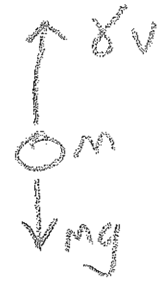
Ch. 1 Introduction

Basic example: Falling object



Two forces acting = gravity, drag

↑ assume drag force $\propto$ velocity

$F_d = \gamma v$ ← const.

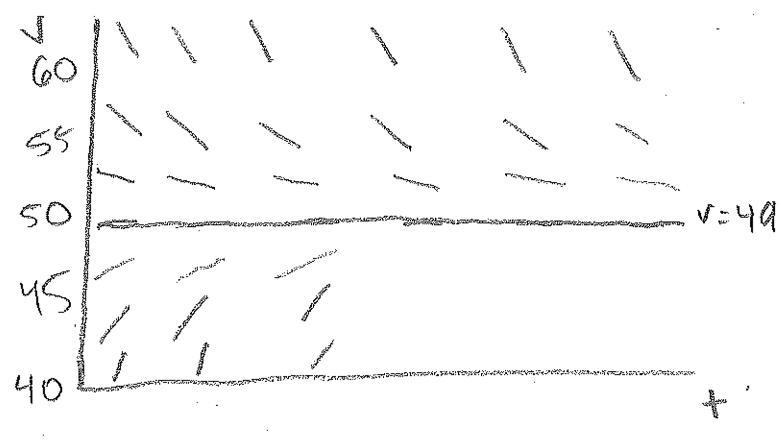
$\Sigma F = ma = m\frac{dv}{dt}$

$m\frac{dv}{dt} = mg - \gamma v$

let $m = 10\,kg$, $\gamma = 2\,\frac{kg}{s}$

$\rightarrow \frac{dv}{dt} = 9.8 - \frac{v}{5}$ gives acceleration at any velocity

We can plot a <u>direction (slope) field</u>



we haven't found a solution $v(t)$ so we cant say anything about v just $\frac{dv}{dt}$

When is $\frac{dv}{dt} = 0$? $v = 49\ m/s$

↑ Terminal Velocity

Forces are equal so v is const!

"equilibrium Solution"

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drug → patient

- Fluid $5 mg/cm^3$ of drug given at $100 cm^3/h$
- drug absorbed or leaves in blood at rate proportional to amount present with rate const. $0.4 (h)^{-1}$

$M(t)$ is amount of drug at any time

$$\frac{dM(t)}{dt} = 5\frac{mg}{cm^3}\left(\frac{100 cm^3}{h}\right) - 0.4M(t) \quad \boxed{mg/h}$$

$$\frac{dM}{dt} = 500 - .4M$$

what is the equilibrium amount after a long time?

$$\frac{dM}{dt} = 0 = 500 - .4M$$

$$\underline{M = 1250 mg}$$

Solutions to some Diff. Eq.

$$m\frac{dv}{dt} = mg - \gamma v$$ falling object

$$\frac{dy}{dt} = ay - b$$ Basic Form

const. coeff.

seperating varicbles and integrating

$$\frac{dy/dt}{y - \left(\frac{b}{a}\right)} = a$$

$$\ln\left|y - \frac{b}{a}\right| = at + c$$

$$y = \left(\frac{b}{a}\right) + ce^{at}$$ <u>general solution</u>

if we have an initial condition we can find the unique solution

$$y(0) = y_0 \rightarrow y = \left(\frac{b}{a}\right) + \left[y_0 - \frac{b}{a}\right]e^{at}$$

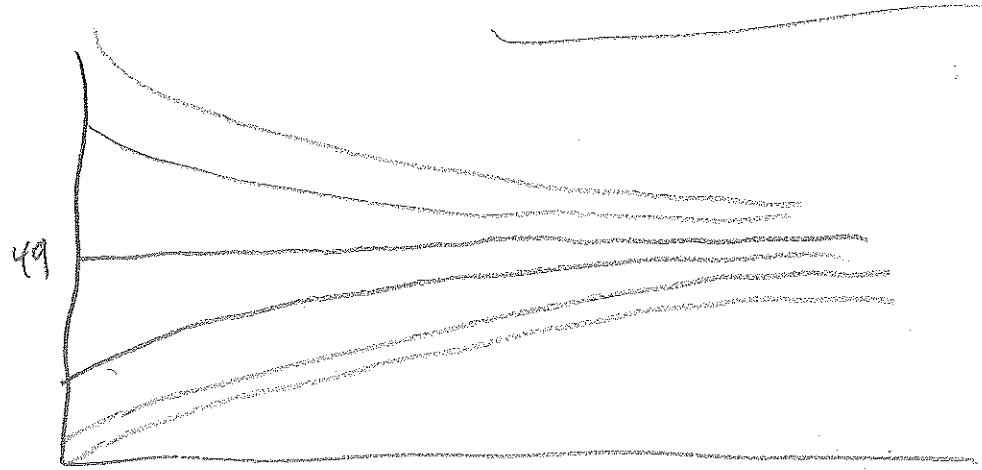
Back to falling object...

$$\frac{dv}{dt} = 9.8 - \frac{v}{5}$$

dropped from 300 m and $v(0) = 0$

$$\frac{\frac{dv}{dt}}{v - 49} = \frac{-1}{5} \rightarrow v = 49 + Ce^{-t/5}$$

apply IC $\rightarrow$ $\underline{v = 49(1 - e^{-t/5})}$



as $t \to \infty$ $v = 49$ terminal velocity

for position we integrate velocity

$$\frac{dx}{dt} = 49(1 - e^{-t/5})$$

$$x = 49t + 245e^{-t/5} + C$$

at $t = 0$ $x = 0$

$$\underline{x = 49t + 245e^{-t/5} - 245}$$

hits ground at $x = 300$ at $t = T$

$\underline{300 = 49T + 245e^{-T/5} - 245}$ need numerical process

Classification of Diff-Eq.

ordinary - dependent variable fn of only one independent variable

$$L \frac{d^2 Q(t)}{dt^2} + R \frac{dQ(t)}{dt} + \frac{1}{C} Q(t) = E(t)$$

partial - $\alpha^2 \frac{\partial u(x,t)}{\partial x^2} = \frac{\partial u(x,t)}{\partial t}$

for ordinary DE

$$F(t, y, y', \ldots y^{(n)}) = 0$$

<u>Order</u> = n - highest derivative

<u>Linear</u> if linear fn of derivatives

<u>t^2 ok</u> y^2, $\sin y'$, yy'' is <u>nonlinear</u>!