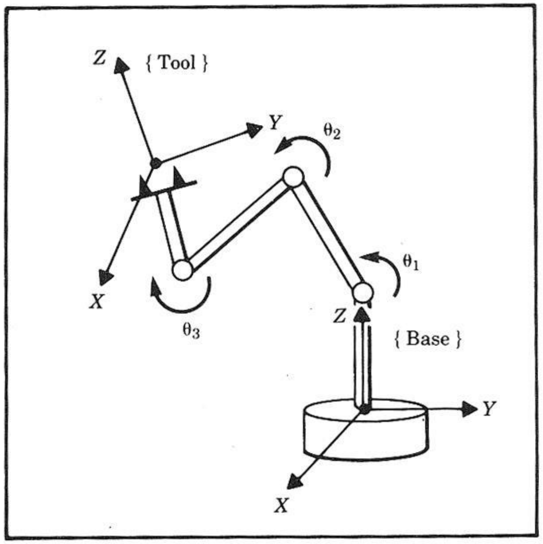


Task Oriented Space
Operational Space

Inverse Kinematics

Problem

Given: Position and orientation of the end effector relative to the base frame
Compute: All possible sets of joint angles and links geometry which could be used to attain the given position and orientation of the end effetor



Solution

There are three approaches for the solution:

- **Analytical Approach** - Kinematic Equations - Linear Transformation (4x4 matrix) which is a function of the joint positions (angles & displacements) and specifies the EE configuration in the base frame. This linear transformation defines 12 non linear equations A subset of these equations are used for obtaining the invers kinematics
- **Geometric Approach** – Projecting the arm configurations on specific planes and using geometrical consideration to obtain the invers kinematics
- **Hybrid Approach** - Synthesizing the analytical and the geometrical approaches

Forward Kinematics – Yasukawa Motoman L-3

And calculating all the products we obtain

$$ {}^0_5T = {}^0_1T\, {}^1_2T\, {}^2_3T\, {}^3_4T\, {}^4_5T. $$

$$ {}^0_5T = \begin{bmatrix} r_{11} & r_{12} & r_{13} & p_x \\ r_{21} & r_{22} & r_{23} & p_y \\ r_{31} & r_{32} & r_{33} & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Where $c_{234} = \cos(\theta_2+\theta_3+\theta_4)$.

$$ \begin{aligned} p_x &= c_1(l_2c_2 + l_3c_{23}), \\ p_y &= s_1(l_2c_2 + l_3c_{23}), \\ p_z &= -l_2s_2 - l_3s_{23}. \end{aligned} $$

$$ \begin{aligned} r_{11} &= c_1c_{234}c_5 - s_1s_5, \\ r_{21} &= s_1c_{234}c_5 + c_1s_5, \\ r_{31} &= -s_{234}c_5, \end{aligned} $$

$$ \begin{aligned} r_{12} &= -c_1c_{234}s_5 - s_1c_5, \\ r_{22} &= -s_1c_{234}s_5 + c_1c_5, \\ r_{32} &= s_{234}s_5, \end{aligned} $$

$$ \begin{aligned} r_{13} &= c_1s_{234}, \\ r_{23} &= s_1s_{234}, \\ r_{33} &= c_{234}, \end{aligned} $$

Inverse Kinematics – Algebraic Solution- PUMA 560

- ***Problem:***

  What are the joint angles ($\theta_1 \cdots \theta_6$) as a function of the wrist position and orientation (or when ${}^0_6T$ is given as numeric values)

$$
{}^0_6T = \underbrace{{}^0_1T(\theta_1)\ {}^1_2T(\theta_2)\ {}^2_3T(\theta_3)\ {}^3_4T(\theta_4)\ {}^4_5T(\theta_5)\ {}^5_6T(\theta_6)}_{\text{Direct Kinematics}} = \underbrace{\begin{bmatrix} r_{11} & r_{12} & r_{13} & p_x \\ r_{21} & r_{22} & r_{23} & p_y \\ r_{31} & r_{32} & r_{33} & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{\text{Goal}}
$$

Inverse Kinematics – Yasukawa Motoman L-3

$$ {}^0_5T = {}^0_1T \, {}^1_2T \, {}^2_3T \, {}^3_4T \, {}^4_5T. $$

and premultiply both sides by ${}^0_1T^{-1}$, we have

$$ {}^0_1T^{-1} \, {}^0_5T = {}^1_2T \, {}^2_3T \, {}^3_4T \, {}^4_5T $$

$$ \begin{bmatrix} c_1r_{11}+s_1r_{21} & c_1r_{12}+s_1r_{22} & c_1r_{13}+s_1r_{23} & c_1p_x+s_1p_y \\ -r_{31} & -r_{32} & -r_{33} & -p_z \\ -s_1r_{11}+c_1r_{21} & -s_1r_{12}+c_1r_{22} & -s_1r_{13}+c_1r_{23} & -s_1p_x+c_1p_y \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} * & * & s_{234} & * \\ * & * & -c_{234} & * \\ s_5 & c_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Inverse Kinematics – Yasukawa Motoman L-3

$$\begin{bmatrix} c_1 r_{11} + s_1 r_{21} & c_1 r_{12} + s_1 r_{22} & c_1 r_{13} + s_1 r_{23} & c_1 p_x + s_1 p_y \\ -r_{31} & -r_{32} & -r_{33} & -p_z \\ -s_1 r_{11} + c_1 r_{21} & -s_1 r_{12} + c_1 r_{22} & -s_1 r_{13} + c_1 r_{23} & \boxed{-s_1 p_x + c_1 p_y} \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} * & * & s_{234} & * \\ * & * & -c_{234} & * \\ s_5 & c_5 & 0 & \boxed{0} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} r_{11} & r_{12} & r_{13} & p_x \\ r_{21} & r_{22} & r_{23} & p_y \\ r_{31} & r_{32} & r_{33} & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

in the latter, several of the elements have not been shown. Equating the (3,4) elements, we get

$$-s_1 p_x + c_1 p_y = 0, \tag{4.95}$$

which gives us[5]

$$\theta_1 = \text{Atan2}(p_y, p_x). \tag{4.96}$$

Atan2 - Definition

Four-quadrant inverse tangent (arctangent) in the range of

$$\mathrm{Atan}\,2(y,x) = \tan^{-1}(y/x)$$

For example

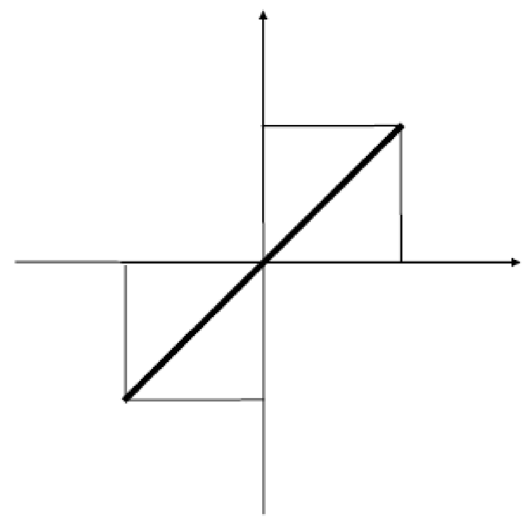
$$[-\pi \quad \pi]$$

$$\mathrm{Atan}(+1,+1) = 45^{o}$$

$$\mathrm{Atan}\,2(+1,+1) = 45^{o}$$

$$\mathrm{Atan}(-1,-1) = 45^{o}$$

$$\mathrm{Atan}\,2(-1,-1) = -135^{o}$$



Inverse Kinematics – Yasukawa Motoman L-3

$$\begin{bmatrix} c_1 r_{11} + s_1 r_{21} & c_1 r_{12} + s_1 r_{22} & c_1 r_{13} + s_1 r_{23} & c_1 p_x + s_1 p_y \\ -r_{31} & -r_{32} & -r_{33} & -p_z \\ \boxed{-s_1 r_{11} + c_1 r_{21}} & \boxed{-s_1 r_{12} + c_1 r_{22}} & -s_1 r_{13} + c_1 r_{23} & -s_1 p_x + c_1 p_y \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} * & * & s_{234} & * \\ * & * & -c_{234} & * \\ \boxed{s_5} & \boxed{c_5} & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Equating the (3,1) and (3,2) elements, we get

$$s_5 = -s_1 r_{11} + c_1 r_{21},$$

$$c_5 = -s_1 r_{12} + c_1 r_{22},$$

from which we calculate θ_5 as

$$\theta_5 = \mathrm{Atan2}(r_{21} c_1 - r_{11} s_1, r_{22} c_1 - r_{12} s_1).$$

Inverse Kinematics – Yasukawa Motoman L-3

$$\begin{bmatrix} c_1 r_{11} + s_1 r_{21} & c_1 r_{12} + s_1 r_{22} & \boxed{c_1 r_{13} + s_1 r_{23}} & c_1 p_x + s_1 p_y \\ -r_{31} & -r_{32} & \boxed{-r_{33}} & -p_z \\ -s_1 r_{11} + c_1 r_{21} & -s_1 r_{12} + c_1 r_{22} & -s_1 r_{13} + c_1 r_{23} & -s_1 p_x + c_1 p_y \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} * & * & \boxed{s_{234}} & * \\ * & * & \boxed{-c_{234}} & * \\ s_5 & c_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Equating the (2,3) and (1,3) elements, we get

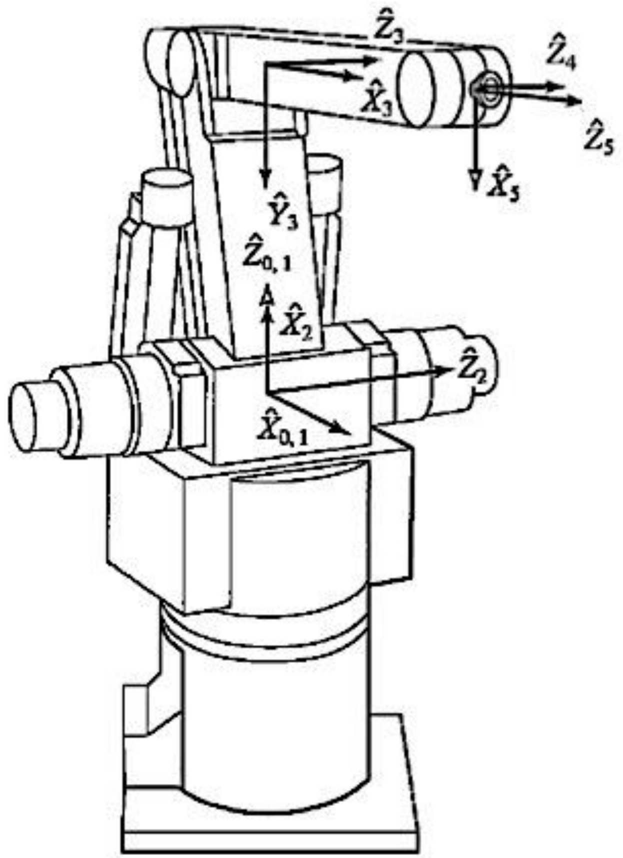
$$c_{234} = r_{33},$$

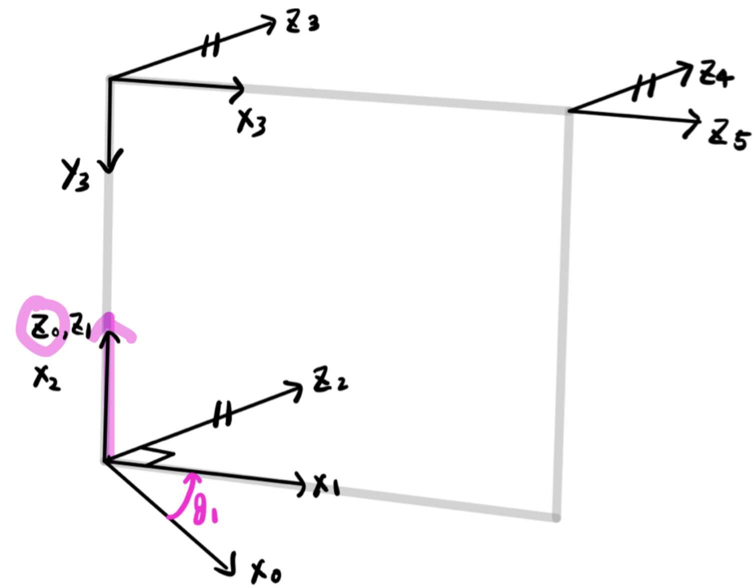
$$s_{234} = c_1 r_{13} + s_1 r_{23},$$

which leads to

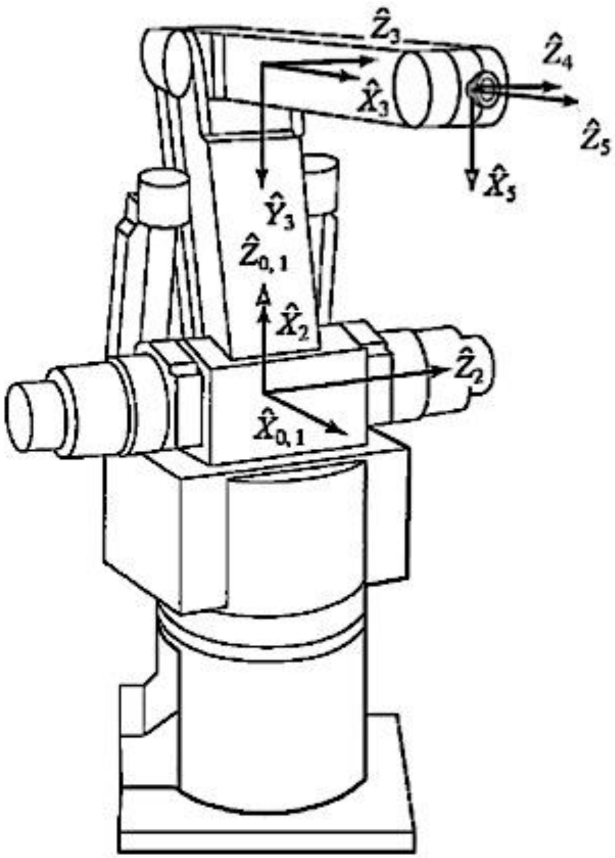
$$\theta_{234} = \text{Atan2}(r_{13}c_1 + r_{23}s_1, r_{33}).$$

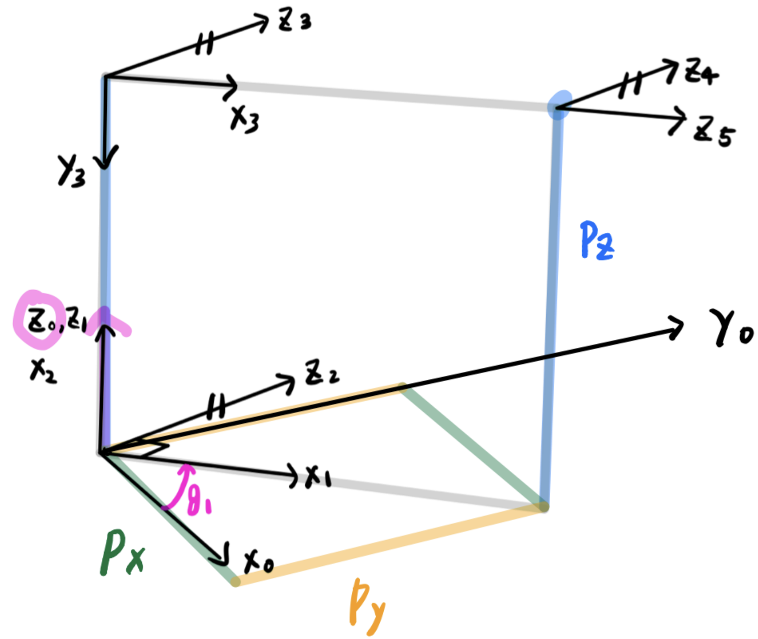
Inverse Kinematics – Yasukawa Motoman L-3



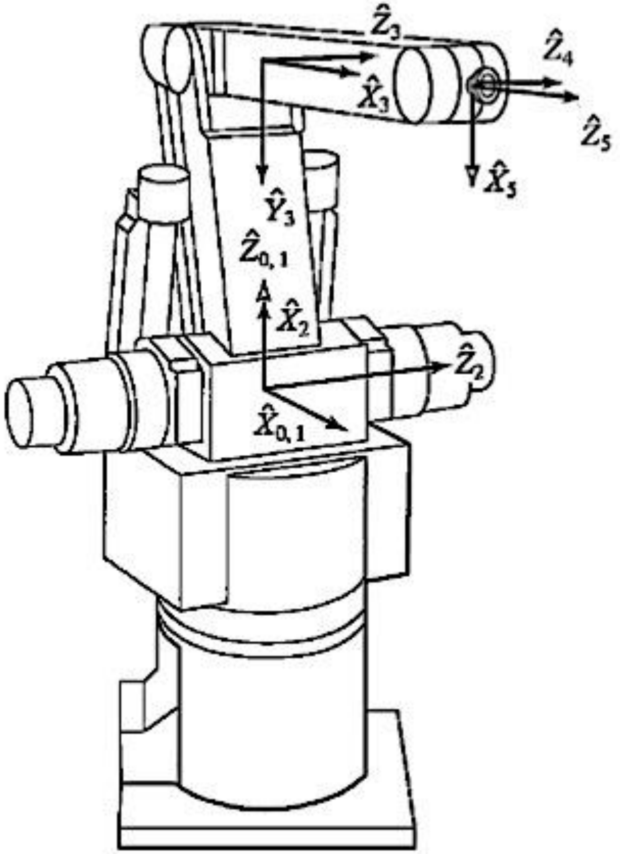


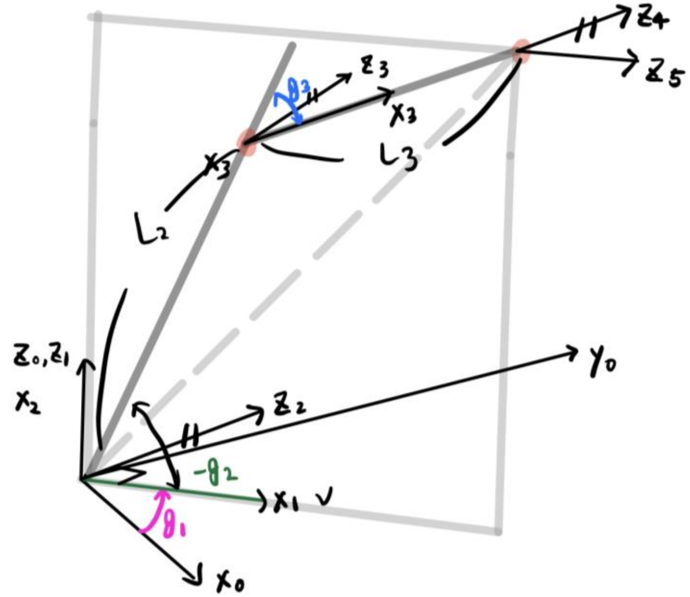
Inverse Kinematics – Yasukawa Motoman L-3



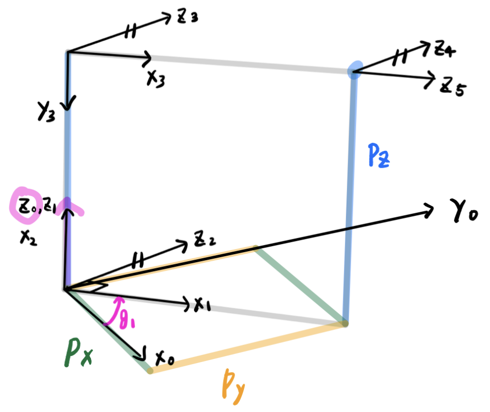


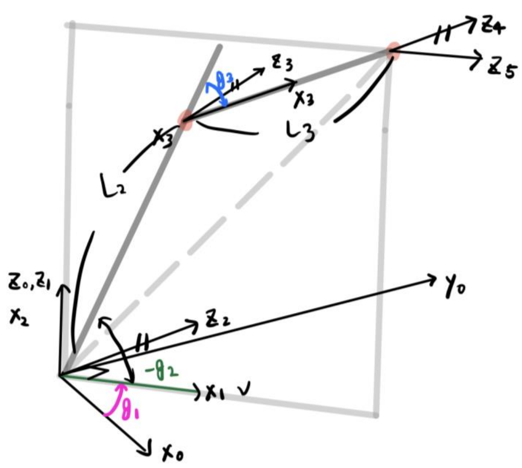
Inverse Kinematics – Yasukawa Motoman L-3

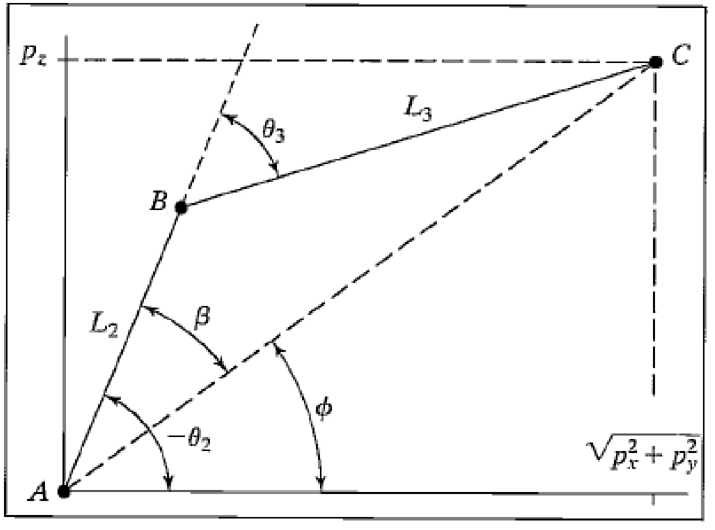




Inverse Kinematics – Yasukawa Motoman L-3

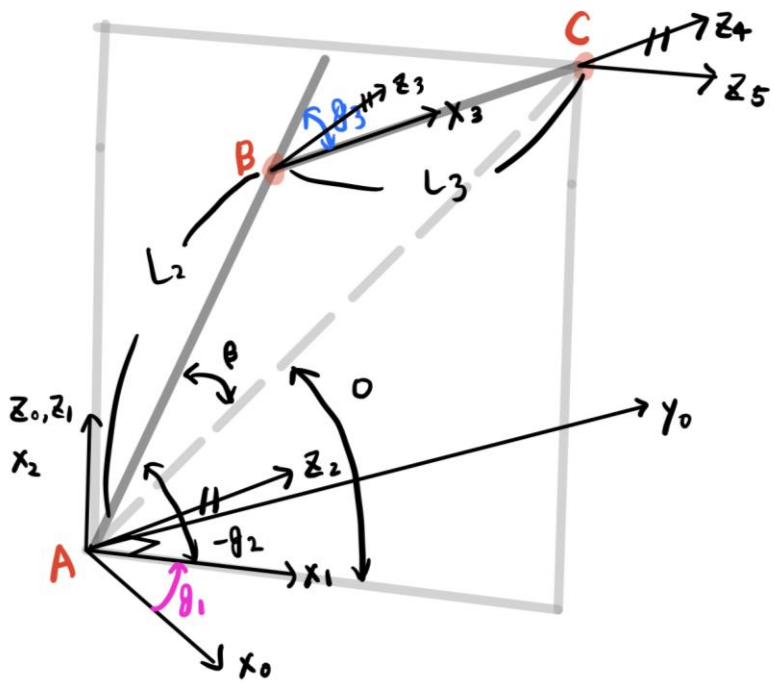






Inverse Kinematics – Yasukawa Motoman L-3

To solve for the individual angles θ_2, θ_3, and θ_4, we will take a geometric approach. Figure 4.10 shows the plane of the arm with point A at joint axis 2, point B at joint axis 3, and point C at joint axis 4.



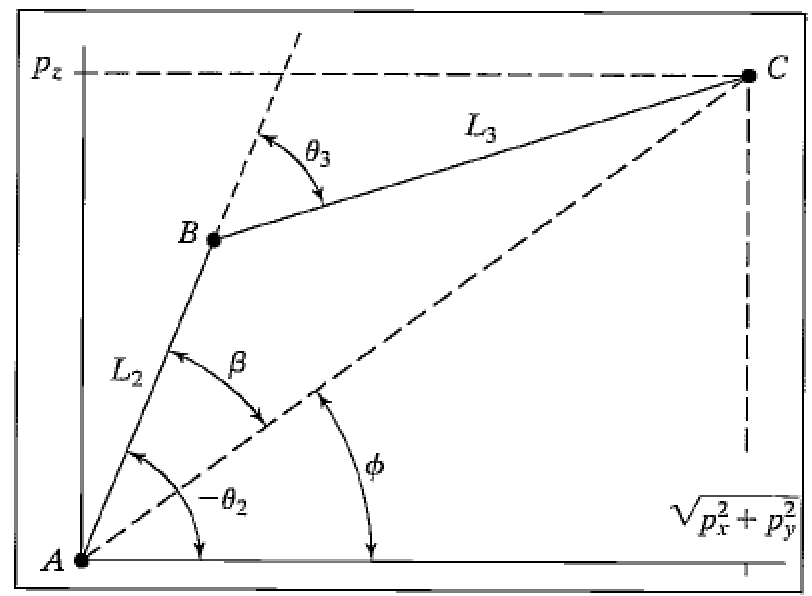
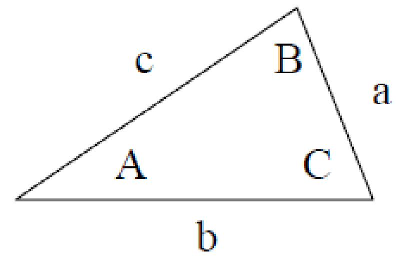


FIGURE 4.10: The plane of the Motoman manipulator.

Mathematical Equations

- Law of Sinus / Cosines - For a general triangle



$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

- Sum of Angles

$$\sin(\theta_1 \pm \theta_2) = s_{12} = c_1 s_2 \pm s_1 c_2$$

$$\cos(\theta_1 \pm \theta_2) = c_{12} = c_1 c_2 \mp s_1 s_2$$

Inverse Kinematics – Yasukawa Motoman L-3

Plane

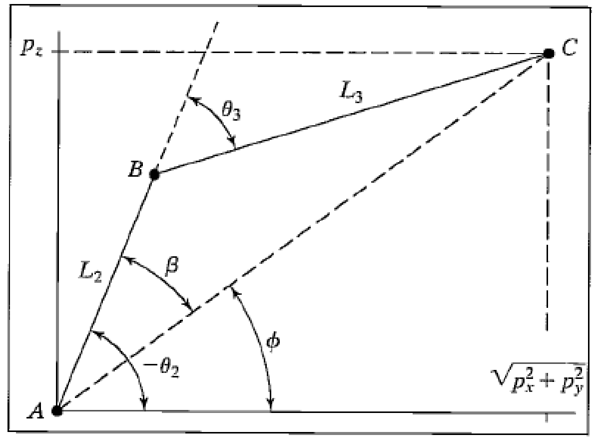


FIGURE 4.10: The plane of the Motoman manipulator.

From the law of cosines applied to triangle ABC, we have

$$\cos\theta_3 = \frac{p_x^2 + p_y^2 + p_z^2 - l_2^2 - l_3^2}{2l_2 l_3}.$$

Next, we have[6]

$$\theta_3 = \text{Atan2}\left(\sqrt{1-\cos^2\theta_3}, \cos\theta_3\right).$$

Inverse Kinematics – Yasukawa Motoman L-3

Plane

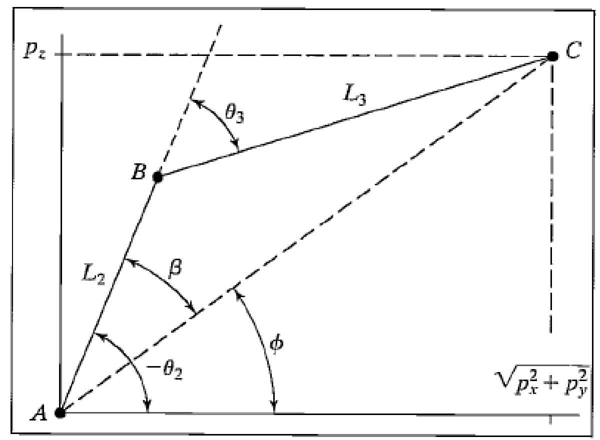


FIGURE 4.10: The plane of the Motoman manipulator.

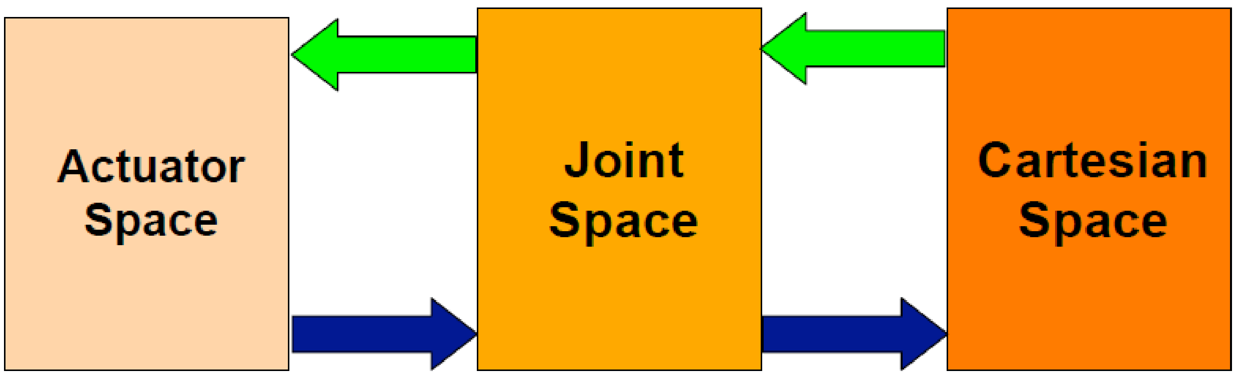
From Fig. 4.10, we see that $\theta_2 = -\phi - \beta$, or

$$\theta_2 = -\text{Atan2}\left(p_z, \sqrt{p_x^2 + p_y^2}\right) - \text{Atan2}(l_3 \sin\theta_3, l_2 + l_3 \cos\theta_3)$$

Finally, we have

$$\theta_4 = \theta_{234} - \theta_2 - \theta_3.$$

Actuator Space - Joint Space - Cartesian Space



Task Oriented Space
Operational Space

Forward Kinematics – Yasukawa Motoman L-3

$$\theta_1 = k_1 A_1 + \lambda_1,$$

$$\theta_2 = \cos^{-1}\left(\frac{(k_2 A_2 + \lambda_2)^2 - \alpha_2^2 - \beta_2^2}{-2\alpha_2\beta_2}\right) + \tan^{-1}\left(\frac{\phi_2}{\gamma_2}\right) + \Omega_2 - 270°,$$

$$\theta_3 = \cos^{-1}\left(\frac{(k_3 A_3 + \lambda_3)^2 - \alpha_3^2 - \beta_3^2}{-2\alpha_3\beta_3}\right) - \theta_2 + \tan^{-1}\left(\frac{\phi_3}{\gamma_3}\right) - 90°,$$

$$\theta_4 = -k_4 A_4 - \theta_2 - \theta_3 + \lambda_4 + 180°,$$

$$\theta_5 = -k_5 A_5 + \lambda_5.$$

Inverse Kinematics – Yasukawa Motoman L-3

Having solved for joint angles, we must perform the further computation to obtain the actuator values. Referring to Section 3.7, we solve equation (3.16) for the A_i:

$$
\begin{aligned}
A_1 &= \frac{1}{k_1}(\theta_1 - \lambda_1), \\
A_2 &= \frac{1}{k_2}\left(\sqrt{-2\alpha_2\beta_2\cos\left(\theta_2 - \Omega_2 - \tan^{-1}\left(\frac{\phi_2}{\gamma_2}\right) + 270^\circ\right) + \alpha_2^2 + \beta_2^2} - \lambda_2\right), \\
A_3 &= \frac{1}{k_3}\left(\sqrt{-2\alpha_3\beta_3\cos\left(\theta_2 + \theta_3 - \tan^{-1}\left(\frac{\phi_3}{\gamma_3}\right) + 90^\circ\right) + \alpha_3^2 + \beta_3^2} - \lambda_3\right), \\
A_4 &= \frac{1}{k_4}(180^\circ + \lambda_4 - \theta_2 - \theta_3 - \theta_4), \\
A_5 &= \frac{1}{k_5}(\lambda_5 - \theta_5).
\end{aligned} \tag{4.105}
$$

The cast that the wrist frame and tool frame differ only by a translation $\widehat{Z_W}$

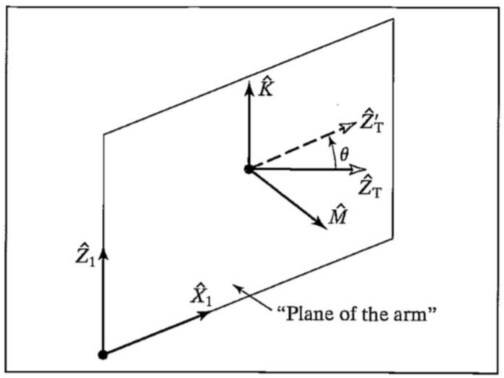


FIGURE 4.9: Rotating a goal frame into the Motoman's subspace.

In Fig. 4.9, we indicate the plane of the arm by its normal, $\hat{M}$, and the desired pointing direction of the tool by $\hat{Z}_T$. This pointing direction must be rotated by angle θ about some vector $\hat{K}$ in order to cause the new pointing direction, $\hat{Z}'_T$, to lie in the plane. It is clear that the $\hat{K}$ that minimizes θ lies in the plane and is orthogonal to both $\hat{Z}_T$ and $\hat{Z}'_T$.

For any given goal frame, $\hat{M}$ is defined as

$$\hat{M} = \frac{1}{\sqrt{p_x^2 + p_y^2}} \begin{bmatrix} -p_y \\ p_x \\ 0 \end{bmatrix}, \tag{4.84}$$

where p_x and p_y are the X and Y coordinates of the desired tool position. Then K is given by

$$K = \hat{M} \times \hat{Z}_T. \tag{4.85}$$

The new $\hat{Z}'_T$ is

$$\hat{Z}'_T = \hat{K} \times \hat{M}. \tag{4.86}$$

The amount of rotation, θ, is given by

$$\cos\theta = \hat{Z}_T \cdot \hat{Z}'_T,$$
$$\sin\theta = (\hat{Z}_T \times \hat{Z}'_T) \cdot \hat{K}. \tag{4.87}$$

Using Rodriques's formula (see Exercise 2.20), we have

$$\hat{Y}'_T = c\theta \hat{Y}_T + s\theta(\hat{K} \times \hat{Y}_T) + (1 - c\theta)(\hat{K} \cdot \hat{Y}_T)\hat{K}. \tag{4.88}$$

Finally, we compute the remaining unknown column of the new rotation matrix of the tool as

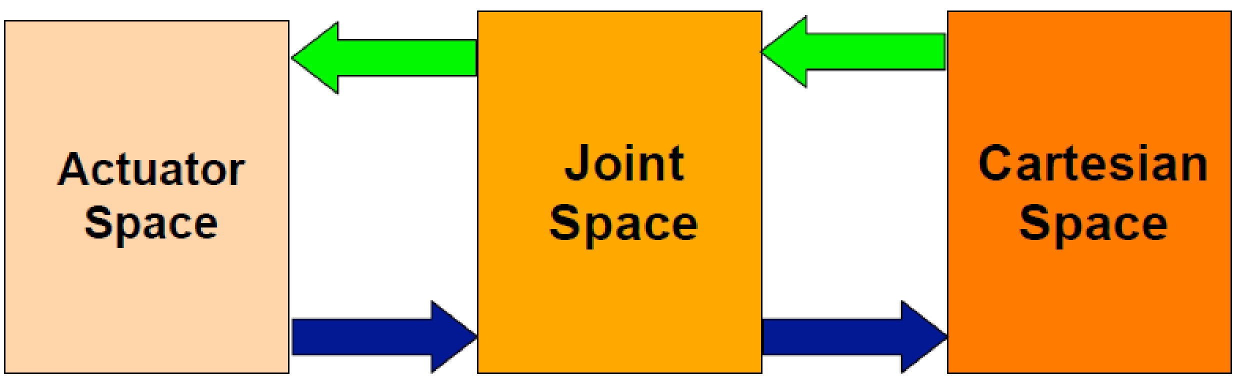
$$\hat{X}'_T = \hat{Y}'_T \times \hat{Z}'_T. \tag{4.89}$$

Equations (4.84) through (4.89) describe a method of projecting a given general goal orientation into the subspace of the Motoman robot.

Summary

- Forward Kinematics
- Inverse Kinematics
- Special case

  (Actuator space $\neq$ Joint space)



Next Discussion Section

- Robotics Toolbox – MATLAB