MAE 182A Project 8 Solution

1. $\frac{dT}{dt} = k(T - T_m)$. $T_m = 50F$, $T(0) = 85F$, $T(-\frac{1}{2}) = 84F$

$$\Rightarrow \int \frac{dT}{T-T_m} = \int k\,dt \Rightarrow \ln|T - T_m| = kt + c'$$

$$\Rightarrow T - T_m = \underbrace{\pm e^{c'}}_{c} e^{kt} = c\,e^{kt}$$

$$\Rightarrow T = T_m + c\,e^{kt}$$

$$\therefore \underline{T(t) = 50 + c\,e^{kt}}$$

1° I.C. 1: $T(0) = 85 = 50 + ce^{0} \Rightarrow \underline{c = 35}$

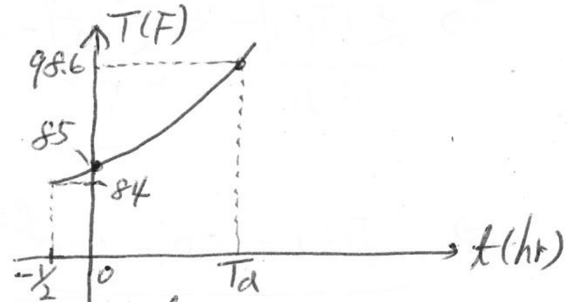
$$\therefore T(t) = 50 + 35e^{kt}$$

2° I.C. 2: $T(-\frac{1}{2}) = 84 = 50 + 35e^{-\frac{1}{2}k} \Rightarrow 34 = 35e^{-\frac{1}{2}k}$

$$\Rightarrow \ln\left(\frac{34}{35}\right) = -\frac{1}{2}k$$

$$\Rightarrow \underline{k = -2\ln\left(\frac{34}{35}\right) = 0.058}$$

$$\therefore \underline{T(t) = 50 + 35e^{0.058t}}$$



3° Assume the time of death was $t = T_d$ hours before 6 a.m.
At the moment Joe was killed, his body temperature was 98.6F.

$$\therefore T(t = T_d) = 98.6 = 50 + 35e^{0.058T_d}$$

$$\Rightarrow 48.6 = 35e^{0.058T_d}$$

$$\Rightarrow T_d = \left(\frac{1}{0.058}\right)\ln\left(\frac{48.6}{35}\right) \approx 5.66 \text{ hrs} \approx \underline{5 \text{ hrs } 40 \text{ mins}}$$

$\therefore$ Joe was killed at about 5 hrs 40 mins before 6 a.m.

or Joe was killed at around <u>12:20 am</u>.

2. $T_m(t) = 50 + 20\,u(t-h)$, $T(0) = 85F$

$$\frac{dT}{dt} = k(T - T_m(t))$$

$$\xrightarrow{\mathcal{L}} \mathcal{L}[T'(t)] = k\mathcal{L}[T - T_m(t)] = k\mathcal{L}[T(t)] - k\mathcal{L}[T_m(t)]$$

$$\Rightarrow sT(s) - T(0) = kT(s) - k\mathcal{L}[50 + 20u_h(t)]$$

$$\Rightarrow (s-k)T(s) = -k\mathcal{L}[50] - k\mathcal{L}[20u_h(t)] + T(0)$$

$$\Rightarrow (s-k)T(s) = \frac{-50k}{s} - \frac{20ke^{-hs}}{s} + 85$$

$$\Rightarrow T(s) = \frac{-50k}{s(s-k)} - \frac{20ke^{-hs}}{s(s-k)} + \frac{85}{s-k}$$

$$\Rightarrow T(s) = \frac{50}{s} - \frac{50}{s-k} + \left[\frac{20}{s} - \frac{20}{s-k}\right]e^{-hs} + \frac{85}{s-k}$$

$$\xrightarrow{\mathcal{L}^{-1}} \underbrace{\mathcal{L}^{-1}[T(s)]}_{T(t)} = 50\underbrace{\mathcal{L}^{-1}\left[\frac{1}{s}\right]}_{1} - 50\underbrace{\mathcal{L}^{-1}\left[\frac{1}{s-k}\right]}_{e^{kt}} + 20\underbrace{\mathcal{L}^{-1}\left[\frac{e^{-hs}}{s}\right]}_{u_h(t)} - 20\underbrace{\mathcal{L}^{-1}\left[\frac{e^{-hs}}{s-k}\right]}_{\mathcal{L}[e^{-hs}F(s)] = u_h(t)f(t-h)} + 85\underbrace{\mathcal{L}^{-1}\left[\frac{1}{s-k}\right]}_{e^{kt}}$$

$$\Rightarrow T(t) = 50 - \underbrace{50e^{kt}} + 20u_h(t) - 20u_h(t)e^{k(t-h)} \underbrace{+ 85e^{kt}}$$

or

$$\Rightarrow T(t) = 50 + 35e^{kt} + 20u_h(t)\left[1 - e^{k(t-h)}\right], \quad k = 0.058$$

3. 1° Time body moved $T_v = 6\,a.m. - h$

2° Time of death

From Pr. 1, the estimated time of death $t = T_d = 5.66$ hrs before 6 a.m.

① if $t = T_d = 5.66 < h \Rightarrow u(t-h) = 0$

From Pr. 2. $T(t) = 50 + 35e^{kt}$

$$\Rightarrow T(t_d) = 98.6 = 50 + 35e^{0.058t_d}$$

$$\Rightarrow t_d = \left(\frac{1}{0.058}\right) \ln\left(\frac{48.6}{35}\right) \approx 5.66 \text{ hrs}$$

∴ Time of death was 12:20 am as that in Pr 1 when $h > 5.66$ hrs.

② if $t = T_d = 5.66 \geq h \Rightarrow u(t-h) = 1$

$$\therefore T(t_d) = 98.6 = 50 + 35e^{0.058t_d} + 20\left[1 - e^{-0.058h} \cdot e^{0.058t_d}\right]$$

$$\Rightarrow 28.6 = \left[35 - 20e^{-0.058h}\right] e^{0.058t_d}$$

$$\Rightarrow t_d = \frac{1}{0.058} \ln\left(\frac{28.6}{35 - 20e^{-0.058h}}\right) \quad (h \leq 5.66\text{hrs})$$

3° ① $h=12$

Time body moved $T_v = 6am - 12 = 6{:}00pm$

$h > 5.66$,

time of death = 12:20 am

② $h=11$

$T_v = 6am - 11 = 7{:}00pm$

$h = 11 > 5.66$, time of death = 12:20 am

③ $h=10$

$T_v = 6am - 10 = 8{:}00pm$

$h = 10 > 5.66$, time of death = 12:20 am

④ $h=9$

$T_v = 6am - 9 = 9{:}00pm$

$h = 9 > 5.66$, time of death = 12:20 am

⑤ $h=8$

$T_v = 6am - 8 = 10{:}00pm$

$h = 8 > 5.66$, time of death = 12:20 am

⑥ $h=7$

$T_v = 6am - 7 = 11{:}00pm$

$h = 7 > 5.66$, time of death = 12:20 am

⑦ $h=6$

$T_v = 6am - 6 = 12{:}00pm$

$h = 6 > 5.66$, time of death = 12:20 am

⑧ $h=5$

$T_v = 6am - 5 = 1{:}00am$

$h = 5 < 5.66$. $t_d = \frac{1}{0.058} \ln\left(\frac{28.6}{35 - 20e^{-0.058 \cdot 5}}\right) = 6.1 hrs \approx 6 hrs\ 6 mins$ before 6am

$\therefore$ time of death = 6am − 6 hrs 6 mins = 11:54 pm

(9) $h=4$

$T_v = 6am - 4 = \underline{2am}$

$h = 4 < 5.66$

$$t_d = \frac{1}{0.058} \ln\left(\frac{28.6}{35-20e^{-0.058 \cdot 4}}\right) = 6.9 \text{ hrs} = 6 \text{ hrs } 54 \text{ mins}$$ before 6am

$\therefore$ time of death $= 6am - 6h\,54m = \underline{11:06pm}$

(10) $h=3$

$T_v = 6am - 3 = \underline{3am}$

$h = 3 < 5.66$

$$t_d = \frac{1}{0.058} \ln\left(\frac{28.6}{35-20e^{-0.058 \cdot 3}}\right) = 7.8 \text{ hrs} = 7 \text{ hrs } 48 \text{ mins}$$ before 6am

$\therefore$ time of death $= 6am - 7h\,48m = \underline{10:12pm}$

(11) $h=2$

$T_v = 6am - 2 = \underline{4am}$

$h = 2 < 5.66$

$$t_d = \frac{1}{0.058} \ln\left(\frac{28.6}{35-20e^{-0.058 \cdot 2}}\right) = 8.8 \text{ hrs} = 8 \text{ hrs } 48 \text{ mins}$$ before 6am

$\therefore$ time of death $= 6am - 8h\,48m = \underline{9:12pm}$

h	time body moved	time of death
12	6:00 pm	12:20 am
11	7:00 pm	12:20 am
10	8:00 pm	12:20 am
9	9:00 pm	12:20 am
8	10:00 pm	12:20 am
7	11:00 pm	12:20 am
6	12:00 pm	12:20 am
5	1:00 am	11:54 pm
4	2:00 am	11:06 pm
3	3:00 am	10:12 pm
2	4:00 am	9:12 pm

4. List the time suspects were seen and left

	Name	time seen	left time
Suspect 1	Twinkles	5 ~ 6 pm	a little after 6pm
Suspect 2	Slim	around 10 pm	around 11 pm
Suspect 3	Shorty (the cook)	~~took a long break~~ at 10:30 pm	2 am

1° The estimated time Joe was killed was around 12:20 am.

Both suspect 1 and 2, left before 12:20 am.

Therefore, suspects 1 & 2 are not on the list to question.

2° For suspect 3. shorty was seen in the restaurant between 10:30 pm and 2:00 am.

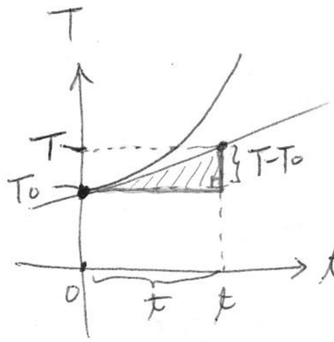
From the table in Pr. 3, 2 situations may be possible.

① Shorty killed Joe at around 11:54 pm and moved his body at around 1:00 am.

② shorty killed Joe at around 11:06 pm and moved his body at around 2:00 am.

Therefore, Daphne should question Shorty (the cook).

5. $$\frac{dT}{dt} = k(T - T_m), \quad T(0) = T_0$$



$$\frac{T - T_0}{t - 0} = \left.\frac{dT}{dt}\right|_{t=0} = k(T_0 - T_m)$$

$$\Rightarrow \quad T - T_0 = k(T_0 - T_m)t$$

$$\Rightarrow \quad t = \frac{T - T_0}{k(T_0 - T_m)}$$

$\boxed{T = 98.4}$

$$\Rightarrow \quad t = \frac{98.4 - T_0}{k(T_0 - T_m)}$$