5/22/15 MAE182A TA Session #8

- Review Chapter 7

Goals:

- systems of 1st order linear D.E.s
- Nth order O.D.E.s & linear 1st order systems
- Review of Matrices

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1. Systems of 1st order linear D.E.s

- system w/ simultaneous 1st order O.D.E.s

$$\begin{cases} x_1' = F_1(t, x_1, x_2, \cdots, x_n) \\ x_2' = F_2(t, x_1, x_2, \cdots, x_n) \\ \vdots \\ x_n' = F_n(t, x_1, x_2, \cdots, x_n) \end{cases}$$

- linear: F_k $(k=1,2,\cdots,n)$ is a linear fun of $(x_1, x_2, \cdots, x_n)$

non linear: F_k $(k=1,2,\cdots,n)$ is a non linear fun of $(x_1, x_2, \cdots, x_n)$

2. Nth order O.D.Es and linear 1st order systems

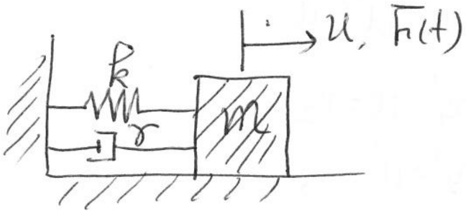
$$y^{(n)} = F(t, y, y', y'', \cdots, y^{(n-1)})$$

$$x_1 = y,\ x_2 = y',\ x_3 = y'',\ \cdots,\ x_n = y^{(n-1)}$$

$$\begin{cases} x_1' = y' = x_2 \\ x_2' = y'' = x_3 \\ x_3' = y''' = x_4 \\ \vdots \\ x_{n-1}' = y^{(n-1)} = x_n \\ x_n' = y^{(n)} = F(t, x_1, x_2, \cdots, x_n) \end{cases}$$

Nth order O.D.E. $\Rightarrow$ n 1st order O.D.E.s

<u>ex</u>



$$\sum_i F_i = m\ddot{u} = -ku - r\dot{u} + F(t)$$

$$\Rightarrow \quad m\ddot{u} + r\dot{u} + ku = F(t)$$

$$\Rightarrow \quad \ddot{u} = -\frac{r}{m}\dot{u} - \frac{k}{m}u + \frac{F(t)}{m}$$

$$\begin{cases} x_1 = u \\ x_2 = u' \end{cases} \Rightarrow \begin{cases} x_1' = u' = x_2 \\ x_2' = u'' = -\frac{k}{m}x_1 - \frac{r}{m}x_2 + \frac{F(t)}{m} \end{cases}$$

state: $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$$\dot{\vec{x}} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ -\frac{k}{m}x_1 - \frac{r}{m}x_2 + \frac{F(t)}{m} \end{bmatrix}$$

$$\Rightarrow \dot{\vec{x}} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{r}{m} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{F(t)}{m} \end{bmatrix}$$

2nd order O.D.E. ⇒ 2 1st order O.D.E.

ex $u^{(4)} - u = 0 \Rightarrow u^{(4)} = u$

$$\begin{cases} x_1 = u \\ x_2 = u' \\ x_3 = u'' \\ x_4 = u''' \end{cases} \Rightarrow \begin{cases} x_1' = u' = x_2 \\ x_2' = u'' = x_3 \\ x_3' = u''' = x_4 \\ x_4' = u^{(4)} = u = x_1 \end{cases}$$

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \Rightarrow \dot{\vec{x}} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} x_2 \\ x_3 \\ x_4 \\ x_1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

ex $u'' + 0.25u' + 4u = 2\cos 3t, \quad u(0) = 1, \ u'(0) = -2$

$$\begin{cases} x_1 = u \\ x_2 = u' \end{cases} \Rightarrow \begin{cases} x_1' = u' = x_2 \\ x_2' = u'' = -4u - 0.25u' + 2\cos 3t \end{cases}$$

$$\Rightarrow x_2' = -4x_1 - 0.25x_2 + 2\cos 3t$$

I.C.s.

$$\begin{cases} x_1(0) = u(0) = 1 \\ x_2(0) = u'(0) = -2 \end{cases}$$

3. Review of Matrices

- $A \in \mathbb{R}^{m\times n}$ or $\mathbb{C}^{m\times n}$

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} = (a_{ij})$$

- Transpose $A^T = (a_{ji})$
- Conjugate $A^T = (\overline{a_{ij}})$
- Adjoint $A^* = \overline{A^T}$

ex $A = \begin{bmatrix} 1 & 2+3i \\ 3-4i & 4 \end{bmatrix}$, $A^T = \begin{bmatrix} 1 & 3-4i \\ 2+3i & 4 \end{bmatrix}$ $A^* = \overline{A^T} = \begin{bmatrix} 1 & 3+4i \\ 2-3i & 4 \end{bmatrix}$

- Square matrix

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2\times2}, \quad B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}_{3\times3}$$

- Vectors

$$\left.\begin{aligned} &\text{column vector } \vec{X} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}_{3\times1} \\ &\text{row vector } \vec{y} = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}_{1\times3}. \end{aligned}\right\} \; y = x^T.$$

- Zero matrix

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

- Matrix equality

$A = (a_{ij})$ & $B = (b_{ij})$ are equal iff $a_{ij} = b_{ij}$, $\forall i, j$

- Scalar Multiplication

$$k \cdot A = (k\,a_{ij})$$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}, \Rightarrow -5A = \begin{bmatrix} -5 & -10 & -15 \\ -20 & -25 & -30 \end{bmatrix}$$

- Matrix addition & Substraction

$$A \pm B = (a_{ij} \pm b_{ij})$$

- Matrix multiplication

$$A = (a_{ij}),\ B = (b_{ij}),\ C = (c_{ij})$$

$$C = AB \Rightarrow c_{ij} = \sum_{k=1}^{n} a_{ik} b_{kj}$$

<u>Q:</u> $AB \overset{?}{=} BA$

$$[A]_{2\times3}\,[B]_{3\times2} = [C_1]_{2\times2} \neq [C_2]_{3\times3} = [B]_{3\times2} \cdot [A]_{2\times3}$$

C_1 & C_2 have different dimensions. $\Rightarrow C_1 \neq C_2$

- Vector multiplication
  - dot product, $x, y \in \mathbb{R}^{n\times1}$, $x^T y = [x_1, x_2, \cdots x_n] \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} = x_1y_1 + x_2y_2 + \cdots + x_ny_n$
  - inner product. $(x, y) = x^T \overline{y}$

ex $x = \begin{bmatrix} 1 \\ 2 \\ 3i \end{bmatrix}$, $y = \begin{bmatrix} -1 \\ 2-3i \\ 5+5i \end{bmatrix}$

dot product : $x^T y = [1\ \ 2\ \ 3i] \begin{bmatrix} -1 \\ 2-3i \\ 5+5i \end{bmatrix} = -12 + 9i$

inner product : $x^T \overline{y} = [1\ 2\ 3i] \begin{bmatrix} -1 \\ 2+3i \\ 5-5i \end{bmatrix} = 18 + 21i$

- Length of vector $\vec{x}$

$$\|\vec{x}\| = (\vec{x}, \vec{x})^{1/2} = \sum_{k=1}^{n} (x_k \bar{x}_k)^{\frac{1}{2}}$$

$$\vec{x} = \begin{bmatrix}1\\2\\3+4i\end{bmatrix} \Rightarrow \|\vec{x}\| = (\underbrace{\vec{x}, \vec{x}}_{x^T\bar{x}})^{\frac{1}{2}} = \sqrt{[1 \;\; 2 \;\; 3+4i]\begin{bmatrix}1\\2\\3-4i\end{bmatrix}} = \sqrt{1+4+(3+4i)(3-4i)} = \sqrt{30}$$

- Orthogonality

x & y are orthogonal if $(\vec{x}, \vec{y}) = 0$

$$\vec{x} = \begin{pmatrix}1\\2\\3\end{pmatrix}, \; \vec{y} = \begin{pmatrix}11\\-4\\-1\end{pmatrix} \Rightarrow (\vec{x}, \vec{y}) = (1 \;\; 2 \;\; 3)\begin{pmatrix}11\\-4\\-1\end{pmatrix} = 11-8-3 = 0.$$

$\vec{x} \perp \vec{y}$

- Identity matrix I : $AI = IA = A$

$$I = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix}$$

- Inverse Matrix A^{-1}

A is singular or invertible if $\exists B \ni AB = BA = I$,

otherwise A is singular.

$$B = A^{-1}$$

$$\begin{aligned} AB = I &\Rightarrow \det(AB) = \det(I) \\ &\Rightarrow \det(A) \cdot \det(B) = 1 \\ &\Rightarrow \det(B) = \frac{1}{\det(A)} \quad \therefore \det(A) \neq 0 \end{aligned}$$

- How to get A^{-1} ⇒ row reduction (Gaussian elimination)

$$(A|I) \Rightarrow (I|A^{-1})$$

- interchange 2 rows
- multiply a row by a nonzero scalar
- add a multiple of one row to another row

ex

$$(A|I) = \left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 2 & 4 & 5 & 0 & 1 & 0 \\ 3 & 5 & 6 & 0 & 0 & 1 \end{array}\right) \begin{matrix} *(-2) \\ *(-3) \end{matrix} \Rightarrow \left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 0 & -1 & -2 & 1 & 0 \\ 0 & -1 & -3 & -3 & 0 & 1 \end{array}\right) \begin{matrix} *(-1) \\ *(-1) \end{matrix}$$

$$\Rightarrow \left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 & -1 & 0 \\ 0 & 1 & 3 & 3 & 0 & -1 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 3 & 3 & 0 & -1 \\ 0 & 0 & 1 & 2 & -1 & 0 \end{array}\right) \begin{matrix} *(-3) \\ *(-3) \end{matrix}$$

$$\Rightarrow \left(\begin{array}{ccc|ccc} 1 & 2 & 0 & -5 & 3 & 0 \\ 0 & 1 & 0 & -3 & 3 & -1 \\ 0 & 0 & 1 & 2 & -1 & 0 \end{array}\right) *(-2) \Rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -3 & 2 \\ 0 & 1 & 0 & -3 & 3 & -1 \\ 0 & 0 & 1 & 2 & -1 & 0 \end{array}\right)$$

$$\underbrace{\begin{matrix} 1 & -3 & 2 \\ -3 & 3 & -1 \\ 2 & -1 & 0 \end{matrix}}_{= A^{-1}}$$

- Matrix differentiation & integration

$$\frac{dA}{dt} = \left(\frac{da_{ij}}{dt}\right), \quad \int_a^b A(t)\,dt = \int_a^b a_{ij}(t)\,dt$$

ex

$$A(t) = \begin{bmatrix} 3t^2 & \sin t \\ \cos t & 4 \end{bmatrix}$$

$$\frac{dA(t)}{dt} = \begin{bmatrix} 6t & \cos t \\ -\sin t & 0 \end{bmatrix}, \quad \int_0^\pi A(t)\,dt = \begin{bmatrix} \int_0^\pi 3t^2 dt & \int_0^\pi \sin t\,dt \\ \int_0^\pi \cos t\,dt & \int_0^\pi 4\,dt \end{bmatrix}$$

$$= \begin{bmatrix} \pi^3 & 2 \\ 0 & 4\pi \end{bmatrix}$$

p9

- System of Linear equations

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2 \\ \quad \vdots \qquad\qquad\qquad\qquad \vdots \\ a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n = b_n \end{cases}$$

$$\Rightarrow \underbrace{\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}}_{A} \cdot \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}}_{b} \qquad \begin{cases} b=0: \text{homogeneous} \\ b\neq 0: \text{nonhomogeneous} \end{cases}$$

ex non singular case: $\det A \neq 0$, A^{-1} exists

$$Ax = b \Rightarrow \underbrace{A^{-1}A}_{I}x = A^{-1}b \Rightarrow x = A^{-1}b$$

$$\begin{cases} x_1 - 2x_2 + 3x_3 = 7 \\ -x_1 + x_2 - 2x_3 = -5 \\ 2x_1 - x_2 - x_3 = 4 \end{cases} \Rightarrow \underbrace{\begin{bmatrix} 1 & -2 & 3 \\ -1 & 1 & -2 \\ 2 & -1 & -1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} 7 \\ -5 \\ 4 \end{bmatrix}}_{b}$$

$$(A|b) = \left(\begin{array}{ccc|c} 1 & -2 & 3 & 7 \\ -1 & 1 & -2 & -5 \\ 2 & -1 & -1 & 4 \end{array}\right) {\scriptstyle *1 \; *(-2)} \Rightarrow \left(\begin{array}{ccc|c} 1 & -2 & 3 & 7 \\ 0 & -1 & 1 & 2 \\ 0 & 3 & -7 & -10 \end{array}\right) {\scriptstyle \to *(-1)} \Rightarrow \left(\begin{array}{ccc|c} 1 & -2 & 3 & 7 \\ 0 & 1 & -1 & -2 \\ 0 & 3 & -7 & -10 \end{array}\right) {\scriptstyle *(-3)}$$

$$\Rightarrow \left(\begin{array}{ccc|c} 1 & -2 & 3 & 7 \\ 0 & 1 & -1 & -2 \\ 0 & 0 & -4 & -4 \end{array}\right) {\scriptstyle \to \div(-4)} \Rightarrow \left(\begin{array}{ccc|c} 1 & -2 & 3 & 7 \\ 0 & 1 & -1 & -2 \\ 0 & 0 & 1 & 1 \end{array}\right) {\scriptstyle *(-3) \; (*1)} \Rightarrow \left(\begin{array}{ccc|c} 1 & -2 & 0 & 4 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{array}\right) {\scriptstyle (*2)}$$

$$\Rightarrow \left(\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & \underbrace{1}_{x} \end{array}\right) \Rightarrow \vec{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$$

ex singular case $\det A = 0$, $\Rightarrow A^{-1}$ dose not exist

$$\begin{cases} x_1 - 2x_2 + 3x_3 = b_1 \\ -x_1 + x_2 - 2x_3 = b_2 \\ 2x_1 - x_2 + 3x_3 = b_3 \end{cases} \Rightarrow \underbrace{\begin{bmatrix} 1 & -2 & 3 \\ -1 & 1 & -2 \\ 2 & -1 & 3 \end{bmatrix}}_{A} \cdot \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}}_{b}$$

$$\det A = \begin{vmatrix} 1 & -2 & 3 \\ -1 & 1 & -2 \\ 2 & -1 & 3 \end{vmatrix} = 0 \Rightarrow A \text{ is singular.}$$

$$(A|b) = \left(\begin{array}{ccc|c} 1 & -2 & 3 & b_1 \\ -1 & 1 & -2 & b_2 \\ 2 & -1 & 3 & b_3 \end{array}\right) \begin{matrix} \\ \times(1) \\ \times(-2) \end{matrix} \Rightarrow \left(\begin{array}{ccc|c} 1 & -2 & 3 & b_1 \\ 0 & -1 & 1 & b_1+b_2 \\ 0 & 3 & -3 & -2b_1+b_3 \end{array}\right) \xrightarrow{\times(-1)} \left(\begin{array}{ccc|c} 1 & -2 & 3 & b_1 \\ 0 & 1 & -1 & -b_1-b_2 \\ 0 & 3 & -3 & -2b_1+b_3 \end{array}\right) \times(-3)$$

$$\Rightarrow \left(\begin{array}{ccc|c} 1 & -2 & 3 & b_1 \\ 0 & 1 & -1 & -b_1-b_2 \\ 0 & 0 & 0 & b_1+3b_2+b_3 \end{array}\right) \Rightarrow \begin{cases} x_1 - 2x_2 + 3x_3 = b_1 \\ x_2 - x_3 = -b_1 - b_2 \\ 0 \cdot x_3 = b_1 + 3b_2 + b_3 \end{cases}$$

1° $b_1 + 3b_2 + b_3 \neq 0 \Rightarrow 0 \cdot x_3 \neq 0 \Rightarrow x_1, x_2, x_3$ have no solutions.

2° $b_1 + 3b_2 + b_3 = 0 \Rightarrow b_1 = 2,\ b_2 = 1,\ b_3 = -5$

$$\begin{cases} x_1 - 2x_2 + 3x_3 = 2 \\ x_2 - x_3 = -3 \\ 0x_3 = 0 \end{cases} \Rightarrow \begin{cases} x_1 = 2(x_3 - 3) - 3x_3 + 2 = -x_3 - 4 \\ x_2 = x_3 - 3 \\ x_3 \end{cases} \Rightarrow \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \text{ have infinite sol'ns}$$

$$\therefore x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -x_3 - 4 \\ x_3 - 3 \\ x_3 \end{pmatrix} = x_3 \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} + \begin{pmatrix} -4 \\ -3 \\ 0 \end{pmatrix}.$$

- Linear dependence & indepedence

- vectors $x_1, x_2, \cdots x_n$ is linearly dept. if

$$c_1x_1 + c_2x_2 + \cdots + c_nx_n = 0, \ni c_1, c_2, \cdots, c_n \text{ are not all zeros.}$$

- $x_1, x_2, \cdots, x_n$ is linearly indept. if

$$c_1x_1 + c_2x_2 + \cdots + c_nx_n = 0 \ni c_1 = c_2 = \cdots = c_n = 0$$

ex $x_1 = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}, \quad x_2 = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}, \quad x_3 = \begin{bmatrix} -4 \\ 1 \\ -11 \end{bmatrix}$

$$c_1x_1 + c_2x_2 + c_3x_3 = 0$$

$$c_1\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} + c_2\begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} + c_3\begin{bmatrix} -4 \\ 1 \\ -11 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \underbrace{\begin{bmatrix} 1 & 2 & -4 \\ 2 & 1 & 1 \\ -1 & 3 & -11 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}}_{c} = \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}}_{b}$$

$$(A|b) = \left(\begin{array}{ccc|c} 1 & 2 & -4 & 0 \\ 2 & 1 & 1 & 0 \\ -1 & 3 & -11 & 0 \end{array}\right) {\scriptstyle *(-2)} \; {\scriptstyle (*1)} \Rightarrow \left(\begin{array}{ccc|c} 1 & 2 & -4 & 0 \\ 0 & -3 & 9 & 0 \\ 0 & 5 & -15 & 0 \end{array}\right) \begin{array}{l} \\ {\scriptstyle /(-3)} \\ {\scriptstyle /(5)} \end{array} \Rightarrow \left(\begin{array}{ccc|c} 1 & 2 & -4 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & 1 & -3 & 0 \end{array}\right) {\scriptstyle *(-1)}$$

$$\Rightarrow \left(\begin{array}{ccc|c} 1 & 2 & -4 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \Rightarrow \left\{\begin{array}{r} c_1 + 2c_2 - 4c_3 = 0 \\ c_2 - 3c_3 = 0 \\ 0 \cdot c_3 = 0 \end{array}\right. \Rightarrow \left\{\begin{array}{l} c_1 = -2c_3 = -2c \\ c_2 = 3c_3 = 3c \\ c_3 = c \end{array}\right.$$

$$C = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix} c \xRightarrow{\boxed{c=-1}} C = \begin{bmatrix} +2 \\ -3 \\ -1 \end{bmatrix} \Rightarrow 2x_1 - 3x_2 - x_3 = 0$$

$c_1, c_2, \& c_3$ are not zeros

$\Rightarrow x_1, x_2, x_3$ are linearly dept.

· Eigenvalues & Eigenvectors

$$Ax = \lambda x \quad \det x \neq 0$$

$$\Rightarrow (A-\lambda I)\cdot x = 0$$

$$\Rightarrow \underbrace{\det(A-\lambda I)}_{\overset{\shortparallel}{0}} \cdot \underbrace{\det x}_{\neq 0} = 0$$

$$\Rightarrow \det(A-\lambda I) = 0 \quad \begin{pmatrix} \lambda: \text{e-Val} \\ x: \text{e-Vec} \end{pmatrix}$$

ex $A = \begin{bmatrix} 3 & -1 \\ 4 & -2 \end{bmatrix}$

$$Ax = \lambda x \Rightarrow \det(A-\lambda I) = 0$$

$$\Rightarrow \det\left(\begin{bmatrix} 3 & -1 \\ 4 & -2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}\right) = \det\begin{bmatrix} 3-\lambda & -1 \\ 4 & -2-\lambda \end{bmatrix} = 0.$$

$$\Rightarrow (3-\lambda)(-2-\lambda)+4 = 0$$

$$\Rightarrow \lambda^2-\lambda-2 = (\lambda-2)(\lambda+1) = 0$$

$$\Rightarrow \lambda = 2, -1$$

1° $\lambda_1 = 2$

$$(A-\lambda I)x = 0 \Rightarrow \begin{bmatrix} 3-2 & -1 \\ 4 & -2-2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0 \Rightarrow \begin{bmatrix} 1 & -1 \\ 4 & -4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow x_1 = x_2 = c$$

$$\vec{X}_1 = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} c \\ c \end{bmatrix} \overset{c=1}{\Longrightarrow} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

2° $\lambda_2 = -1$

$$(A-\lambda I)x = 0 \Rightarrow \begin{bmatrix} 3+1 & -1 \\ 4 & -2+1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0 \Rightarrow \begin{bmatrix} 4 & -1 \\ 4 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow 4x_1 - x_2 = 0$$

$$\Rightarrow x_2 = 4\underset{c}{x_1} = 4c$$

$$\vec{X}_2 = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} c \\ 4c \end{bmatrix} \overset{c=1}{=} \begin{bmatrix} 1 \\ 4 \end{bmatrix}.$$