

Murder at the Mayfair Diner

by Tom LoFaro



The Mayfair diner in Philadelphia, PA

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Dawn at the Mayfair Diner. The amber glow of streetlights mixed with the violent red flash of police cruisers begins to fade with the rising of a furnace orange sun. Detective Daphne Marlow exits the diner holding a steaming cup of hot joe in one hand and a summary of the crime scene evidence in the other. Taking a seat on the bumper of her tan LTD, Detective Marlow begins to review the evidence.

At 5:30 a.m. the body of one Joe D. Wood was found in the walk in refrigerator in the diner's basement. At 6:00 a.m. the coroner arrived and determined that the core body temperature of the corpse was 85 degrees Fahrenheit. Thirty minutes later the coroner again measured the core body temperature. This time the reading was 84 degrees Fahrenheit. The thermostat inside the refrigerator reads 50 degrees Fahrenheit.

Daphne takes out a fading yellow legal pad and ketchup-stained calculator from the front seat of her cruiser and begins to compute. She knows that Newton's Law of Cooling says that the rate at which an object cools is proportional to the difference between the temperature T of the body at time t and the temperature T_m of the environment surrounding the body. She jots down the equation

$$\frac{dT}{dt} = k(T - T_m), \quad t > 0, \quad (1)$$

where k is a constant of proportionality, T and T_m are measured in degrees Fahrenheit, and t is time measured in hours. Because Daphne wants to investigate the past using positive values of time, she decides to correspond $t = 0$ with 6:00 a.m., and so, for example, $t = 4$ is 2:00 a.m. After a few scratches on her yellow pad, Daphne realizes that with this time convention the constant k in (1) will turn out to be *positive*. She jots a reminder to herself that 6:30 a.m. is now $t = -1/2$.

As the cool and quiet dawn gives way to the steamy midsummer morning, Daphne begins to sweat and wonders aloud, "But what if the corpse was moved into the fridge in a feeble attempt to hide the body? How does this change my estimate?" She re-enters the restaurant and finds the grease-streaked thermostat above the empty cash register. It reads 70 degrees Fahrenheit.

"But when was the body moved?" Daphne asks. She decides to leave this question unanswered for now, simply letting h denote the number of hours the body has been in the refrigerator prior to 6:00 a.m. For example, if $h = 6$, then the body was moved at midnight.

Daphne flips a page on her legal pad and begins calculating. As the rapidly cooling coffee begins to do its work, she realizes that the way to model the environmental temperature change caused by the move is with the unit step function $\mathcal{U}(t)$. She writes

$$T_m(t) = 50 + 20\mathcal{U}(t - h) \quad (2)$$

and below it the differential equation

$$\frac{dT}{dt} = k(T - T_m(t)). \quad (3)$$

Daphne's mustard-stained polyester blouse begins to drip sweat under the blaze of a midmorning sun. Drained from the heat and the mental exercise, she fires up her cruiser and motors to Boodle's Café for another cup of java and a heaping plate

of scrapple and fried eggs. She settles into the faux leather booth. The intense air-conditioning conspires with her sweat-soaked blouse to raise goose flesh on her rapidly cooling skin. The intense chill serves as a gruesome reminder of the tragedy that occurred earlier at the Mayfair.

While Daphne waits for her breakfast, she retrieves her legal pad and quickly reviews her calculations. She then carefully constructs a table that relates refrigeration time h to time of death while eating her scrapple and eggs.

Shoving away the empty platter, Daphne picks up her cell phone to check in with her partner Marie. “Any suspects?” Daphne asks.

“Yeah,” she replies, “we got three of ’em. The first is the late Mr. Wood’s ex-wife, a dancer by the name of Twinkles. She was seen in the Mayfair between 5 and 6 p.m. in a shouting match with Wood.”

“When did she leave?”

“A witness says she left in a hurry a little after six. The second suspect is a South Philly bookie who goes by the name of Slim. Slim was in around 10 last night having a whispered conversation with Joe. Nobody overheard the conversation, but witnesses say there was a lot of hand gesturing, like Slim was upset or something.”

“Did anyone see him leave?”

“Yeah. He left quietly around 11. The third suspect is the cook.”

“The cook?”

“Yep, the cook. Goes by the name of Shorty. The cashier says he heard Joe and Shorty arguing over the proper way to present a plate of veal scaloppine. She said that Shorty took an unusually long break at 10:30 p.m. He took off in a huff when the restaurant closed at 2:00 a.m. Guess that explains why the place was such a mess.”

“Great work, partner. I think I know who to bring in for questioning.”

Related Problems

1. Solve equation (1), which models the scenario in which Joe Wood is killed in the refrigerator. Use this solution to estimate the time of death (recall that normal living body temperature is 98.6 degrees Fahrenheit).
2. Solve the differential equation (3) using Laplace transforms. Your solution $T(t)$ will depend on both t and h . (Use the value of k found in Problem 1.)
3. (CAS) Complete Daphne’s table. In particular, explain why large values of h give the same time of death.

| h | time body moved | time of death |
|-----|-----------------|---------------|
| 12 | 6:00 p.m. | |
| 11 | | |
| 10 | | |
| 9 | | |
| 8 | | |
| 7 | | |
| 6 | | |
| 5 | | |
| 4 | | |
| 3 | | |
| 2 | | |

4. Who does Daphne want to question and why?
5. **Still Curious?** The process of temperature change in a dead body is known as *algor mortis* (*rigor mortis* is the process of body stiffening), and although it is not

perfectly described by Newton's Law of Cooling, this topic is covered in most forensic medicine texts. In reality, the cooling of a dead body is determined by more than just Newton's Law. In particular, chemical processes in the body continue for several hours after death. These chemical processes generate heat, and thus a near constant body temperature may be maintained during this time before the exponential decay due to Newton's Law of Cooling begins.

A linear equation, known as the *Glaister equation*, is sometimes used to give a preliminary estimate of the time t since death. The Glaister equation is

$$t = \frac{98.4 - T_0}{1.5} \quad (4)$$

where T_0 is measured body temperature (98.4°F is used here for normal living body temperature instead of 98.6°F). Although we do not have all of the tools to derive this equation exactly (the 1.5 degrees per hour was determined experimentally), we can derive a similar equation via linear approximation.

Use equation (1) with an initial condition of $T(0) = T_0$ to compute the equation of the tangent line to the solution through the point $(0, T_0)$. Do not use the values of T_m or k found in Problem 1. Simply leave these as parameters. Next, let $T = 98.4$ and solve for t to get

$$t = \frac{98.4 - T_0}{k(T_0 - T_m)}. \quad (5)$$

ABOUT THE AUTHOR



Courtesy of Tom LoFaro

Tom LoFaro is a professor and chair of the Mathematics and Computer Science Department at Gustavus Adolphus College in St. Peter, Minnesota. He has been involved in developing differential modeling projects for over 10 years, including being a principal investigator of the NSF-funded IDEA project (<http://www.sci.wsu.edu/idea/>) and a contributor to CODEE's ODE Architect (Wiley and Sons). Dr. LoFaro's nonacademic interests include fly fishing and coaching little league soccer. His oldest daughter (age 12) aspires to be a forensic anthropologist much like Detective Daphne Marlow.