MAE 163B / 263B – Dynamics of Robotic System

Project No. 4

Jacobian Matrix & Design

In this HW use the same SCARA robot/gripper defined in HW 2 (SCARA Mitsubishi Arm - Model RH-3FRH5515 - Yamaha YRG-4220W

1. **Jacobian Derivation** - Derive the Jacobian in a parametric fashion (i.e. do not use numerical values for the DH parameters) using the following methods and express all the results in the base frame (i.e. frame {0}). Verify that all the expressions are identical
   - Velocity Propagation
   - Force Propagation
   - Explicit Method

2. **Singularities**
   a. **Pose Definition** - Define all the configurations in which the manipulator will reach singularity utilizing the Jacobean matrix. For defining these theoretical configurations ignore the joints' range of motion of the SCARA Mitsubishi Arm.
   a. **Implications** - Demonstrate analytically and explain in writing the implications of singularities from the following perspectives:
      i. Velocity
      ii. Force
      iii. Mathematical

3. **Jacobian Ellipsoid -** Given a 2R manipulator assume that the total length of the arm is 1m. Create 3 arm configurations:

   Configuration 1: L1=0.25m; L2-0.75m

   Configuration 2: L1=0.5m; L2=0.5m

   Configuration 3: L1=0.75m; L2=0.25m

   Position the end effector along the following points along the X axis starting with X=0m till X=1m with increments of 0.1m while maintaining Y=0 for all the points

   a. Define the Jacobian Matrix
   b. Define the Eigenvalues and eigenvectors of the JJ^T
   c. Write a Matlab script that calculate the Eigenvectors and Eigenvalues and plots the manipulability matrix for the velocity mapping (JJ^T) and for the force torque mapping $(JJ^T)^T$ You will need to plot 11 ellipses describing JJ^T and 11 ellipses describing $(JJ^T)^T$ for the following points $X = 0,0.1,0.2,0.3,0.4,0.5,0.6,0.7,0.8,0.9,1$ and $Y = 0$. Feel free to use the matlab function vellipse and fellipse, refer to TA's notes or the following link.
   https://robotacademy.net.au/lesson/velocity-ellipsoid-in-3d-and-manipulability/
   d. What is the best pose for each arm configuration?

e. What is the geometrical relationship between the velocities and the force torque ellipses?

4. **Design - Arm Optimization** - As a robotic designer you have the freedom to select link lengths of the SCARA along with its position with respect to the task and optimize for the task defined in HW2. For simplifying the optimization process, you are asked to optimize three parameters

The length of link 1 (L_1)

The length of link 2 (L_2)

The Goal function (C) is defined as follows

$$C = \max_{L_1,L_2} \frac{\left(\sum_{i=1}^{i=n} \kappa_i\right)\kappa_{min}}{(L_1)^3 + (L_2)^3}$$

Where:

L_1, L_2 – are the lengths of the first and the second links of the manipulator

$\kappa_i = \frac{\sqrt{\lambda_{min}}}{\sqrt{\lambda_{max}}}$ - The ration between the min and max squares roots of the eigenvalues of JJ^T

$\sum_{i=1}^{i=n} \kappa_i$ – is the sum of all the κ_i across the workspace and i is the number of selected discreet points in the workspace.

κ_{min} - is the minimal value of all the κ_i within workspace

$(L_1)^3$+$(L_2)^3$ – is the sum of the cube length of each link of the arm based on the fact that the stiffness of a cantilever beam is defined as $K = \frac{3EI}{L^3}$

a. Using a brute force approach calculate the Goal function (C) across the entire workspace i.e. 15x11 points i=165 using the length of L_1, L_2 to be in the range of 0 to 500 mm with increments of 10mm. Note that not all the 2500 (50x50) combinations of L_1, L_2 are possible. If the length of the two links cannot reach all the points in the workspace the pair of link lengths should not be considered at all as part of the optimization process.
b. Plot the values of the Goal function (C) as a function of L_1, L_2
c. Find the optimal values of L_1, L_2 that maximize the goal function across the workspace
d. For the best combination and the worst combination of L_1, L_2 plot the value of $\kappa_i = \frac{\sqrt{\lambda_{min}}}{\sqrt{\lambda_{max}}}$ for the entire workspace and explain
   i. Are the plots symmetric and if so why?

ii. Where are best and the worst manipulability points within the workspaces and why.

iii. Which points are closer to singularity within the workspace

e. The selected arm used for HW 2 was one among a family of several arms with different arm lengths. Check which one of the arms has a similar value to the optimal configuration and select the one which is most similar to the one proposed by the optimization process. See pages 44,46,48 in the following URL https://us.mitsubishielectric.com/fa/en/support/technical-support/knowledge-base/getdocument/?docid=3E26SJWH3ZZR-38-2664

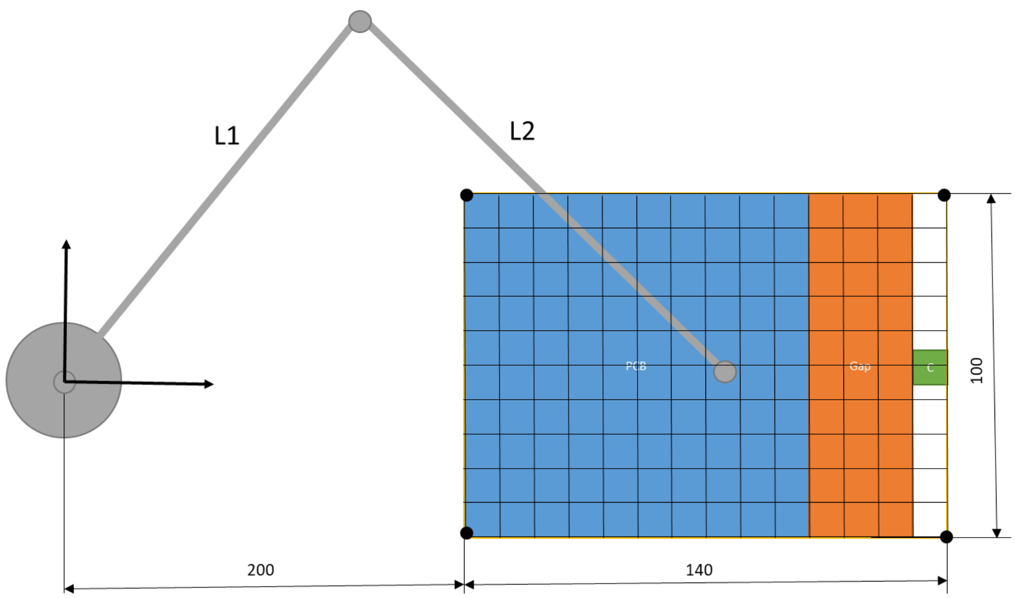

Figure: Simplified 2D version of the SCARA manipulator and the corresponding workspace. The workspace of 140x10 is discretized into 165 points (15x11). For each combination of f L_1, L_2 the goal function is calculated across the 165 points i=165 to produce a single value of the Goal function C.