MAE 163B / 263B – Dynamics of Robotic System

Project No. 5

Newton-Euler Equations / Lagrange Equations – Dynamics – 2D

MAE 163B (Undergraduate Class) – Solve 1a, 1c, 1d, 2

MAE 263B (Graduate Class) – Solve All Parts

1. **Equations of Motion – Derivation** – Given the 2R manipulator architecture depicted in figure 1 and external force and external torque are applied on its end effector. Each link has a diagonal tensor of inertia diagonal. For sections a, b, c assume that gravity is pointing down.

    a. Use the **Newton-Euler formulation** and derive the equations of motion of the manipulator
    b. Use the **Lagrange formulation** and derive the equations of motion of the manipulator
    c. Using the equation of motion, assuming that the end effector needs to move from point (0.1,0) to (0.9,0) in 4s starting and ending the trajectory at zero velocity under horizontal force resisting the motion of 10N and a positive torque of 10N/m. Assume the link to have a cylindrical geometry with a length of 0.5m long each, outer diameter of 0.1m and a wall thickness of 0.005m, and they are made of aluminum with density of 2,710kg/m3
        i. Design the trajectory at the joint space.
        ii. Compute the joint torques as a function of time that are needed to complete the desired trajectory.
        iii. Plot:
            1. The end effector position, velocity and acceleration as a function of time
            2. The following joint torques as a function of time
                a. Joint torque resulted from the inertial element
                b. Joint torque resulted from the centrifugal element
                c. Joint torque resulted from the curiolis element
                d. Joint torque resulted from the gravitational element
                e. Total joint toques
    d. Repeat section c but this time the mechanism is placed sideways such the gravity is parallel to the rotation axis z_0 , z_1, z_2, z_3. The practical implication of this configuration is that gravity is no longer affect the dynamics – a similar situation to the first 2 DOF of the SCARA. For this scenario eliminate the gravity terms form the equations of motion. In addition to the plots in 1.c.iii add plot including the ratio of the total joint toques as a function of time defined by $\tau_{i\ witout\ gravity}/\tau_{i\ with\ gravity}$. Describe and comment on the results.

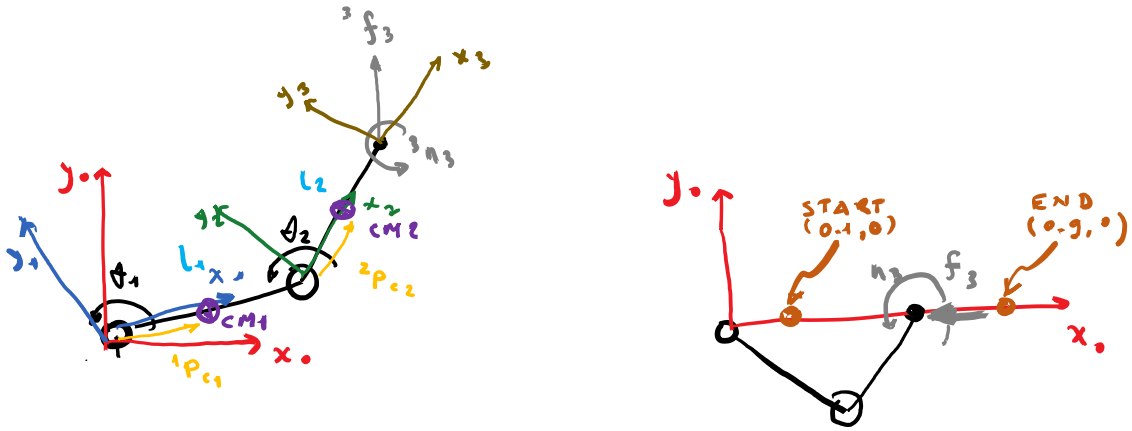

Figure 1 – 2R manipulator

2. Given the gemometry of the link depicted in figure 2, calculate the tensor of inertia of the link at the center of mass using a frame located at the CM with axes that are parallel to the axes of the frame at Joint i.

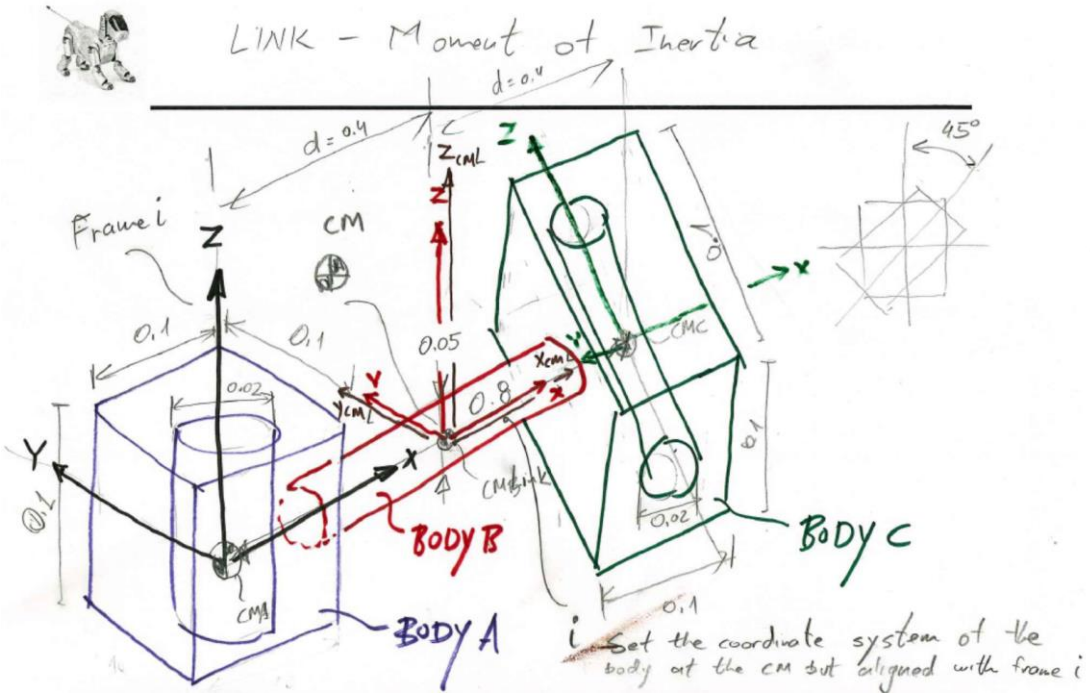

Figure 2 – Robotic link i