MAE 182A Project Solution

1. \[
\frac{dT}{dt} = k(T - T_m) \quad T_m = 50\,\text{F}, \quad T(0) = 85\,\text{F}, \quad T(\frac{1}{2}) = 84\,\text{F}
\]

\[
\Rightarrow \int \frac{dT}{T - T_m} = \int k \, dt \Rightarrow \ln |T - T_m| = kt + c'
\]

\[
\Rightarrow T - T_m = \pm e^{c'} e^{kt} = c e^{kt}
\]

\[
\Rightarrow T = T_m + c e^{kt}
\]

\[
T(t) = 50 + ce^{kt}
\]

1° I.C. 1. \[T(0) = 85 = 50 + ce^0 \Rightarrow c = 35\]

\[T(t) = 50 + 35e^{kt}\]

2° I.C. 2. \[T(\frac{1}{2}) = 84 = 50 + 35e^{-\frac{1}{2}k} \Rightarrow 34 = 35e^{-\frac{1}{2}k}\]

\[\Rightarrow \ln \left(\frac{34}{35}\right) = -\frac{1}{2}k\]

\[\Rightarrow k = -2 \ln \left(\frac{34}{35}\right) = 0.058\]

\[T(t) = 50 + 35e^{0.058t}\]

3° Assume the time of death was \(t = T_a\) hours before 6 a.m. At the moment Joe was killed, his body temperature was 98.6 F.

\[T(t = T_a) = 98.6 = 50 + 35e^{0.058T_a}\]

\[\Rightarrow 48.6 = 35e^{0.058T_a}\]

\[\Rightarrow T_a = \left(\frac{1}{0.058}\ln \left(\frac{48.6}{35}\right)\right) \approx 5.66 \text{ hrs} \approx 5 \text{ hrs 40 mins}\]

Joe was killed at about 5 hrs 40 mins before 6 a.m. or Joe was killed at around 12:20 a.m.
2. \( T_m(t) = 50 + 20 \left( e^{lt} - h \right) \), \( T(0) = 85 \)

\[ \frac{dT}{dt} = k (T - T_m(t)) \]

\[ \Rightarrow \mathcal{L} \left[ T'(s) \right] = k \mathcal{L} \left[ T - T_m(t) \right] = k \mathcal{L} \left[ T(t) \right] - k \mathcal{L} \left[ T_m(t) \right] \]

\[ \Rightarrow sT(s) - T(0) = kT(s) - k \mathcal{L} \left[ 50 + 20 e^{lt} \right] \]

\[ \Rightarrow (s - k)T(s) = -k \mathcal{L} \left[ 50 \right] - k \mathcal{L} \left[ 20 e^{lt} \right] + T(0) \]

\[ \Rightarrow (s - k)T(s) = -\frac{50k}{s} - \frac{20ke^{-hs}}{s} + 85 \]

\[ \Rightarrow T(s) = \frac{-50k}{s(s - k)} - \frac{20ke^{-hs}}{s(s - k)} + \frac{85}{s - k} \]

\[ \Rightarrow \mathcal{L} \left[ T(s) \right] = \frac{50}{s} - \frac{50}{s - k} + \left[ \frac{20}{s} - \frac{20}{s - k} \right] e^{-hs} + \frac{85}{s - k} \]

\[ \Rightarrow \mathcal{L} \left[ T(t) \right] = 50 \left[ \frac{1}{s} \right] - 50 \left[ \frac{e^{-hs}}{s - k} \right] + 20 \left[ \frac{e^{-hs}}{s} \right] - 20 \left[ \frac{e^{-hs}}{s - k} \right] + 85 \left[ \frac{e^{-hs}}{s - k} \right] \]

\[ \Rightarrow T(t) = 50 - 50e^{-kt} + 20u_h(t) - 20u_h(t)e^{kt} + 85e^{kt} \]

or

\[ T(t) = 50 + 35e^{kt} + 20u_h(t) \left[ 1 - e^{-kt} \right], \quad k = 0.05 \text{ s}^{-1} \]
3. 1° Time body moved \( T_v = 6 \text{ a.m.} - h \)

2° Time of death

From Pr. 1, the estimated time of death \( t = T_d = 5.66 \text{ hrs} \) before 6 a.m.

\[ U(t - h) = 0 \]

From Pr. 2, \( T(t) = 50 + 35e^{kt} \)

\[ T(T_d) = 98.6 = 50 + 35 \cdot 0.058 \cdot t_d \]

\[ t_d = \frac{1}{0.058} \ln \left( \frac{48.6}{35} \right) \approx 5.66 \text{ hrs} \]

Time of death was 12:30 a.m. as that in Pr 1 when \( h > 5.66 \text{ hrs} \).

4° If \( t = T_d = 5.66 \geq h \) \( \Rightarrow U(t - h) = 1 \)

\[ T(T_d) = 98.6 = 50 + 35e^{\frac{1}{0.058} \ln \left( \frac{28.6}{35-20e^{-0.058h}} \right)} \]

\[ 28.6 = \left[ 35 - 20e^{-0.058} \right] e^{-0.058 \cdot t_d} \]

\[ t_d = \frac{1}{0.058} \ln \left( \frac{28.6}{35 - 20e^{-0.058h}} \right) \quad (h \leq 5.66 \text{ hrs}) \]
3° ① \( h = 12 \)

Time body moved \( T_v = 6\text{am} - 12 = 6:00\text{pm} \)
\( h > 5.66 \), 

*Time of death = 12:20 am*

② \( h = 11 \)

\( T_v = 6\text{am} - 11 = 7:00\text{pm} \)
\( h = 11 > 5.66 \), 

*Time of death = 12:20 am*

③ \( h = 10 \)

\( T_v = 6\text{am} - 10 = 8:00\text{pm} \)
\( h = 10 > 5.66 \), 

*Time of death = 12:20 am*

④ \( h = 9 \)

\( T_v = 6\text{am} - 9 = 9:00\text{pm} \)
\( h = 9 > 5.66 \), 

*Time of death = 12:20 am*

⑤ \( h = 8 \)

\( T_v = 6\text{am} - 8 = 10:00\text{pm} \)
\( h = 8 > 5.66 \), 

*Time of death = 12:20 am*

⑥ \( h = 7 \)

\( T_v = 6\text{am} - 7 = 11:00\text{pm} \)
\( h = 7 > 5.66 \), 

*Time of death = 12:20 am*

⑦ \( h = 6 \)

\( T_v = 6\text{am} - 6 = 12:00\text{pm} \)
\( h = 6 > 5.66 \), 

*Time of death = 12:20 am*

⑧ \( h = 5 \)

\( T_v = 6\text{am} - 5 = 1:00\text{pm} \)
\( h = 5 < 5.66 \), 

\[ t_d = \frac{28.6}{0.058} \ln \left( \frac{55 - 20e^{0.058}}{55 - 20e^{0.058}} \right) = 6.1 \text{hrs} = 6\text{hrs} 36\text{mins} \text{ before 6am} \]

\( \therefore \text{Time of death} = 6\text{am} - 6\text{hrs} 36\text{mins} = 11:54\text{pm} \)
9. \( h = 4 \)
   \( T_v = 6\text{am} - 4 = 2\text{am} \)
   \( h = 4 < 5.66 \)
   \( t_d = \frac{1}{0.058 \ln \left( \frac{2.86}{35 - 20 e^{-0.058 \cdot 4}} \right)} = 6.9 \text{hrs} = 6 \text{hrs 54 mins} \)
   before 6am
   \[ \text{Time of death} = 6\text{am} - 6\text{h 54m} = 11:06 \text{pm} \]

10. \( h = 3 \)
    \( T_v = 6\text{am} - 3 = 3\text{am} \)
    \( h = 3 < 5.66 \)
    \( t_d = \frac{1}{0.058 \ln \left( \frac{2.86}{35 - 20 e^{-0.058 \cdot 3}} \right)} = 7.8 \text{hrs} = 7 \text{hrs 48 mins} \)
    before 6am
    \[ \text{Time of death} = 6\text{am} - 7\text{h 48m} = 10:12 \text{pm} \]

11. \( h = 2 \)
    \( T_v = 6\text{am} - 2 = 4\text{am} \)
    \( h = 2 < 5.66 \)
    \( t_d = \frac{1}{0.058 \ln \left( \frac{2.86}{35 - 20 e^{-0.058 \cdot 2}} \right)} = 8.8 \text{hrs} = 8 \text{hrs 48 mins} \)
    before 6am
    \[ \text{Time of death} = 6\text{am} - 8\text{h 48m} = 9:12 \text{pm} \]

<table>
<thead>
<tr>
<th>( h )</th>
<th>Time body moved</th>
<th>Time of death</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>6:00 pm</td>
<td>12:20 am</td>
</tr>
<tr>
<td>11</td>
<td>7:00 pm</td>
<td>12:20 am</td>
</tr>
<tr>
<td>10</td>
<td>8:00 pm</td>
<td>12:20 am</td>
</tr>
<tr>
<td>9</td>
<td>9:00 pm</td>
<td>12:20 am</td>
</tr>
<tr>
<td>8</td>
<td>10:00 pm</td>
<td>12:20 am</td>
</tr>
<tr>
<td>7</td>
<td>11:00 pm</td>
<td>12:20 am</td>
</tr>
<tr>
<td>6</td>
<td>12:00 pm</td>
<td>12:20 am</td>
</tr>
<tr>
<td>5</td>
<td>1:00 am</td>
<td>11:54 pm</td>
</tr>
<tr>
<td>4</td>
<td>2:00 am</td>
<td>11:06 pm</td>
</tr>
<tr>
<td>3</td>
<td>3:00 am</td>
<td>10:12 pm</td>
</tr>
<tr>
<td>2</td>
<td>4:00 am</td>
<td>9:12 pm</td>
</tr>
</tbody>
</table>
4. List the time suspects were seen and left:

<table>
<thead>
<tr>
<th>Suspect</th>
<th>Name</th>
<th>Time seen</th>
<th>Left time</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Twinkles</td>
<td>5-6 pm</td>
<td>a little after 6 pm</td>
</tr>
<tr>
<td>2</td>
<td>Slim</td>
<td>around 10 pm</td>
<td>around 11 pm</td>
</tr>
<tr>
<td>3</td>
<td>Shorty (the cook)</td>
<td>took a long break at 10:30 pm</td>
<td>2 am</td>
</tr>
</tbody>
</table>

1° The estimated time Joe was killed was around 12:20 am.
Both suspect 1 and 2 left before 12:20 am.
Therefore, suspects 1 & 2 are not on the list to question.

2° For suspect 3, Shorty was seen in the restaurant between 10:30 pm and 2:00 am.

From the table in Pr. 3, 2 situations may be possible.
1. Shorty killed Joe at around 11:54 pm and moved his body at around 1:00 am.
2. Shorty killed Joe at around 11:06 pm and moved his body at around 2:00 am.

Therefore, Daphne should question Shorty (the cook).
5. \( \frac{dT}{dt} = k(T - T_m), \quad T(0) = T_0 \)

\[
\frac{T-T_0}{t-0} = \frac{dT}{dt} \bigg|_{t=0} = k(T_0 - T_m)
\]

\[\Rightarrow T - T_0 = k(T_0 - T_m) t\]

\[\Rightarrow t = \frac{T - T_0}{k(T_0 - T_m)}\]

\[T = 98.4, \quad t = \frac{98.4 - T_0}{k(T_0 - T_m)}\]