MAE256B - Final Exam

The submission of the final exam must place in a folder including hard copy of the following material in the following order

- The following cover page with your name on it
- Solution for the Final Exam
- Corrected solution for Midterm
- Correct solution for HW1
- Correct solution for HW2
- Correct solution for HW3
- Correct solution for HW4

MAE256B - Final Exam

Name: ______________________________

	Criterion	Score	Maximum
1	**Correctness of Analysis** (Errors in work)		
1a	Newton Euler Approach		22
1b	Lagrange Approach		22
2a	Interpolation at the joint space		22
2b	Interpolation at the end effector tool space		22
2	**Thoroughness of Work** (Could someone else figure out what you did?)		
1a	Newton Euler Approach		3
1b	Lagrange Approach		3
2a	Interpolation at the joint space		3
2b	Interpolation at the end effector tool space		3
	Total		100
3	**Joint Torque**		20
4	**Matlab Simulation (Extra Question)**		20

Robotic Arm Configuration

Add the following gripper (Fig. 1) and tool to the Puma 560 (Fig.2). The circle is attached to the last link of the robotic arm. The gripper includes two fingers holding a laser cutter tool (Fig. 1). Given the coordinate system (in red) the position of the tip of the tool where the origin of the tool frame is attached expressed in the gripper coordinate system is ${}^{G}P_{T} = [-0.1, 0, 0.08]m$. The position of the origin of the gripper coordinate system expressed in the last coordinate system of the robot (frame 6) is ${}^{6}P_{G} = [0, 0, 0.05625]m$

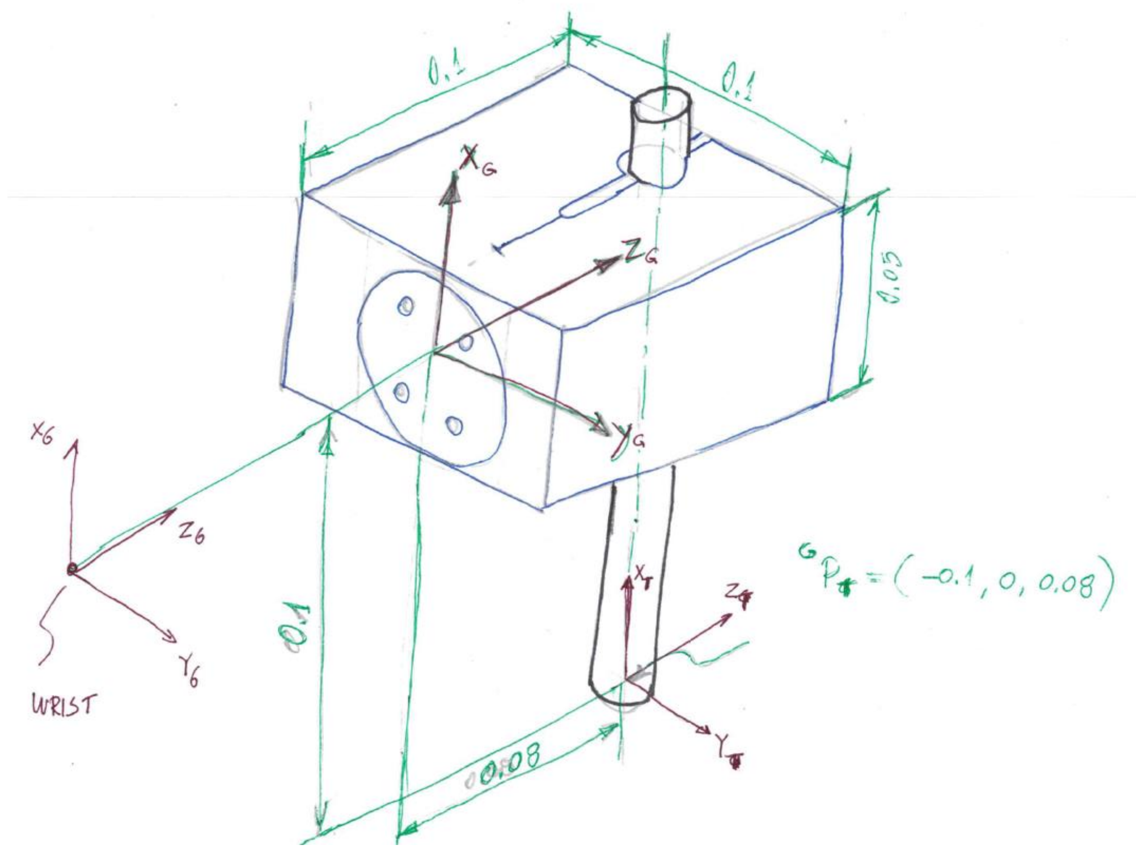

Fig.1 End Effector (gripper) Holding a plasma cutting tool.

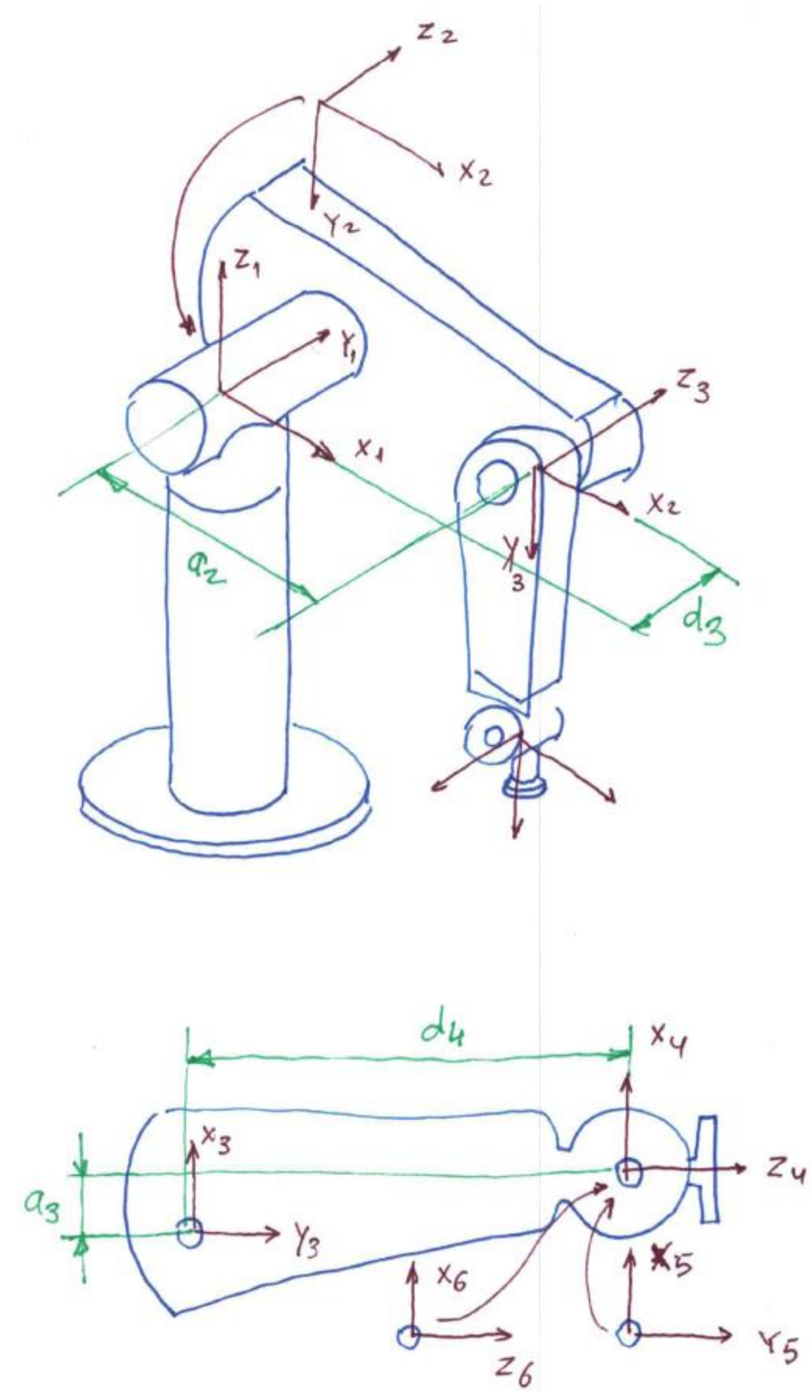

Fig. 2 - The PUMA 560 Coordinate system and geometrical dimensions - $a_2 = 0.4318m$; $a_3 = 0.0191m$ $d_3 = 0.1254m$ $d_2 = 0.4318m$

1) **Dynamics Equation** - Derive the dynamic equations of the robotic arm using only the **<u>first 3 DOF</u>** and formulate the three equations of motion using the two following methods. Use the standard form to express the final result. Note that the two methods should generate the same sets of three differential equations. Assume that The external forces and torques are acting on the tool in three orthogonal directions

$$\tau = M(\theta)\ddot{\theta} + V(\theta, \dot{\theta}) + G(\theta) + F(\theta, \dot{\theta})$$

a) Newton Euler Approach

b) Lagrange Approach

Mass and Inertia - Parameter

(1) **Mass Links 1,2,3** - Assume that every link has a mass and it center of mass is expressed with respect to its coordinate system

(2) **Inertia Links 1,2,3** - Assume that every link has an inertia matrix expressed with respect to the center of mass

$$^{i}I_c = \begin{bmatrix} ^{i}I_{c_{xx}} & 0 & 0 \\ 0 & ^{i}I_{c_{yy}} & 0 \\ 0 & 0 & ^{i}I_{c_{zz}} \end{bmatrix}$$

(3) Mass / Inertia Links 4,5,6, gripper - Lump the mass of the last 3 DOF with the end effector and the tool and assume that the manipulator carries it at the origin of frame 4. Assume no inertia for the lumped mass

(4) For a numerical assessment of the joint torque the a numerical value for the mass and inertia will be provided for section 3 (extra credit)

2) Trajectory Generation – The tool tip need to follow the parametric trajectory defined by the red shape as if the tip cut without touch a part from a sheep of metal. The dimension L is the length of the robotic arm when it is fully stretched.

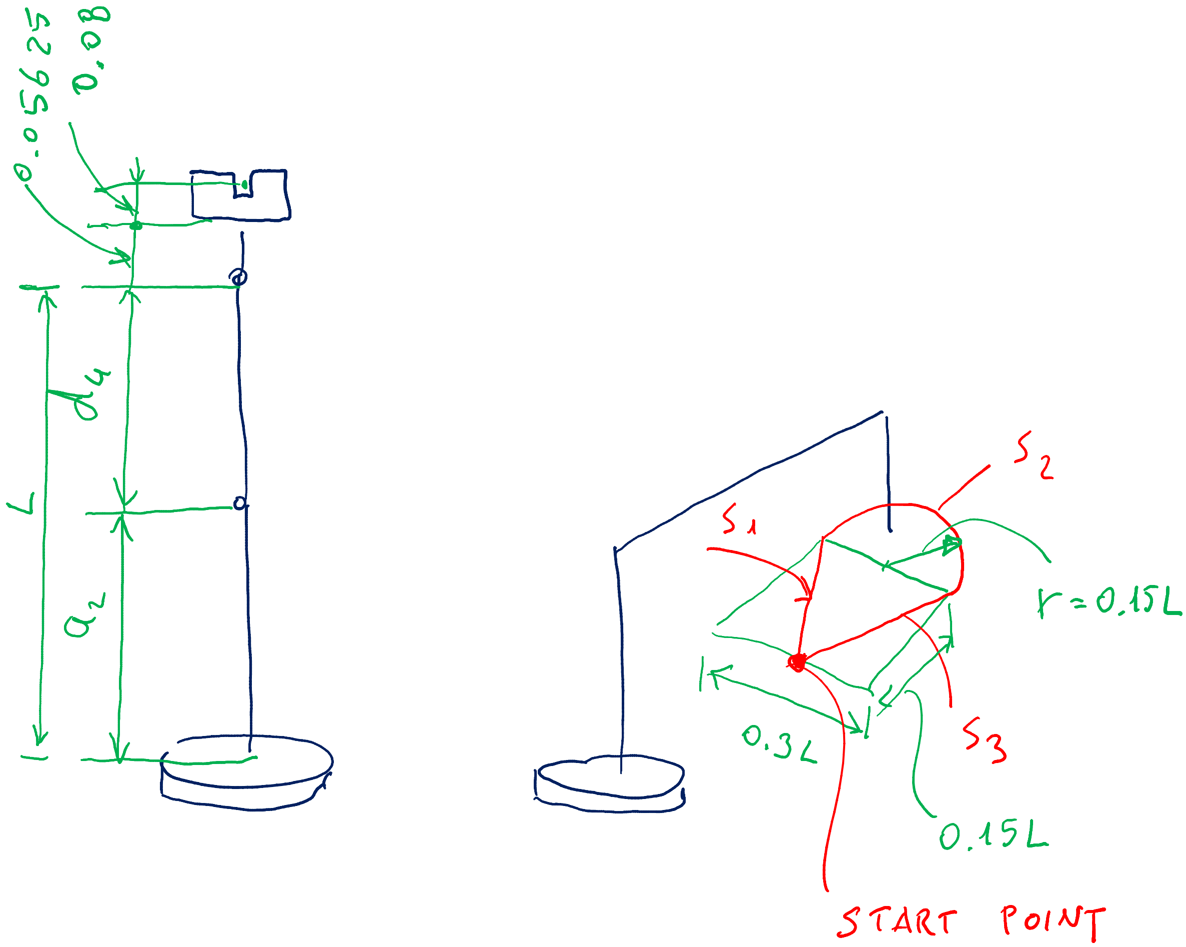

Figure 3 Trajectory definitions – Use $L = 0.8636m$

Specifications

- **Trajectory Location** - The trajectory can be translated but not rotated with respect to the base frame as long as it is flat as depicted in the diagram. You may start with the arm configuration in an L shape and check if it is within the workspace of the manipulator if not adjust it by translate it with respect to the base.

- **Trajectory Sequence –** The trajectory starts and ends at the same point as marked moving clockwise it should follow s1,s2,s3
- **Orientation of the Tool Tip** - Note that since the tool is cylindrical the tool can rotate along its long axis without any effect on the trajectory.
- **Accuracy and Via Point -** Add as many via points as needed such that the tip of the tool will not deviate from the expected trajectory by more than 0.5 mm assuming no errors in the joint angles or compliance of the links.
- **Completion Time -** The cut must be completed in 20s. Divide this time interval between the various segments base on the length of each segment
- **Velocity –** The tool starts/ends following the trajectory with a zero velocity. As the tool is making the transition between S1 to S2 as well as between S4 and S5 the velocity should be set to zero. However as the tool is making the transition between S2 to S3 as well as S3 to S4 the velocity should be continues.
- **Sampling Frequency –** The sampling frequency is 100Hz meaning that you need to calculate new position and joint angle evert 0.01 sec

Use the invers kinematics to calculate and plot the joint angles as a function of time with the following methods

a) Interpolation at the joint space

b) Interpolation at the end effector tool space

For both cases make sure that you meet the accuracy requirements

3) **Joint Torque (Extra Credit) –** Assuming that there are not external force or torque applied on the tip of the manipulator as it follows the given trajectory calculate the joint torques applied on the first three axes as a function of time while the tip followed the given trajectory. Plot the individual elements joint torques of each joint as well as well as the total joint torque Plot as a function of time

   a) $M(\theta)\ddot{\theta}$

   b) $V(\theta,\dot{\theta})$

   c) $G(\theta)$

   d) τ

Assume the following numerical values

Link	**Mass [Kg]**	**Center of Mass [m] [x,y,z]**	**Moment of Inertia [Kgm^2] [Icxx, Icyy, Iczz]**
1	0	[0,0,0]	[0,0,0.35]
2	17.4	[0.068, 0.006,-0.016]	[0.13,0.524,0.539]
3	4.8	[0,-0.070,0.014]	[0.066,0.0125,0.066]
4	0.82	*	*
5	0.34	*	*
6	0.09	*	*
Gripper	3	*	*

4) **Matlab Simulation (Extra Credit and or Extra Question)** – Develop a Matlab simulation for the problem 1 and 2 using the robotic tool box