7. Suppose that it is desired to construct a set of polynomials $f_0(x)$, $f_1(x)$, $f_2(x)$, ..., $f_k(x)$, ..., where $f_k(x)$ is of degree k, that are orthonormal on the interval $0 \le x \le 1$. That is, the set of polynomials must satisfy

$$(f_j, f_k) = \int_0^1 f_j(x) f_k(x)\,dx = \delta_{jk}.$$

a. Find $f_0(x)$ by choosing the polynomial of degree zero such that $(f_0, f_0) = 1$.
b. Find $f_1(x)$ by determining the polynomial of degree one such that $(f_0, f_1) = 0$ and $(f_1, f_1) = 1$.
c. Find $f_2(x)$.
d. The normalization condition $(f_k, f_k) = 1$ is somewhat awkward to apply. Let $g_0(x)$, $g_1(x)$, ..., $g_k(x)$, ... be the sequence of polynomials that are orthogonal on $0 \le x \le 1$ and that are normalized by the condition $g_k(1) = 1$. Find $g_0(x)$, $g_1(x)$, and $g_2(x)$, and compare them with $f_0(x)$, $f_1(x)$, and $f_2(x)$.

8. Suppose that it is desired to construct a set of polynomials $P_0(x)$, $P_1(x)$, ..., $P_k(x)$, ..., where $P_k(x)$ is of degree k, that are orthogonal on the interval $-1 \le x \le 1$; see Problem 7. Suppose further that $P_k(x)$ is normalized by the condition $P_k(1) = 1$. Find $P_0(x)$, $P_1(x)$, $P_2(x)$, and $P_3(x)$. Note that these are the first four Legendre polynomials (see Problem 19 of Section 5.3).

9. This problem develops some further results associated with mean convergence. Define $R_n(a_1, \dots, a_n)$, $S_n(x)$, and a_i by equations (2), (6), and (9), respectively.

a. Show that

$$R_n = \int_0^1 r(x) f^2(x)\,dx - \sum_{i=1}^{n} a_i^2.$$

Hint: Substitute for $S_n(x)$ in equation (6) and integrate, using the orthogonality relation (1).

b. Show that $\sum_{i=1}^{n} a_i^2 \le \int_0^1 r(x) f^2(x)\,dx$. This result is known as **Bessel's inequality**.

c. Show that $\sum_{i=1}^{\infty} a_i^2$ converges.

d. Show that $\lim_{n\to\infty} R_n = \int_0^1 r(x) f^2(x)\,dx - \sum_{i=1}^{\infty} a_i^2$.

e. Show that $\sum_{i=1}^{\infty} a_i \phi_i(x)$ converges to $f(x)$ in the mean if and only if

$$\int_0^1 r(x) f^2(x)\,dx = \sum_{i=1}^{\infty} a_i^2.$$

This result is known as **Parseval's equation**.

In Problems 10 through 12, let $\phi_1, \phi_2, \dots, \phi_n, \dots$ be the normalized eigenfunctions of the Sturm-Liouville problem (11), (12).

10. Show that if a_n is the nth Fourier coefficient of a square integrable function f, then $\lim_{n\to\infty} a_n = 0$.

Hint: Use Bessel's inequality, Problem 9b.

11. Show that the series

$$\phi_1(x) + \phi_2(x) + \cdots + \phi_n(x) + \cdots$$

cannot be the eigenfunction series for any square integrable function.
Hint: See Problem 10.

12. Show that the series

$$\phi_1(x) + \frac{\phi_2(x)}{\sqrt{2}} + \cdots + \frac{\phi_n(x)}{\sqrt{n}} + \cdots$$

is not the eigenfunction series for any square integrable function.
Hint: Use Bessel's inequality, Problem 9b.

13. Show that Parseval's equation in Problem 9e is obtained formally by squaring the series (10) corresponding to f, multiplying by the weight function r, and integrating term by term.

References

The following books were mentioned in the text in connection with certain theorems about Sturm-Liouville problems:

Birkhoff, G., and Rota, G.-C., *Ordinary Differential Equations* (4th ed.) (New York: John Wiley & Sons, 1989).

Sagan, H., *Boundary and Eigenvalue Problems in Mathematical Physics* (New York: John Wiley & Sons, 1961; New York: Dover, 1989).

Weinberger, H. F., *A First Course in Partial Differential Equations with Complex Variables and Transform Methods* (New York: Blaisdell, 1965; New York: Dover, 1995).

Yosida, K., *Lectures on Differential and Integral Equations* (New York: Interscience Publishers, 1960; New York: Dover, 1991).

The following book is a convenient source of numerical and graphical data about Bessel and Legendre functions:

Abramowitz, M., and Stegun, I. A. (eds.), *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables* (New York: Dover, 1965); originally published by the National Bureau of Standards, Washington, DC, 1964.

The digital successor to Abramowitz and Stegun is

Digital Library of Mathematical Functions. Release date August 29, 2011. National Institute of Standards and Technology from http://dlmf.nist.gov/.

The following books also contain much information about Sturm-Liouville problems:

Cole, R. H., *Theory of Ordinary Differential Equations* (New York: Appleton-Century-Crofts, 1968).

Hochstadt, H., *Differential Equations: A Modern Approach* (New York: Holt, Rinehart, and Winston, 1964; New York: Dover, 1975).

Miller, R. K., and Michel, A. N., *Ordinary Differential Equations* (New York: Academic Press, 1982; Mineola, NY: Dover, 2007).

Tricomi, F. G., *Differential Equations* (translated by Elizabeth A. McHarg) (New York: Hafner, 1961; Mineola, NY: Dover, 2012).

Answers to Problems

Chapter 1

Section 1.1, page 8

1. $y \to 3/2$ as $t \to \infty$

2. y diverges from $3/2$ as $t \to \infty$

3. $y \to -1/2$ as $t \to \infty$

4. y diverges from $-1/2$ as $t \to \infty$

5. $y' = 2 - 3y$

6. $y' = y - 2$

7. $y = 0$ and $y = 4$ are equilibrium solutions; $y \to 4$ if initial value is positive; y diverges from 0 if initial value is negative.

8. $y = 0$ and $y = 5$ are equilibrium solutions; y diverges from 5 if initial value is greater than 5; $y \to 0$ if initial value is less than 5.

9. $y = 0$ is equilibrium solution; $y \to 0$ if initial value is negative; y diverges from 0 if initial value is positive.

10. $y = 0$ and $y = 2$ are equilibrium solutions; y diverges from 0 if initial value is negative; $y \to 2$ if initial value is between 0 and 2; y diverges from 2 if initial value is greater than 2.

11. (j)

12. (c)

13. (g)

14. (b)

15. (h)

16. (e)

17. **a.** $da/dt = 3 - 3 \times 10^{-4} a$; a in g, t in h
b. $a \to 10^4$ g; no
c. $\dfrac{dc}{dt} = 3 \times 10^{-6} - 3 \times 10^{-4} c$, with initial condition $c(10) = a(0)/10^6 = a_0/10^6$

18. $dV/dt = -kV^{2/3}$ for some $k > 0$.

19. $du/dt = -0.05(u - 70)$; u in °F, t in min

20. **a.** $dq/dt = 500 - 0.4q$; q in mg, t in h
b. $q \to 1250$ mg

21. **a.** $mv' = mg - kv^2$
b. $v \to \sqrt{mg/k}$
c. $k = 2/49$

22. y is asymptotic to $t - 3$ as $t \to \infty$

23. $y \to \infty$, 0, or $-\infty$ depending on the initial value of y

24. $y \to \infty$ or $-\infty$ or y oscillates depending on the initial value of y

25. $y(t_f) = 0$ and then y fails to exist after some $t_f \ge 0$

Section 1.2, page 15

1. **a.** $y = 5 + (y_0 - 5)e^{-t}$
b. $y = (5/2) + (y_0 - 5/2)e^{-2t}$
c. $y = 5 + (y_0 - 5)e^{-2t}$
Equilibrium solution is $y = 5$ in (a) and (c), $y = 5/2$ in (b); solution approaches equilibrium faster in (b) and (c) than in (a).

2. **a.** $y = 5 + (y_0 - 5)e^{t}$
b. $y = 5/2 + (y_0 - 5/2)e^{2t}$
c. $y = 5 + (y_0 - 5)e^{2t}$
Equilibrium solution is $y = 5$ in (a) and (c), $y = 5/2$ in (b); solution diverges from equilibrium faster in (b) and (c) than in (a).

3. **a.** $y = ce^{-at} + b/a$
b. (i) Equilibrium is lower and is approached more rapidly. (ii) Equilibrium is higher. (iii) Equilibrium remains the same and is approached more rapidly.

4. **a.** $y_e = (b/a)$
b. $Y' = aY$

5. **a.** $y_1(t) = ce^{at}$
b. $y = ce^{at} + b/a$

6. $y = ce^{-at} + b/a$

7. **a.** $T = 2\ln 18 \cong 5.78$ months
b. $T = 2\ln(900/(900 - p_0))$ months
c. $p_0 = 900(1 - e^{-6}) \cong 897.8$

8. **a.** $T = 5\ln 50 \cong 19.56$ s
b. 718.34 m

9. **b.** $v = 49\tanh(t/5)$ m/s
e. $x = 245\ln\cosh(t/5)$ m
f. $T \cong 9.48$ s

10. **a.** $r \cong 0.02828$ day^{-1}
b. $Q(t) = 100e^{-0.02828t}$
c. $T \cong 24.5$ d

12. **a.** $u = T + (u_0 - T)e^{-kt}$
b. $k\tau = \ln 2$

13. **a.** $Q(t) = CV(1 - e^{-t/RC})$
b. $Q(t) \to CV = Q_L$
c. $Q(t) = CV\exp(-(t - t_1)/(RC))$

14. **a.** $Q' = 3(1 - 10^{-4}Q)$, $Q(0) = 0$
b. $Q(t) = 10^4\left(1 - e^{-3t/10^4}\right)$, t in h; after 1 year $Q \cong 9277.77$ g
c. $Q' = -3Q/10^4$, $Q(0) = 9277.77$
d. $Q(t) = 9277.77e^{-3t/10^4}$, t in h; after 1 year $Q \cong 670.07$ g
e. $T \cong 2.60$ yr

Section 1.3, page 22

1. Second order, linear

2. Second order, nonlinear

3. Fourth order, linear

4. Second order, nonlinear

11. $r = -2$

12. $r = 2, -3$

13. $r = 0, 1, 2$

14. $r = -1, -2$

15. $r = 1, 4$

16. Second order, linear

17. Fourth order, linear

18. Second order, nonlinear

Chapter 2

Section 2.1, page 31

1. c. $y = ce^{-3t} + t/3 - 1/9 + e^{-2t}$; y is asymptotic to $t/3 - 1/9$ as $t \to \infty$

2. c. $y = ce^{2t} + t^3e^{2t}/3$; $y \to \infty$ as $t \to \infty$

3. c. $y = ce^{-t} + 1 + t^2e^{-t}/2$; $y \to 1$ as $t \to \infty$

4. c. $y = c/t + 3\cos(2t)/(4t) + 3\sin(2t)/2$; y is asymptotic to $3\sin(2t)/2$ as $t \to \infty$

5. c. $y = ce^{2t} - 3e^t$; $y \to \infty$ or $-\infty$ as $t \to \infty$

6. c. $y = -te^{-t} + ct$; $y \to \infty$, 0, or $-\infty$ as $t \to \infty$

7. c. $y = ce^{-t} + \sin(2t) - 2\cos(2t)$; y is asymptotic to $\sin(2t) - 2\cos(2t)$ as $t \to \infty$

8. c. $y = ce^{-t/2} + 3t^2 - 12t + 24$; y is asymptotic to $3t^2 - 12t + 24$ as $t \to \infty$

9. $y = 3e^t + 2(t-1)e^{2t}$

10. $y = (t^2 - 1)e^{-2t}/2$

11. $y = (\sin t)/t^2$

12. $y = (t - 1 + 2e^{-t})/t$, $t \neq 0$

13. b. $y = -\frac{4}{5}\cos t + \frac{8}{5}\sin t + \left(a + \frac{4}{5}\right)e^{t/2}$; $a_0 = -\frac{4}{5}$
 c. y oscillates for $a = a_0$

14. b. $y = \left(2 + a(3\pi + 4)e^{2t/3} - 2e^{-\pi t/2}\right)/(3\pi + 4)$; $a_0 = -2/(3\pi + 4)$
 c. $y \to 0$ for $a = a_0$

15. b. $y = te^{-t} + (ea - 1)e^{-t}/t$; $a_0 = 1/e$
 c. $y \to 0$ as $t \to 0$ for $a = a_0$

16. b. $y = \left(e^t - e + a\sin 1\right)/\sin t$; $a_0 = (e-1)/\sin 1$
 c. $y \to 1$ for $a = a_0$

17. $(t, y) = (1.364312, 0.820082)$

18. $y_0 = -1.642876$

19. a. $y = 12 + \frac{8}{65}\cos 2t + \frac{64}{65}\sin 2t - \frac{788}{65}e^{-t/4}$; y oscillates about 12 as $t \to \infty$
 b. $t = 10.065778$

20. $y_0 = -5/2$

21. $y_0 = -16/3$; $y \to -\infty$ as $t \to \infty$ for $y_0 = -16/3$

29. See Problem 2.

30. See Problem 4.

Section 2.2, page 38

1. $3y^2 - 2x^3 = c$; $y \neq 0$

2. $y^{-1} + \cos x = c$ if $y \neq 0$; also $y = 0$; everywhere

3. $2\tan(2y) - 2x - \sin(2x) = c$ if $\cos(2y) \neq 0$; also $y = \pm(2n+1)\pi/4$ for any integer n; everywhere

4. $y = \sin(\ln|x| + c)$ if $x \neq 0$ and $|y| < 1$; also $y = \pm 1$

5. $y^2 - x^2 + 2(e^y - e^{-x}) = c$; $y + e^y \neq 0$

6. $3y + y^3 - x^3 = c$; everywhere

7. $y = kx$

8. $y = \pm\sqrt{x^2 + c}$

9. a. $y = 1/(x^2 - x - 6)$
 c. $-2 < x < 3$

10. a. $y = -\sqrt{2x - 2x^2 + 4}$
 c. $-1 < x < 2$

11. a. $y = (2(1-x)e^x - 1)^{1/2}$
 c. $-1.68 < x < 0.77$ approximately

12. a. $r = 2/(1 - 2\ln\theta)$
 c. $0 < \theta < \sqrt{e}$

13. a. $y = \left(3 - 2\sqrt{1 + x^2}\right)^{-1/2}$
 c. $|x| < \frac{1}{2}\sqrt{5}$

14. a. $y = -\frac{1}{2} + \frac{1}{2}\sqrt{4x^2 - 15}$
 c. $x > \frac{1}{2}\sqrt{15}$

15. a. $y = 5/2 - \sqrt{x^3 - e^x + 13/4}$
 c. $-1.4445 < x < 4.6297$ approximately

16. a. $y = \left(\pi - \arcsin\left(3\cos^2 x\right)\right)/3$
 c. $|x - \pi/2| < 0.6155$

17. $y^3 - 3y^2 - x - x^3 + 2 = 0$, $|x| < 1$

18. $y^3 - 4y - x^3 = -1$, $|x^3 - 1| < 16/3\sqrt{3}$ or $-1.28 < x < 1.60$

19. $y = -1/(x^2/2 + 2x - 1)$; $x = -2$

20. $y = -3/2 + \sqrt{2x - e^x + 13/4}$; $x = \ln(2)$

21. a. $y \to 4$ if $y_0 > 0$; $y = 0$ if $y_0 = 0$; $y \to -\infty$ if $y_0 < 0$
 b. $T = 3.29527$

22. a. $y \to 4$ as $t \to \infty$
 b. $T = 2.84367$
 c. $3.6622 < y_0 < 4.4042$

23. $x = \frac{c}{a}y + \frac{ad - bc}{a^2}\ln|ay + b| + k$; $a \neq 0$, $ay + b \neq 0$

25. c. $|y + 2x|^3|y - 2x| = c$

26. b. $\arctan(y/x) - \ln|x| = c$

27. b. $x^2 + y^2 - cx^3 = 0$

28. b. $|y - x| = c|y + 3x|^5$; also $y = x$

29. b. $|y + x|\,|y + 4x|^2 = c$

30. b. $|x|^3|x^2 - 5y^2| = c$

31. b. $c|x|^3 = |y^2 - x^2|$

Section 2.3, page 47

1. $t = 100\ln 100$ min $\cong 460.5$ min

2. $Q(t) = 120\gamma(1 - \exp(-t/60))$; 120γ

3. a. $Q(t) = \frac{63,150}{2501}e^{-t/50} + 25 - \frac{625}{2501}\cos t + \frac{25}{5002}\sin t$
 c. level = 25; amplitude = $25\sqrt{2501}/5002 \cong 0.24995$

4. c. 130.41 s

5. a. $(\ln 2)/r$ yr
 b. 9.90 yr
 c. 8.66%

6. a. $k\left(e^{rt} - 1\right)/r$
 b. $k \cong \$3930$
 c. 9.77%

7. $k = \$3086.64$/yr; \$1259.92

8. a. $t \cong 146.54$ months
 b. \$246,758.02

9. a. 0.00012097 yr^{-1}
 b. $Q_0\exp(-0.00012097t)$, t in yr
 c. 13,305 yr

10. a. $\tau \cong 2.9632$; no
 b. $\tau = 10\ln 2 \cong 6.9315$
 c. $\tau = 6.3805$

11. b. $y_c \cong 0.83$

12. $t = \frac{\ln(13/8)}{\ln(13/12)}$ min $\cong 6.07$ min

13. a. $u(t) = 2000/(1 + 0.048t)^{1/3}$
 c. $\tau \cong 750.77$ s

14. a. $u(t) = ce^{-kt} + T_0 + kT_1(k\cos(\omega t) + \omega\sin(\omega t))/(k^2 + \omega^2)$
 b. $R \cong 9.11°$F; $\tau \cong 3.51$ h
 c. $R = kT_1/\sqrt{k^2 + \omega^2}$; $\tau = (1/\omega)\arctan(\omega/k)$

15. a. $c = k + P/r + (c_0 - k - P/r)e^{-rt/V}$; $\lim_{t\to\infty} c = k + P/r$
 b. $T = (V\ln 2)/r$; $T = (V\ln 10)/r$
 c. Superior, $T = 431$ yr; Michigan, $T = 71.4$ yr; Erie, $T = 6.05$ yr; Ontario, $T = 17.6$ yr

16. a. 50.408 m
 b. 5.248 s

17. a. 45.783 m
 b. 5.129 s

18. a. 48.562 m
 b. 5.194 s

19. a. $x_m = -\frac{m^2g}{k^2}\ln\left(1 + \frac{kv_0}{mg}\right) + \frac{mv_0}{k}$; $t_m = \frac{m}{k}\ln\left(1 + \frac{kv_0}{mg}\right)$

20. a. $v = -mg/k + (v_0 + (mg/k))\exp(-kt/m)$
 b. $v = v_0 - gt$; yes
 c. $v = 0$ for $t > 0$

21. a. $v_L = 2a^2g(\rho - \rho')/9\mu$
 b. $e = 4\pi a^3g(\rho - \rho')/3E$

22. b. $x = ut\cos A$, $y = -gt^2/2 + ut\sin A + h$
 d. $-16L^2/(u^2\cos^2 A) + L\tan A + 3 \geq H$
 e. 0.63 rad $\leq A \leq$ 0.96 rad
 f. $u = 106.89$ ft/s, $A = 0.7954$ rad

23. a. $v = (u\cos A)e^{-rt}$, $w = -g/r + (u\sin A + g/r)e^{-rt}$
 b. $x = (u\cos A)(1 - e^{-rt})/r$, $y = -gt/r + (u\sin A + g/r)(1 - e^{-rt})/r + h$
 d. $u = 145.3$ ft/s, $A = 0.644$ rad

24. d. $k = 2.193$

Section 2.4, page 57

1. $0 < t < 3$

2. $\pi/2 < t < 3\pi/2$

3. $-\infty < t < -2$

4. $1 < t < \pi$

5. $t^2 + y^2 < 1$

6. $1 - t^2 + y^2 > 0$ or $1 - t^2 + y^2 < 0$, $t \neq 0$, $y \neq 0$

7. Everywhere

8. $y \neq 0$, $y \neq 3$

9. $y = \pm\sqrt{y_0^2 - 4t^2}$ if $y_0 \neq 0$; $|t| < |y_0|/2$

10. $y = (1/y_0 - t^2)^{-1}$ if $y_0 \neq 0$; $y = 0$ if $y_0 = 0$; interval is $|t| < 1/\sqrt{y_0}$ if $y_0 > 0$; $-\infty < t < \infty$ if $y_0 \leq 0$

11. $y = y_0/\sqrt{2ty_0^2 + 1}$ if $y_0 \neq 0$; $y = 0$ if $y_0 = 0$; interval is $-1/2y_0^2 < t < \infty$ if $y_0 \neq 0$; $-\infty < t < \infty$ if $y_0 = 0$

12. $y = \pm\sqrt{\frac{2}{3}\ln(1 + t^3) + y_0^2}$; $-(1 - \exp(-3y_0^2/2))^{1/3} < t < \infty$

13. $y \to 3$ if $y_0 > 0$; $y = 0$ if $y_0 = 0$; $y \to -\infty$ if $y_0 < 0$

14. $y \to -\infty$ if $y_0 < 0$; $y \to 0$ if $y_0 \geq 0$

15. $y \to 0$ if $y_0 \leq 9$; $y \to \infty$ if $y_0 > 9$

16. $y \to -\infty$ if $y_0 < y_c \approx -0.019$; otherwise y is asymptotic to $\sqrt{t-1}$

17. a. No
 b. Yes; set $t_0 = 1/2$ in equation (19) in text.
 c. $|y| \leq (4/3)^{3/2} \cong 1.5396$

18. a. $y_1(t)$ is a solution for $t \geq 2$; $y_2(t)$ is a solution for all t
 b. f_y is not continuous at $(2, -1)$

22. a. $y = c\frac{1}{\mu(t)} + \frac{1}{\mu(t)}\int_{t_0}^{t}\mu(s)g(s)ds$

23. a. $n = 0$: $y = \frac{1}{\mu(t)}\int_{t_0}^{t}\mu(s)q(s)\,ds$ where $\mu(t) = \exp\left(\int_{t_0}^{t}p(t)\,dt\right)$
 $n = 1$: $y = c\exp\left(\int_{t_0}^{t}q(t) - p(t)\,dt\right)$

24. $y = r/(k + cre^{-rt})$

25. $y = \pm\left(\epsilon/\left(\sigma + c\epsilon e^{-2\epsilon t}\right)\right)^{1/2}$

26. $y = \frac{1}{2}\left(1 - e^{-2t}\right)$ for $0 \leq t \leq 1$; $y = \frac{1}{2}\left(e^2 - 1\right)e^{-2t}$ for $t > 1$

27. $y = e^{-2t}$ for $0 \leq t \leq 1$; $y = e^{-(t+1)}$ for $t > 1$

Section 2.5, page 67

1. $y = -a/b$ is asymptotically stable, $y = 0$ is unstable

2. $y = 1$ is asymptotically stable, $y = 0$ and $y = 2$ are unstable

3. $y = 0$ is unstable

4. $y = 0$ is asymptotically stable

5. c. $y = (y_0 + (1 - y_0)kt)/(1 + (1 - y_0)kt)$

6. $y = -1$ is asymptotically stable, $y = 0$ is semistable, $y = 1$ is unstable

7. $y = -1$ and $y = 1$ are asymptotically stable, $y = 0$ is unstable

8. $y = 2$ is asymptotically stable, $y = 0$ is semistable, $y = -2$ is unstable

9. $y = 0$ and $y = 1$ are semistable

16. a. $y = 0$ is unstable, $y = K$ is asymptotically stable
 b. Concave up for $0 < y \leq K/e$, concave down for $K/e \leq y < K$

17. a. $y = K\exp((\ln(y_0/K))e^{-rt})$
 b. $y(2) \cong 0.7153K \cong 57.6 \times 10^6$ kg
 c. $\tau \cong 2.215$ yr

18. b. $V^* = (k/(\alpha\pi))^{3/2}\,\pi h/(3a)$; yes
 c. $k < \alpha\pi a^2$

19. c. $Y = Ey_2 = KE(1 - (E/r))$
 d. $Y_m = Kr/4$ for $E = r/2$

20. a. $y_{1,2} = K(1 \mp \sqrt{1 - (4h/rK)})/2$

21. a. $y = 0$ is unstable, $y = 1$ is asymptotically stable
 b. $y = y_0/(y_0 + (1 - y_0)e^{-\alpha t})$

22. a. $y = y_0e^{-\beta t}$
 b. $x = x_0\exp(-\alpha y_0(1 - e^{-\beta t})/\beta)$
 c. $x_0\exp(-\alpha y_0/\beta)$

23. b. $z = 1/(v + (1 - v)e^{\beta t})$
c. 0.0927

24. a, b. $a = 0$: $y = 0$ is semistable. $a > 0$: $y = \sqrt{a}$ is asymptotically stable and $y = -\sqrt{a}$ is unstable.

25. a. $a \le 0$: $y = 0$ is asymptotically stable. $a > 0$: $y = 0$ is unstable; $y = \sqrt{a}$ and $y = -\sqrt{a}$ are asymptotically stable.

26. a. $a < 0$: $y = 0$ is asymptotically stable and $y = a$ is unstable.
$a = 0$: $y = 0$ is semistable.
$a > 0$: $y = 0$ is unstable and $y = a$ is asymptotically stable.

27. a. $\lim_{t\to\infty} x(t) = \min(p, q)$; $x(t) = \dfrac{pq(e^{\alpha(q-p)t} - 1)}{qe^{\alpha(q-p)t} - p}$
b. $\lim_{t\to\infty} x(t) = p$; $x(t) = \dfrac{p^2\alpha t}{p\alpha t + 1}$

Section 2.6, page 75

1. $x^2 + 3x + y^2 - 2y = c$
2. Not exact
3. $x^3 - x^2y + 2x + 2y^3 + 3y = c$
4. $ax^2 + 2bxy + cy^2 = k$
5. Not exact
6. $e^{xy}\cos 2x + x^2 - 3y = c$
7. $y \ln x + 3x^2 - 2y = c$
8. $x^2 + y^2 = c$
9. $y = \left(x + \sqrt{28 - 3x^2}\right)/2$, $|x| < \sqrt{28/3}$
10. $y = (x - (24x^3 + x^2 - 8x - 16)^{1/2})/4$, $x > 0.9846$
11. $b = 3$; $x^2y^2 + 2x^3y = c$
12. $b = 1$; $e^{2xy} + x^2 = c$
15. $x^2 + 2\ln|y| - y^{-2} = c$; also $y = 0$
16. $x^2e^x \sin y = c$
18. $\mu(x) = e^{3x}$; $(3x^2y + y^3)e^{3x} = c$
19. $\mu(x) = e^{-x}$; $y = ce^x + 1 + e^{2x}$
20. $\mu(y) = y$; $xy + y\cos y - \sin y = c$
21. $\mu(y) = e^{2y}/y$; $xe^{2y} - \ln|y| = c$; also $y = 0$

Section 2.7, page 82

1. a. 1.2, 1.39, 1.571, 1.7439
b. 1.1975, 1.38549, 1.56491, 1.73658
c. 1.19631, 1.38335, 1.56200, 1.73308
d. 1.19516, 1.38127, 1.55918, 1.72968

2. a. 1.1, 1.22, 1.364, 1.5368
b. 1.105, 1.23205, 1.38578, 1.57179
c. 1.10775, 1.23873, 1.39793, 1.59144
d. 1.1107, 1.24591, 1.41106, 1.61277

3. a. 1.25, 1.54, 1.878, 2.2736
b. 1.26, 1.5641, 1.92156, 2.34359
c. 1.26551, 1.57746, 1.94586, 2.38287
d. 1.2714, 1.59182, 1.97212, 2.42554

4. a. 0.3, 0.538501, 0.724821, 0.866458
b. 0.284813, 0.513339, 0.693451, 0.831571
c. 0.277920, 0.501813, 0.678949, 0.815302
d. 0.271428, 0.490897, 0.665142, 0.799729

5. Converge for $y \ge 0$; undefined for $y < 0$
6. Converge for $y \ge 0$; diverge for $y < 0$
7. Converge for $|y(0)| < 2.37$ (approximately); diverge otherwise
8. Diverge

9. a. 2.30800, 2.49006, 2.60023, 2.66773, 2.70939, 2.73521
b. 2.30167, 2.48263, 2.59352, 2.66227, 2.70519, 2.73209
c. 2.29864, 2.47903, 2.59024, 2.65958, 2.70310, 2.73053
d. 2.29686, 2.47691, 2.58830, 2.65798, 2.70185, 2.72959

10. a. 1.70308, 3.06605, 2.44030, 1.77204, 1.37348, 1.11925
b. 1.79548, 3.06051, 2.43292, 1.77807, 1.37795, 1.12191
c. 1.84579, 3.05769, 2.42905, 1.78074, 1.38017, 1.12328
d. 1.87734, 3.05607, 2.42672, 1.78224, 1.38150, 1.12411

11. a. −0.166134, −0.410872, −0.804660, 4.15867
b. −0.174652, −0.434238, −0.889140, −3.09810

12. A reasonable estimate for y at $t = 0.8$ is between 5.5 and 6. No reliable estimate is possible at $t = 1$ from the specified data.

13. b. $2.37 < \alpha_0 < 2.38$

14. b. $0.67 < \alpha_0 < 0.68$

Section 2.8, page 90

1. $dw/ds = (s+1)^2 + (w+2)^2$, $w(0) = 0$
2. $dw/ds = 1 - (w+3)^3$, $w(0) = 0$

3. a. $\phi_n(t) = \sum_{k=1}^{n} \dfrac{2^k t^k}{k!}$
c. $\lim_{n\to\infty} \phi_n(t) = e^{2t} - 1$

4. a. $\phi_n(t) = \sum_{k=1}^{n} (-1)^{k+1} t^{k+1}/(k+1)!2^{k-1}$
c. $\lim_{n\to\infty} \phi_n(t) = 4e^{-t/2} + 2t - 4$

5. a. $\phi_n(t) = \sum_{k=1}^{n} \dfrac{t^{2k-1}}{1 \cdot 3 \cdot 5 \cdots (2k-1)}$

6. a. $\phi_n(t) = -\sum_{k=1}^{n} \dfrac{t^{3k-1}}{2 \cdot 5 \cdot 8 \cdots (3k-1)}$

7. a. $\phi_1(t) = \dfrac{t^3}{3}$; $\phi_2(t) = \dfrac{t^3}{3} + \dfrac{t^7}{7 \cdot 9}$;

$$\phi_3(t) = \frac{t^3}{3} + \frac{t^7}{7 \cdot 9} + \frac{2t^{11}}{3 \cdot 7 \cdot 9 \cdot 11} + \frac{t^{15}}{(7 \cdot 9)^2 \cdot 15}$$

8. a. $\phi_1(t) = t$; $\phi_2(t) = t - \dfrac{t^4}{4}$;

$$\phi_3(t) = t - \frac{t^4}{4} + \frac{3t^7}{4 \cdot 7} - \frac{3t^{10}}{16 \cdot 10} + \frac{t^{13}}{64 \cdot 13}$$

9. a. $\phi_1(t) = t$, $\phi_2(t) = t - \dfrac{t^2}{2!} + \dfrac{t^4}{4!} - \dfrac{t^6}{6!} + O(t^8)$,

$$\phi_3(t) = t - \frac{t^2}{2!} + \frac{t^3}{3!} + \frac{t^4}{4!} - \frac{7t^5}{5!} + \frac{14t^6}{6!} + O(t^7),$$

$$\phi_4(t) = t - \frac{t^2}{2!} + \frac{t^3}{3!} - \frac{7t^5}{5!} + \frac{31t^6}{6!} + O(t^7)$$

10. a. $\phi_1(t) = -t - t^2 - \dfrac{t^3}{2}$,

$$\phi_2(t) = -t - \frac{t^2}{2} + \frac{t^3}{6} + \frac{t^4}{4} - \frac{t^5}{5} - \frac{t^6}{24} + O(t^7),$$

$$\phi_3(t) = -t - \frac{t^2}{2} + \frac{t^4}{12} - \frac{3t^5}{20} + \frac{4t^6}{45} + O(t^7),$$

$$\phi_4(t) = -t - \frac{t^2}{2} + \frac{t^4}{8} - \frac{7t^5}{60} + \frac{t^6}{15} + O(t^7)$$

Section 2.9, page 99

1. $y_n = (-1)^n(0.9)^n y_0$; $y_n \to 0$ as $n \to \infty$
2. $y_n = y_0\sqrt{(n+2)(n+1)/2}$; $y_n \to \infty$ as $n \to \infty$
3. $y_n = \begin{cases} y_0, & \text{if } n = 4k \text{ or } n = 4k-1; \\ -y_0, & \text{if } n = 4k-2 \text{ or } n = 4k-3; \end{cases}$
y_n has no limit as $n \to \infty$
4. $y_n = (0.5)^n(y_0 - 12) + 12$; $y_n \to 12$ as $n \to \infty$
5. \$2283.63
6. \$258.14
7. 30 yrs: \$804.62/mo; \$289,663.20 total;
20 yrs: \$899.73/mo; \$215,935.20 total
8. \$103,624.62
9. 9.73%

12. b. $u_n \to -\infty$ as $n \to \infty$

15. a. 4.7263
b. 1.223%
c. 3.5643
e. 3.5699

Miscellaneous Problems, page 100

1. $y = c/x^2 + x^3/5$
2. $2y + \cos y - x - \sin x = c$
3. $x^2 + xy - 3y - y^3 = 0$
4. $y = -3 + ce^{x - x^2}$
5. $x^2y + xy^2 + x = c$
6. $y = x^{-1}(1 - e^{1-x})$
7. $y = (4 + \cos 2 - \cos x)/x^2$
8. $x^2y + x + y^2 = c$
9. $x + \ln|x| + x^{-1} + y - 2\ln|y| = c$; also $y = 0$
10. $x^3/3 + xy + e^y = c$
11. $x^2 + 2xy + 2y^2 = 34$
12. $y = c/\cosh^2(x/2)$
13. $e^{-x}\cos y + e^{2y}\sin x = c$
14. $y = ce^{3x} - e^{2x}$
15. $y = e^{-2x}\int_0^x e^{-s^2}\,ds + 3e^{-2x}$
16. $2xy + xy^3 - x^3 = c$
17. $e^x + e^{-y} = c$
18. $2xy^2 + 3x^2y - 4x + y^3 = c$
19. $y = e^{2t}/(3t) + ce^{-t}/t$
20. $e^{-y/x} + \ln|x| = c$
21. $(x^2 + y^2 + 1)e^{-y^2} = c$
22. $\arctan(y/x) - \ln\sqrt{x^2 + y^2} = c$
23. $y^2/x^3 + y/x^2 = c$
24. $\dfrac{1}{y} = -x\int_1^x \dfrac{e^{2s}}{s^2}\,ds + \dfrac{x}{2}$

26. a. $y = t + (c - t)^{-1}$
b. $y = t^{-1} + 2t(c - t^2)^{-1}$
c. $y = \sin t + \left(c\cos t - \frac{1}{2}\sin t\right)^{-1}$

27. a. $v' - (x(t) + b)v = b$
b. $v = \left(b\int \mu(t)\,dt + c\right)/\mu(t)$, $\mu(t) = \exp(-at^2/2 - bt)$

28. $y = c_1t^{-1} + c_2 + \ln t$
29. $y = c_1 \ln t + c_2 + t$
30. $y = 1/k \ln|(k - t)/(k + t)| + c_2$ if $c_1 = k^2 > 0$;
$y = 2/k \arctan(t/k) + c_2$ if $c_1 = -k^2 < 0$;
$y = -2t^{-1} + c_2$ if $c_1 = 0$; also $y = c$
31. $y = \pm\frac{2}{3}(t - 2c_1)\sqrt{t + c_1} + c_2$; also $y = c$
Hint: $\mu(v) = v^{-3}$ is an integrating factor.
32. $y^2 = c_1t + c_2$
33. $y = c_1\sin(t + c_2) = k_1\sin t + k_2\cos t$
34. $y\ln|y| - y + c_1y + t = c_2$; also $y = c$
35. $e^y = (t + c_2)^2 + c_1$
36. $y = \frac{4}{3}(t+1)^{3/2} - \frac{1}{3}$
37. $y = 3\ln t - \frac{3}{2}\ln(t^2 + 1) - 5\arctan t + 2 + \frac{3}{2}\ln 2 + \frac{5}{4}\pi$

Chapter 3

Section 3.1, page 109

1. $y = c_1e^t + c_2e^{-3t}$
2. $y = c_1e^{-t} + c_2e^{-2t}$
3. $y = c_1e^{t/2} + c_2e^{-t/3}$
4. $y = c_1 + c_2e^{-5t}$
5. $y = c_1e^{3t/2} + c_2e^{-3t/2}$
6. $y = c_1\exp\left((1 + \sqrt{3})t\right) + c_2\exp\left((1 - \sqrt{3})t\right)$
7. $y = e^t$; $y \to \infty$ as $t \to \infty$
8. $y = \frac{5}{2}e^{-t} - \frac{1}{2}e^{-3t}$; $y \to 0$ as $t \to \infty$
9. $y = -1 - e^{-3t}$; $y \to -1$ as $t \to \infty$
10. $y = (2/\sqrt{33})\exp\left((-1 + \sqrt{33})t/4\right)$
$- (2/\sqrt{33})\exp\left((-1 - \sqrt{33})t/4\right)$; $y \to \infty$ as $t \to \infty$
11. $y = \frac{1}{10}e^{-9(t-1)} + \frac{9}{10}e^{t-1}$; $y \to \infty$ as $t \to \infty$
12. $y = -\frac{1}{2}e^{(t+2)/2} + \frac{3}{2}e^{-(t+2)/2}$; $y \to -\infty$ as $t \to \infty$
13. $y'' + y' - 6y = 0$
14. $y = \frac{1}{4}e^t + e^{-t}$; minimum is $y = 1$ at $t = \ln 2$
15. $y = -e^t + 3e^{t/2}$; maximum is $y = 9/4$ at $t = \ln(9/4)$,
$y = 0$ at $t = \ln 9$
16. $\alpha = -2$
17. $y \to 0$ for $\alpha < 0$; y becomes unbounded for $\alpha > 1$
18. $y \to 0$ for $\alpha < 1$; there is no α for which all nonzero solutions become unbounded

19. a. $y = (6 + \beta)e^{-2t} - (4 + \beta)e^{-3t}$
b. $t_m = \ln((12 + 3\beta)/(12 + 2\beta))$, $y_m = \frac{4}{27}(6 + \beta)^3/(4 + \beta)^2$
c. $\beta = 6(1 + \sqrt{3}) \cong 16.3923$
d. $t_m \to \ln(3/2)$, $y_m \to \infty$

20. a. $y = d/c$
b. $aY'' + bY' + cY = 0$

21. a. $b > 0$ and $0 < c < b^2/(4a)$
b. $c < 0$
c. $b < 0$ and $0 < c < b^2/(4a)$

Section 3.2, page 119

1. $-\frac{7}{2}e^{t/2}$
2. 1
3. e^{-4t}
4. $-e^{2t}$
5. 0
6. $0 < t < \infty$
7. $0 < t < 4$
8. $0 < t < \infty$

9. $2 < x < 3\pi/2$

11. The equation is nonlinear.

12. The equation is nonhomogeneous.

13. No

14. $3te^{2t} + ce^{2t}$

15. $-4(t\cos t - \sin t)$

16. y_3 and y_4 are a fundamental set of solutions if and only if $a_1b_2 - a_2b_1 \neq 0$.

17. $y_1(t) = \frac{1}{3}e^{-2t} + \frac{2}{3}e^t, \quad y_2(t) = -\frac{1}{3}e^{-2t} + \frac{1}{3}e^t$

18. $y_1(t) = -\frac{1}{2}e^{-3(t-1)} + \frac{3}{2}e^{-(t-1)}$, $y_2(t) = -\frac{1}{2}e^{-3(t-1)} + \frac{1}{2}e^{-(t-1)}$

19. Yes

20. Yes

21. Yes

22. **b.** Yes
    c. $\{y_1(t), y_3(t)\}$ and $\{y_1(t), y_4(t)\}$ are fundamental sets of solutions; $\{y_2(t), y_3(t)\}$ and $\{y_4(t), y_5(t)\}$ are not

23. ct^2e^t

24. $c\cos t$

25. $c/(1-x^2)$

27. $2/25$

28. $p(t) = 0$ for all t

30. If t_0 is an inflection point, and $y = \phi(t)$ is a solution, then from the differential equation $p(t_0)\phi'(t_0) + q(t_0)\phi(t_0) = 0$.

32. Yes, $y = c_1e^{-x^2/2}\int_{x_0}^{x} e^{t^2/2}dt + c_2e^{-x^2/2}$

33. Yes, $y = \frac{1}{\mu(x)}\left(c_1\int_{x_0}^{x}\frac{\mu(t)}{t}dt + c_2\right)$, where $\mu(x) = \exp\left(-\int\left(\frac{1}{x} + \frac{\cos x}{x}\right)dx\right)$

34. Yes, $y = c_1x^{-1} + c_2x$

36. $x^2\mu'' + 3x\mu' + (1 + x^2 - \nu^2)\mu = 0$

37. $\mu'' - x\mu = 0$

38. The Legendre and Airy equations are self-adjoint.

Section 3.3, page 125

1. $e^2\cos 3 - ie^2\sin 3 \cong -7.3151 - 1.0427i$

2. -1

3. $e^2\cos(\pi/2) - ie^2\sin(\pi/2) = -e^2 i \cong -7.3891i$

4. $2\cos(\ln 2) - 2i\sin(\ln 2) \cong 1.5385 - 1.2779i$

5. $y = c_1e^t\cos t + c_2e^t\sin t$

6. $y = c_1e^t\cos\left(\sqrt{5}\,t\right) + c_2e^t\sin\left(\sqrt{5}\,t\right)$

7. $y = c_1e^{-t}\cos t + c_2e^{-t}\sin t$

8. $y = c_1e^{-3t}\cos(2t) + c_2e^{-3t}\sin(2t)$

9. $y = c_1e^{-t}\cos(t/2) + c_2e^{-t}\sin(t/2)$

10. $y = c_1e^{t/3} + c_2e^{-4t/3}$

11. $y = c_1e^{-2t}\cos(3t/2) + c_2e^{-2t}\sin(3t/2)$

12. $y = \frac{1}{2}\sin(2t)$; steady oscillation

13. $y = -e^{t-\pi/2}\sin 2t$; growing oscillation

14. $y = \left(1 + 2\sqrt{3}\right)\cos t - \left(2 - \sqrt{3}\right)\sin t$; steady oscillation

15. $y = \sqrt{2}\,e^{-(t-\pi/4)}\cos t + \sqrt{2}\,e^{-(t-\pi/4)}\sin t$; decaying oscillation

16. **a.** $u = 2e^{t/6}\cos\left(\sqrt{23}\,t/6\right) - \left(2/\sqrt{23}\right)e^{t/6}\sin\left(\sqrt{23}\,t/6\right)$
    b. $t = 10.7598$

17. **a.** $u = 2e^{-t/5}\cos\left(\sqrt{34}\,t/5\right) + \left(7/\sqrt{34}\right)e^{-t/5}\sin\left(\sqrt{34}\,t/5\right)$
    b. $T = 14.5115$

18. **a.** $y = 2e^{-t}\cos\left(\sqrt{5}\,t\right) + \left((\alpha+2)/\sqrt{5}\right)e^{-t}\sin\left(\sqrt{5}\,t\right)$
    b. $\alpha = 1.50878$
    c. $t = \left(\pi - \arctan\left(2\sqrt{5}/(2+\alpha)\right)\right)/\sqrt{5}$
    d. $\pi/\sqrt{5}$

26. $y = c_1\cos(\ln t) + c_2\sin(\ln t)$

27. $y = c_1t^{-1} + c_2t^{-2}$

28. $y = c_1t^6 + c_2t^{-1}$

29. $y = c_1t^2 + c_2t^3$

30. $y = c_1t + c_2t^{-3}$

31. $y = c_1t^{-3}\cos(\ln t) + c_2t^{-3}\sin(\ln t)$

32. **e.** $\frac{q'(t) + 2p(t)q(t)}{2(-q(t))^{3/2}}$ must be a constant

33. Yes, $y = c_1\cos x + c_2\sin x, \quad x = \int e^{-t^2/2}dt$

34. No

35. Yes, $y = c_1e^{-t^2/4}\cos\left(\sqrt{3}\,t^2/4\right) + c_2e^{-t^2/4}\sin\left(\sqrt{3}\,t^2/4\right)$

36. $y = c_1\exp\left(\int e^{-t^2/2}\,dt\right) + c_2\exp\left(-\int e^{-t^2/2}\,dt\right)$

Section 3.4, page 132

1. $y = c_1e^t + c_2te^t$

2. $y = c_1e^{-t/3} + c_2te^{-t/3}$

3. $y = c_1e^{-t/2} + c_2e^{3t/2}$

4. $y = c_1e^t\cos(3t) + c_2e^t\sin(3t)$

5. $y = c_1e^{3t} + c_2te^{3t}$

6. $y = c_1e^{-t/4} + c_2e^{-4t}$

7. $y = c_1e^{-3t/4} + c_2te^{-3t/4}$

8. $y = e^{-t/2}\cos(t/2) + c_2e^{-t/2}\sin(t/2)$

9. $y = 2e^{2t/3} - \frac{7}{3}te^{2t/3}, \quad y \to -\infty$ as $t \to \infty$

10. $y = 2te^{3t}, \quad y \to \infty$ as $t \to \infty$

11. $y = 7e^{-2(t+1)} + 5te^{-2(t+1)}, \quad y \to 0$ as $t \to \infty$

12. $y = 2e^{t/2} + (b-1)te^{t/2}; \quad b = 1$

13. **a.** $y = e^{-t/2} + \frac{5}{2}te^{-t/2}$
    b. $t_M = \frac{8}{5}, \quad y_M = 5e^{-4/5} \cong 2.24664$
    c. $y = e^{-t/2} + \left(b + \frac{1}{2}\right)te^{-t/2}$
    d. $t_M = 4b/(1+2b) \to 2$ as $b \to \infty$; $y_M = (1+2b)\exp(-2b/(1+2b)) \to \infty$ as $b \to \infty$

18. $y_2(t) = t^3$

19. $y_2(t) = t^{-2}$

20. $y_2(t) = t^{-1}\ln t$

21. $y_2(x) = \cos x^2$

22. $y_2(x) = x^{-1/2}\cos x$

23. $y = c_1e^{-\delta x^2/2}\int_0^x e^{\delta s^2/2}ds + c_2e^{-\delta x^2/2}$

24. $y_2(t) = y_1(t)\int_{t_0}^{t} y_1^{-2}(s)\exp\left(-\int_{s_0}^{s} p(r)\,dr\right)ds$

25. $y_2(t) = t^{-1}\ln t$

26. $y_2(t) = \cos t^2$

27. $y_2(x) = x^{-1/2}\cos x$

29. **b.** $y_0 + (a/b)y_0'$

31. $y = c_1t^2 + c_2t^2\ln t$

32. $y = c_1t^{-1/2} + c_2t^{-1/2}\ln t$

33. $y = c_1t^{-1} + c_2t^{-1}\ln t$

34. $y = c_1t^{3/2} + c_2t^{3/2}\ln t$

Section 3.5, page 141

1. $y = c_1e^{3t} + c_2e^{-t} - e^{2t}$

2. $y = c_1e^{-t} + c_2e^{2t} - \frac{7}{2} + 3t - 2t^2$

3. $y = c_1e^{2t} + c_2e^{-3t} + 2e^{3t} - 3e^{-2t}$

4. $y = c_1e^{3t} + c_2e^{-t} + \frac{3}{16}te^{-t} + \frac{3}{8}t^2e^{-t}$

5. $y = c_1 + c_2e^{-2t} + \frac{3}{2}t - \frac{1}{2}\sin(2t) - \frac{1}{2}\cos(2t)$

6. $y = c_1e^{-t} + c_2te^{-t} + t^2e^{-t}$

7. $y = c_1\cos t + c_2\sin t - \frac{1}{3}t\cos(2t) - \frac{5}{9}\sin(2t)$

8. $u = c_1\cos(\omega_0 t) + c_2\sin(\omega_0 t) + \left(\omega_0^2 - \omega^2\right)^{-1}\cos(\omega t)$

9. $u = c_1\cos(\omega_0 t) + c_2\sin(\omega_0 t) + 1/\left(2\omega_0\right)t\sin(\omega_0 t)$

10. $y = c_1e^{-t/2}\cos\left(\sqrt{15}\,t/2\right) + c_2e^{-t/2}\sin\left(\sqrt{15}\,t/2\right) + \frac{1}{6}e^t - \frac{1}{4}e^{-t}$

11. $y = e^t - \frac{1}{2}e^{-2t} - t - \frac{1}{2}$

12. $y = \frac{7}{10}\sin(2t) - \frac{19}{40}\cos(2t) + \frac{1}{4}t^2 - \frac{1}{8} + \frac{3}{5}e^t$

13. $y = 4te^t - 3e^t + \frac{1}{6}t^3e^t + 4$

14. $y = 2\cos(2t) - \frac{1}{8}\sin(2t) - \frac{3}{4}t\cos(2t)$

15. $y = e^{-t}\cos(2t) + \frac{1}{2}e^{-t}\sin(2t) + te^{-t}\sin(2t)$

16. **a.** $Y(t) = t(A_0t^4 + A_1t^3 + A_2t^2 + A_3t + A_4) + t(B_0t^2 + B_1t + B_2)e^{-3t} + D\sin 3t + E\cos 3t$
    b. $A_0 = 2/15,\ A_1 = -2/9,\ A_2 = 8/27,\ A_3 = -8/27,\ A_4 = 16/81,\ B_0 = -1/9,\ B_1 = -1/9,\ B_2 = -2/27,\ D = -1/18,\ E = -1/18$

17. **a.** $Y(t) = e^t(A\cos 2t + B\sin 2t) + (D_0t + D_1)e^{2t}\sin t + (E_0t + E_1)e^{2t}\cos t$
    b. $A = -1/20,\ B = -3/20,\ D_0 = -3/2,\ D_1 = -5,\ E_0 = 3/2,\ E_1 = 1/2$

18. **a.** $Y(t) = Ae^{-t} + t(B_0t^2 + B_1t + B_2)e^{-t}\cos t + t(D_0t^2 + D_1t + D_2)e^{-t}\sin t$
    b. $A = 3,\ B_0 = -2/3,\ B_1 = 0,\ B_2 = 1,\ D_0 = 0,\ D_1 = 1,\ D_2 = 1$

19. **a.** $Y(t) = t(A_0t^2 + A_1t + A_2)\sin(2t) + t(B_0t^2 + B_1t + B_2)\cos(2t)$
    b. $A_0 = 0,\ A_1 = 13/16,\ A_2 = 7/4,\ B_0 = -1/12,\ B_1 = 0,\ B_2 = 13/32$

20. **a.** $Y(t) = (A_0t^2 + A_1t + A_2)e^t\sin(2t) + (B_0t^2 + B_1t + B_2)e^t\cos(2t) + e^{-t}(D\cos t + E\sin t) + Fe^t$
    b. $A_0 = 1/52,\ A_1 = 10/169,\ A_2 = -1233/35{,}152,\ B_0 = -5/52,\ B_1 = 73/676,\ B_2 = -4105/35{,}152,\ D = -3/2,\ E = 3/2,\ F = 2/3$

21. **a.** $Y(t) = t(A_0t + A_1)e^{-t}\cos 2t + t(B_0t + B_1)e^{-t}\sin 2t + (D_0t + D_1)e^{-2t}\cos t + (E_0t + E_1)e^{-2t}\sin t$
    b. $A_0 = 0,\ A_1 = 3/16,\ B_0 = 3/8,\ B_1 = 0,\ D_0 = -2/5,\ D_1 = -7/25,\ E_0 = 1/5,\ E_1 = 1/25$

22. **b.** $w = -\frac{2}{5} + c_1e^{5t}$

23. $y = c_1\cos\lambda t + c_2\sin\lambda t + \sum_{m=1}^{N}\left(a_m/\left(\lambda^2 - m^2\pi^2\right)\right)\sin(m\pi t)$

24. $y = \begin{cases} t, & 0 \le t \le \pi \\ -(1+\pi/2)\sin t - (\pi/2)\cos t + (\pi/2)e^{\pi-t}, & t > \pi \end{cases}$

25. No

28. $y = c_1e^{4t} + c_2e^{-t} - \frac{1}{2}e^{2t}$

Section 3.6, page 146

1. $Y(t) = e^t$

2. $Y(t) = -\frac{2}{3}te^{-t}$

3. $Y(t) = 2t^2e^{t/2}$

4. $y = c_1\cos t + c_2\sin t - (\cos t)\ln(\tan t + \sec t)$

5. $y = c_1\cos(3t) + c_2\sin(3t) + \sin(3t)\ln(\tan(3t) + \sec(3t) - 1$

6. $y = c_1e^{-2t} + c_2te^{-2t} - e^{-2t}\ln t$

7. $y = c_1\cos(t/2) + c_2\sin(t/2) + t\sin(t/2) + 2(\ln(\cos(t/2)))\cos(t/2)$

8. $y = c_1e^t + c_2te^t - \frac{1}{2}e^t\ln(1+t^2) + te^t\arctan t$

9. $y = c_1e^{2t} + c_2e^{3t} + \int\left(e^{3(t-s)} - e^{2(t-s)}\right)g(s)ds$

10. $Y(t) = \frac{1}{2} + t^2\ln(t)$

11. $Y(t) = -2t^2$

12. $Y(t) = \frac{1}{2}(t-1)e^{2t}$

13. $Y(x) = \frac{1}{6}x^2(\ln x)^3$

14. $Y(x) = -\frac{3}{2}x^{1/2}\cos x$

15. $Y(x) = x^{-1/2}\int t^{-3/2}\sin(x-t)g(t)dt$

18. **b.** $y = y_0\cos t + y_0'\sin t + \int_{t_0}^{t}\sin(t-s)g(s)ds$

19. $y = (b-a)^{-1}\int_{t_0}^{t}[e^{b(t-s)} - e^{a(t-s)}]\,g(s)ds$

20. $y = \mu^{-1}\int_{t_0}^{t} e^{\lambda(t-s)}\sin\mu(t-s)g(s)ds$

21. $y = \int_{t_0}^{t}(t-s)e^{a(t-s)}g(s)ds$

24. $y = c_1t + c_2t^2 + 4t^2\ln t$

25. $y = c_1t^{-1} + c_2t^{-5} + \frac{1}{12}t$

26. $y = c_1(1+t) + c_2e^t + \frac{1}{2}(t-1)e^{2t}$

Section 3.7, page 157

1. $u = 5\cos(2t - \delta), \quad \delta = \arctan(4/3) \cong 0.9273$

2. $u = \sqrt{13}\cos(\pi t - \delta), \quad \delta = \pi + \arctan(3/2) \cong 4.1244$

3. $u = \frac{5}{7}\sin(14t)$ cm, t in s; $\quad t = \pi/14$ s

4. $u = \frac{1}{4\sqrt{2}}\sin\left(8\sqrt{2}\,t\right) - \frac{1}{12}\cos\left(8\sqrt{2}\,t\right)$ ft, t in s; $\omega = 8\sqrt{2}$ rad/s, $T = \pi/(4\sqrt{2})$ s, $\quad R = \sqrt{11/288} \cong 0.1954$ ft, $\delta = \pi - \arctan(3/\sqrt{2}) \cong 2.0113$

5. $u = e^{-10t}\left(2\cos\left(4\sqrt{6}\,t\right) + \left(5/\sqrt{6}\right)\ \sin\left(4\sqrt{6}\,t\right)\right)$ cm, t in s; $\mu = 4\sqrt{6}$ rad/s, $T_d = \pi/2\sqrt{6}$ s, $T_d/T = 7/(2\sqrt{6}) \cong 1.4289$, $\tau \cong 0.4045$ s

6. $u \cong 0.057198e^{-0.15t}\cos(3.87008\,t - 0.50709)$ m, t in s; $\mu = 3.87008$ rad/s, $\mu/\omega_0 = 3.87008/\sqrt{15} \cong 0.99925$

7. $Q = 10^{-6}\left(2e^{-500t} - e^{-1000t}\right)$ C; t in s

8. $\gamma = \sqrt{20/9} \cong 1.4907$

11. **a.** $r = \sqrt{A^2 + B^2},\ r\cos\theta = B,\ r\sin\theta = -A$
b. $R = r;\quad \delta = \theta + (4n+1)\pi/2, n = 0, 1, 2, \dots$

12. $R = 10^3\ \Omega$

14. $v_0 < -\gamma u_0/(2m)$

16. $2\pi/\sqrt{31}$

17. $k = 6,\ v = \pm 2\sqrt{5}$

18. **a.** $u(t) = e^{-\gamma t/2m}$ $\times\left(u_0\sqrt{4km - \gamma^2}\cos(\mu t) + (2mv_0 + \gamma u_0)\sin(\mu t)\right)/\sqrt{4km - \gamma^2}$
b. $R^2 = 4m\left(ku_0^2 + \gamma u_0 v_0 + mv_0^2\right)/\left(4km - \gamma^2\right)$

19. $\rho l u'' + \rho_0 g u = 0,\quad T = 2\pi\sqrt{\rho l/\rho_0 g}$

20. **a.** $u = \sqrt{2}\sin\left(\sqrt{2}\,t\right)$
c. clockwise

21. **a.** $u = (16/\sqrt{127})e^{-t/8}\sin\left(\sqrt{127}\,t/8\right)$
c. clockwise

22. **b.** $u = a\cos\left(\sqrt{k/m}\,t\right) + b\sqrt{m/k}\sin\left(\sqrt{k/m}\,t\right)$

24. **b.** $u = \sin t,\ A = 1,\ T = 2\pi$
c. $A = 0.98,\ T = 6.07$
d. $\epsilon = 0.2,\ A = 0.96,\ T = 5.90;\quad \epsilon = 0.3,\ A = 0.94,$ $T = 5.74$
f. $\epsilon = -0.1,\ A = 1.03,\ T = 6.55;\quad \epsilon = -0.2,$ $A = 1.06,\ T = 6.90;\quad \epsilon = -0.3, A = 1.11,\ T = 7.41$

Section 3.8, page 167

1. $2\sin(t/2)\cos(13t/2)$

2. $2\cos(3\pi t/2)\cos(\pi t/2)$

3. $2\sin(7t/2)\cos(t/2)$

4. $u'' + 10u' + 98u = 2\sin(t/2),\quad u(0) = 0,\quad u'(0) = 0.03,$ u in m, t in s

5. **a.** $u = \dfrac{1}{153,281}\left(160e^{-5t}\cos\left(\sqrt{73}\,t\right) + \dfrac{383,443}{7300}e^{-5t}\sin(\sqrt{73}\,t) - 160\cos(t/2) + 3128\sin(t/2)\right)$
b. The first two terms are the transient solution.
d. $\omega = 4\sqrt{3}$ rad/s

6. $u = \cos(8t) + \sin(8t) - 8t\cos(8t)/4$ ft, t in sec; $1/8, \pi/8, \pi/4, 3\pi/8$ s

7. **a.** $\dfrac{8}{901}\left(30\cos(2t) + \sin(2t)\right)$ ft, t in s
b. $m = 4$ slugs

8. $u = \left(\sqrt{2}/6\right)\cos(3t - 3\pi/4)$ m, t in s

11. $u = \begin{cases} F_0(t - \sin t), & 0 \le t \le \pi \\ F_0((2\pi - t) - 3\sin t), & \pi < t \le 2\pi \\ -4F_0\sin t, & 2\pi < t < \infty \end{cases}$

13. **a.** $u = 3(\cos t - \cos(\omega t))/(\omega^2 - 1)$

14. **a.** $u = ((\omega^2 + 2)\cos t - 3\cos(\omega t))/(\omega^2 - 1) + \sin t$

Chapter 4

Section 4.1, page 173

1. $-\infty < t < \infty$

2. $t > 1$, or $0 < t < 1$, or $t < 0$

3. $\dots, -3\pi/2 < x < -\pi/2,\ -\pi/2 < x < 1,$ $1 < x < \pi/2,\ \pi/2 < x < 3\pi/2, \dots$

4. $-\infty < x < -2,\ -2 < x < 2,\ 2 < x < \infty$

5. Linearly independent

6. Linearly dependent; $f_1(t) + 3f_2(t) - 2f_3(t) = 0$

7. Linearly dependent; $2f_1(t) + 13f_2(t) - 3f_3(t) - 7f_4(t) = 0$

8. 1

9. $-6e^{-2t}$

10. $6x$

11. $6/x$

12. $\sin^2 t = \dfrac{1}{2} - \dfrac{1}{2}\cos(2t)$

14. **a.** $a_0(n(n-1)(n-2)\cdots 1) + a_1(n(n-1)\cdots 2)t + \cdots + a_n t^n$
b. $(a_0 r^n + a_1 r^{n-1} + \cdots + a_n)e^{rt}$
c. $e^t, e^{-t}, e^{2t}, e^{-2t}$; yes, $W[e^t, e^{-t}, e^{2t}, e^{-2t}] \ne 0,$ $-\infty < t < \infty$

16. $W(t) = ce^{-2t}$

17. $W(t) = c/t^2$

20. $y = c_1e^t + c_2t + c_3te^t$

21. $y = c_1t^2 + c_2t^3 + c_3(t+1)$

Section 4.2, page 180

1. $\sqrt{2}\exp(i(\pi/4) + 2m\pi))$

2. $2\exp(i(2\pi/3 + 2m\pi))$

3. $3\exp(i(\pi + 2m\pi))$

4. $2\exp(i(11\pi/6) + 2m\pi)$

5. $1,\ \frac{1}{2}\left(-1 + i\sqrt{3}\right),\ \frac{1}{2}\left(-1 - i\sqrt{3}\right)$

6. $2^{1/4}e^{-\pi i/8},\ 2^{1/4}e^{7\pi i/8}$

7. $\left(\sqrt{3} + i\right)/\sqrt{2},\ -\left(\sqrt{3} + i\right)/\sqrt{2}$

8. $y = c_1e^t + c_2te^t + c_3e^{-t}$

9. $y = c_1e^t + c_2te^t + c_3t^2e^t$

10. $y = c_1 + c_2t + c_3e^{2t} + c_4te^{2t}$

11. $y = c_1\cos t + c_2\sin t + e^{\sqrt{3}t/2}\left(c_3\cos\left(\frac{t}{2}\right) + c_4\sin\left(\frac{t}{2}\right)\right)$ $+ e^{-\sqrt{3}t/2}\left(c_5\cos\left(\frac{t}{2}\right) + c_6\sin\left(\frac{t}{2}\right)\right)$

12. $y = c_1e^t + c_2te^t + c_3t^2e^t + c_4e^{-t} + c_5te^{-t} + c_6t^2e^{-t}$

13. $y = c_1 + c_2t + c_3e^t + c_4e^{-t} + c_5\cos t + c_6\sin t$

14. $y = c_1 + c_2e^t + c_3e^{2t} + c_4\cos t + c_5\sin t$

15. $y = e^t((c_1 + c_2t)\cos t + (c_3 + c_4t)\sin t)$ $+ e^{-t}((c_5 + c_6t)\cos t + (c_7 + c_8t)\sin t)$

16. $y = (c_1 + c_2t)\cos t + (c_3 + c_4t)\sin t$

17. $y = c_1e^{-t} + c_2e^{\left(-2+\sqrt{2}\right)t} + c_3e^{\left(-2-\sqrt{2}\right)t}$

18. $y = c_1e^{3t} + c_2e^{-2t} + c_3e^{\left(3+\sqrt{3}\right)t} + c_4e^{\left(3-\sqrt{3}\right)t}$

19. $y = c_1e^{-t/3} + c_2e^{-t/4} + c_3e^{-t}\cos(2t) + c_4e^{-t}\sin(2t)$

20. $y = 2 - 2\cos t + \sin t$

21. $y = \frac{1}{2}e^{-t/\sqrt{2}}\sin\left(t/\sqrt{2}\right) - \frac{1}{2}e^{t/\sqrt{2}}\sin\left(t/\sqrt{2}\right)$

22. $y = 2t - 3$

23. $y = -\frac{2}{3}e^t - \frac{1}{10}e^{2t} - \frac{1}{6}e^{-2t} - \frac{16}{15}e^{-t/2}$

24. $y = \frac{2}{13}e^{-t} + \frac{24}{13}e^{t/2}\cos t + \frac{3}{13}e^{t/2}\sin t$

25. $y = 8 - 18e^{-t/3} + 8e^{-t/2}$

26. $y = \frac{95}{32}e^{-t} + \frac{1}{32}e^t + \frac{1}{2}\cos t - \frac{17}{16}\sin t$

27. $y = \frac{1}{2}(\cosh t - \cos t) + \frac{1}{2}(\sinh t - \sin t)$

28. **a.** $W(t) = c$, a constant
b. $W(t) = -8$
c. $W(t) = 4$

29. **b.** $u_1 = c_1\cos t + c_2\sin t + c_3\cos\left(\sqrt{6}\,t\right) + c_4\sin\left(\sqrt{6}\,t\right)$

Section 4.3, page 184

1. $y = c_1e^t + c_2te^t + c_3e^{-t} + \frac{1}{2}te^{-t} + 3$

2. $y = c_1e^t + c_2e^{-t} + c_3\cos t + c_4\sin t - 3t - \frac{1}{4}t\sin t$

3. $y = c_1e^{-t} + c_2\cos t + c_3\sin t + \frac{1}{2}te^{-t} + 4(t-1)$

4. $y = c_1 + c_2t + c_3e^{-2t} + c_4e^{2t} - \frac{1}{3}e^t - \frac{1}{48}t^4 - \frac{1}{16}t^2$

5. $y = c_1\cos t + c_2\sin t + c_3t\cos t + c_4t\sin t + 3 + \frac{1}{9}\cos 2t$

6. $y = c_1 + c_2t + c_3t^2 + c_4e^{-t}$ $+ e^{t/2}\left(c_5\cos\left(\sqrt{3}\,t/2\right) + c_6\sin\left(\sqrt{3}\,t/2\right)\right) + \frac{1}{24}t^4$

7. $y = \frac{3}{16}(1 - \cos(2t)) + \frac{1}{8}t^2$

8. $y = (t-4)\cos t - \left(\frac{3}{2}t + 4\right)\sin t + 3t + 4$

9. $y = -\frac{2}{5}\cos t - \frac{4}{5}\sin t + \frac{1}{20}e^{-t} + \frac{81}{40}e^t + \frac{73}{520}e^{-3t}$ $+ \frac{77}{65}\cos(2t) - \frac{49}{130}\sin(2t)$

10. $Y(t) = t(A_0t^3 + A_1t^2 + A_2t + A_3) + Bt^2e^t$

11. $Y(t) = t(A_0t + A_1)e^{-t} + B\cos t + C\sin t$

12. $Y(t) = t(A_0t^2 + A_1t + A_2) + (B_0t + B_1)\cos t + (C_0t + C_1)\sin t$

13. $Y(t) = Ae^t + (B_0t + B_1)e^{-t} + te^{-t}(C\cos t + D\sin t)$

14. $k_0 = a_0,\quad k_n = a_0\alpha^n + a_1\alpha^{n-1} + \cdots + a_{n-1}\alpha + a_n$

Section 4.4, page 188

1. $y = c_1 + c_2\cos t + c_3\sin t - \ln\cos t - (\sin t)\ln(\sec t + \tan t)$

2. $y = c_1 + c_2e^t + c_3e^{-t} - \frac{1}{2}t^2$

3. $y = c_1e^t + c_2e^{-t} + c_3e^{2t} + \frac{1}{30}e^{4t}$

4. $y = c_1e^t + c_2\cos t + c_3\sin t - \frac{1}{5}e^{-t}\cos t$

5. $y = c_1e^t + c_2\cos t + c_3\sin t - \frac{1}{2}(\cos t)\ln\cos t$ $+ \frac{1}{2}(\sin t)\ln\cos t - \frac{1}{2}t\cos t - \frac{1}{2}t\sin t + \frac{1}{2}e^t\int_{t_0}^{t}(e^{-s}/\cos s)ds$

6. $y(t) = y_c(t) + y_p(t)$ $= c_1 + c_2e^t + c_3e^{-t} + \ln\left|\dfrac{\csc(t) + \cot(t)}{\csc(t_0) + \cot(t_0)}\right| + \dfrac{e^t}{2}\int_{t_0}^{t} e^{-s}\csc(s)ds +$ $\dfrac{e^{-t}}{2}\int_{t_0}^{t} e^s\csc(s)ds$

7. $c_1 = \frac{3}{2},\ c_2 = \frac{1}{2},\ c_3 = -\frac{5}{2},\ t_0 = 0$

8. $y = 3 + \ln(2)/2 - e^{-(t-\pi/4)} - \ln|\sec(t)|$ $+ \dfrac{e^t}{2}\int_{\pi/4}^{t} e^{-s}\tan(s)ds + \dfrac{e^{-t}}{2}\int_{\pi/4}^{t} e^s\tan(s)ds$

9. $Y(x) = x^4/15$

10. $Y(t) = \frac{1}{2}\int_{t_0}^{t}(e^{t-s} - \sin(t-s) - \cos(t-s))g(s)ds$

11. $Y(t) = \frac{1}{2}\int_{t_0}^{t}(\sinh(t-s) - \sin(t-s))g(s)ds$

12. $Y(t) = \frac{1}{2}\int_{t_0}^{t} e^{(t-s)}(t-s)^2g(s)ds;\quad Y(t) = -te^t\ln|t|$

Chapter 5

Section 5.1, page 195

1. $\rho = 1$

2. $\rho = 2$

3. $\rho = \infty$

4. $\rho = \frac{1}{2}$

5. $\rho = 1$

6. $\rho = 3$

7. $\sum_{n=0}^{\infty}\dfrac{(-1)^n x^{2n+1}}{(2n+1)!},\quad \rho = \infty$

8. $\sum_{n=0}^{\infty}\dfrac{x^n}{n!},\quad \rho = \infty$

9. $1 + (x-1),\quad \rho = \infty$

10. $1 - 2(x+1) + (x+1)^2,\quad \rho = \infty$

11. $\sum_{n=1}^{\infty}(-1)^{n+1}\dfrac{(x-1)^n}{n},\quad \rho = 1$

12. $\sum_{n=0}^{\infty}x^n,\quad \rho = 1$

13. $\sum_{n=0}^{\infty}(-1)^{n+1}(x-2)^n,\quad \rho = 1$

14. **a.** $y' = 1 + 2^2x + 3^2x^2 + 4^2x^3 + \cdots + (n+1)^2x^n + \cdots,$
b. $y'' = 2^2 + 3^2\cdot 2x + 4^2\cdot 3x^2 + 5^2\cdot 4x^3 + \cdots + (n+2)^2(n+1)x^n + \cdots$

15. **a.** $y' = a_1 + 2a_2x + 3a_3x^2 + 4a_4x^3 + \cdots + (n+1)a_{n+1}x^n + \cdots$ $= \sum_{n=1}^{\infty}na_nx^{n-1} = \sum_{n=0}^{\infty}(n+1)a_{n+1}x^n,$
$y'' = 2a_2 + 6a_3x + 12a_4x^2 + 20a_5x^3 + \cdots + (n+2)(n+1)a_{n+2}x^n + \cdots$ $= \sum_{n=2}^{\infty}n(n-1)a_nx^{n-2} = \sum_{n=0}^{\infty}(n+2)(n+1)a_{n+2}x^n$

18. $\sum_{n=0}^{\infty}(n+2)(n+1)a_{n+2}x^n$

19. $\sum_{n=0}^{\infty}(n+1)a_nx^n$

20. $\sum_{n=0}^{\infty}((n+2)(n+1)a_{n+2} + na_n)x^n$

21. $a_1 + \sum_{n=1}^{\infty}((n+1)a_{n+1} + a_{n-1})x^n$

22. $\sum_{n=0}^{\infty}((n+2)(n+1)a_{n+2} + a_n)x^n$

23. $a_n = (-2)^na_0/n!,\quad n = 1, 2, \dots;\quad a_0e^{-2x}$

Section 5.2, page 204

1. a. $a_{n+2} = a_n/(n+2)(n+1)$

b, d. $y_1(x) = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \cdots = \sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!} = \cosh x,$

$y_2(x) = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \cdots = \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!} = \sinh x$

2. a. $a_{n+2} = \frac{-3}{n+2} a_{n+1}$

b, d. $y_1 = 1, y_2 = x + \frac{(-3)^1}{2!}x^2 + \frac{(-3)^2}{3!}x^3 + \frac{(-3)^3}{4!}x^4 + \cdots$

$= \sum_{n=1}^{\infty} \frac{(-3)^{n-1}x^n}{n!} = \frac{1}{3}(1 - e^{3x})$

3. a. $a_{n+2} = a_n/(n+2)$

b, d. $y_1(x) = 1 + \frac{x^2}{2} + \frac{x^4}{2 \cdot 4} + \frac{x^6}{2 \cdot 4 \cdot 6} + \cdots = \sum_{n=0}^{\infty} \frac{x^{2n}}{2^n n!},$

$y_2(x) = x + \frac{x^3}{3} + \frac{x^5}{3 \cdot 5} + \frac{x^7}{3 \cdot 5 \cdot 7} + \cdots = \sum_{n=0}^{\infty} \frac{2^n n! x^{2n+1}}{(2n+1)!}$

4. a. $(n+2)a_{n+2} - a_{n+1} - a_n = 0$

b. $y_1(x) = 1 + \frac{1}{2}(x-1)^2 + \frac{1}{6}(x-1)^3 + \frac{1}{6}(x-1)^4 + \cdots,$

$y_2(x) = (x-1) + \frac{1}{2}(x-1)^2 + \frac{1}{2}(x-1)^3 + \frac{1}{4}(x-1)^4 + \cdots$

5. a. $a_{n+4} = -k^2 a_n/((n+4)(n+3)); \quad a_2 = a_3 = 0$

b, d. $y_1(x) = 1 - \frac{k^2x^4}{3 \cdot 4} + \frac{k^4x^8}{3 \cdot 4 \cdot 7 \cdot 8} - \frac{k^6x^{12}}{3 \cdot 4 \cdot 7 \cdot 8 \cdot 11 \cdot 12} + \cdots$

$= 1 + \sum_{m=0}^{\infty} \frac{(-1)^{m+1}(k^2x^4)^{m+1}}{3 \cdot 4 \cdot 7 \cdot 8 \cdots (4m+3)(4m+4)},$

$y_2(x) = x - \frac{k^2x^5}{4 \cdot 5} + \frac{k^4x^9}{4 \cdot 5 \cdot 8 \cdot 9} - \frac{k^6x^{13}}{4 \cdot 5 \cdot 8 \cdot 9 \cdot 12 \cdot 13} + \cdots$

$= x\left(1 + \sum_{m=0}^{\infty} \frac{(-1)^{m+1}(k^2x^4)^{m+1}}{4 \cdot 5 \cdot 8 \cdot 9 \cdots (4m+4)(4m+5)}\right)$

Hint: Let $n = 4m$ in the recurrence relation, $m = 1, 2, 3, \ldots$.

6. a. $(n+2)(n+1)a_{n+2} - n(n+1)a_{n+1} + a_n = 0, \quad n \geq 1;$
$a_2 = -\frac{1}{2}a_0$

b. $y_1(x) = 1 - \frac{1}{2}x^2 - \frac{1}{6}x^3 - \frac{1}{24}x^4 + \cdots,$

$y_2(x) = x - \frac{1}{6}x^3 - \frac{1}{12}x^4 - \frac{1}{24}x^5 + \cdots$

7. a. $a_{n+2} = -a_n/(n+1), \quad n = 0, 1, 2, \ldots$

b. $y_1(x) = 1 - \frac{x^2}{1} + \frac{x^4}{1 \cdot 3} - \frac{x^6}{1 \cdot 3 \cdot 5} + \cdots$

$= 1 + \sum_{n=1}^{\infty} \frac{(-1)^n x^{2n}}{1 \cdot 3 \cdot 5 \cdots (2n-1)},$

$y_2(x) = x - \frac{x^3}{2} + \frac{x^5}{2 \cdot 4} - \frac{x^7}{2 \cdot 4 \cdot 6} + \cdots$

$= x + \sum_{n=1}^{\infty} \frac{(-1)^n x^{2n+1}}{2 \cdot 4 \cdot 6 \cdots (2n)}$

8. a. $a_{n+2} = -((n+1)^2 a_{n+1} + a_n + a_{n-1})/((n+1)(n+2)),$
$n = 1, 2, \ldots, \quad a_2 = -(a_0 + a_1)/2$

b. $y_1(x) = 1 - \frac{1}{2}(x-1)^2 + \frac{1}{6}(x-1)^3 - \frac{1}{12}(x-1)^4 + \cdots,$

$y_2(x) = (x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{6}(x-1)^3 - \frac{1}{6}(x-1)^4 + \cdots$

9. a. $3(n+2)a_{n+2} - (n+1)a_n = 0, \quad n = 0, 1, 2, \ldots$

b, d. $y_1(x) = 1 + \frac{x^2}{6} + \frac{x^4}{24} + \frac{5}{432}x^6$
$+ \cdots + \frac{3 \cdot 5 \cdots (2n-1)}{3^n \cdot 2 \cdot 4 \cdots (2n)} x^{2n} + \cdots,$

$y_2(x) = x + \frac{2}{9}x^3 + \frac{8}{135}x^5 + \frac{16}{945}x^7$
$+ \cdots + \frac{2 \cdot 4 \cdots (2n)}{3^n \cdot 3 \cdot 5 \cdots (2n+1)} x^{2n+1} + \cdots$

10. a. $2(n+2)(n+1)a_{n+2} + (n+3)a_n = 0, \quad n = 0, 1, 2, \ldots$

b, d. $y_1(x) = 1 - \frac{3}{4}x^2 + \frac{5}{32}x^4 - \frac{7}{384}x^6$
$+ \cdots + (-1)^n \frac{3 \cdot 5 \cdots (2n+1)}{2^n (2n)!} x^{2n} + \cdots,$

$y_2(x) = x - \frac{x^3}{3} + \frac{x^5}{20} - \frac{x^7}{210}$
$+ \cdots + (-1)^n \frac{4 \cdot 6 \cdots (2n+2)}{2^n (2n+1)!} x^{2n+1} + \cdots$

11. a. $2(n+2)(n+1)a_{n+2} + 3(n+1)a_{n+1} + (n+3)a_n = 0,$
$n = 0, 1, 2, \ldots$

b. $y_1(x) = 1 - \frac{3}{4}(x-2)^2 + \frac{3}{8}(x-2)^3 + \frac{1}{64}(x-2)^4 + \cdots,$

$y_2(x) = (x-2) - \frac{3}{4}(x-2)^2 + \frac{1}{24}(x-2)^3 + \frac{9}{64}(x-2)^4 + \cdots$

12. a. $y = 2 + x + x^2 + \frac{1}{3}x^3 + \frac{1}{4}x^4 + \cdots$
c. about $|x| < 0.7$

13. a. $y = 4 - x - 4x^2 + \frac{1}{2}x^3 + \frac{4}{3}x^4 + \cdots$
c. about $|x| < 0.5$

14. a. $y = -3 + 2x - \frac{3}{2}x^2 - \frac{1}{2}x^3 - \frac{1}{8}x^4 + \cdots$
c. about $|x| < 0.9$

15. a. $y_1(x) = 1 - \frac{1}{3}(x-1)^3 - \frac{1}{12}(x-1)^4 + \frac{1}{18}(x-1)^6 + \cdots,$

$y_2(x) = (x-1) - \frac{1}{4}(x-1)^4 - \frac{1}{20}(x-1)^5 + \frac{1}{28}(x-1)^7 + \cdots$

16. *Hint:* Consider using induction.

18. a. $y_1(x) = 1 - \frac{\lambda}{2!}x^2 + \frac{\lambda(\lambda-4)}{4!}x^4 - \frac{\lambda(\lambda-4)(\lambda-8)}{6!}x^6$
$+ \cdots, y_2(x) = x - \frac{\lambda-2}{3!}x^3 + \frac{(\lambda-2)(\lambda-6)}{5!}x^5$
$- \frac{(\lambda-2)(\lambda-6)(\lambda-10)}{7!}x^7 + \cdots$

b. $1,\ x,\ 1 - 2x^2,\ x - \frac{2}{3}x^3,\ 1 - 4x^2 + \frac{4}{3}x^4,\ x - \frac{4}{3}x^3 + \frac{4}{15}x^5$

c. $1,\ 2x,\ 4x^2 - 2,\ 8x^3 - 12x,\ 16x^4 - 48x^2 + 12,$
$32x^5 - 160x^3 + 120x$

19. b. $y = x - x^3/6 + \cdots$

Section 5.3, page 209

1. $\phi''(0) = -1, \quad \phi'''(0) = 0, \quad \phi^{(4)}(0) = 3$

2. $\phi''(1) = 0, \quad \phi'''(1) = -6, \quad \phi^{(4)}(1) = 42$

3. $\phi''(0) = 0, \quad \phi'''(0) = -a_0, \quad \phi^{(4)}(0) = -4a_1$

4. $\rho = \infty, \quad \rho = \infty$

5. $\rho = 1, \quad \rho = 3, \quad \rho = 1$

6. $\rho = 1, \quad \rho = \sqrt{3}$

7. a. $\rho = \infty$
b. $\rho = \infty$
c. $\rho = \infty$
d. $\rho = \infty$
e. $\rho = 1$
f. $\rho = \sqrt{2}$
g. $\rho = \infty$
h. $\rho = 1$
i. $\rho = 1$
j. $\rho = 2$
k. $\rho = \sqrt{3}$
l. $\rho = 1$
m. $\rho = \infty$
n. $\rho = \infty$

8. a. $y_1(x) = 1 - \frac{\alpha^2}{2!}x^2 - \frac{(2^2 - \alpha^2)\alpha^2}{4!}x^4 - \frac{(4^2 - \alpha^2)(2^2 - \alpha^2)\alpha^2}{6!}x^6 - \cdots$
$- \frac{((2m-2)^2 - \alpha^2) \cdots (2^2 - \alpha^2)\alpha^2}{(2m)!}x^{2m} - \cdots,$

$y_2(x) = x + \frac{1 - \alpha^2}{3!}x^3 + \frac{(3^2 - \alpha^2)(1 - \alpha^2)}{5!}x^5 + \cdots$
$+ \frac{((2m-1)^2 - \alpha^2) \cdots (1 - \alpha^2)}{(2m+1)!}x^{2m+1} + \cdots$

b. $y_1(x)$ or $y_2(x)$ terminates with x^n as $\alpha = n$ is even or odd

c. $n = 0,\ y = 1; \quad n = 1,\ y = x; \quad n = 2,\ y = 1 - 2x^2;$
$n = 3,\ y = x - \frac{4}{3}x^3$

9. $y_1(x) = 1 - \frac{1}{6}x^3 + \frac{1}{120}x^5 + \frac{1}{180}x^6 + \cdots,$
$y_2(x) = x - \frac{1}{12}x^4 + \frac{1}{180}x^6 + \frac{1}{504}x^7 + \cdots, \rho = \infty$

10. $y_1(x) = 1 - \frac{1}{6}x^3 + \frac{1}{12}x^4 - \frac{1}{40}x^5 + \cdots,$
$y_2(x) = x - \frac{1}{12}x^4 + \frac{1}{20}x^5 - \frac{1}{60}x^6 + \cdots, \quad \rho = \infty$

11. $y_1(x) = 1 + x^2 + \frac{1}{12}x^4 + \frac{1}{120}x^6 + \cdots,$
$y_2(x) = x + \frac{1}{6}x^3 + \frac{1}{60}x^5 + \frac{1}{560}x^7 + \cdots, \rho = \pi/2$

12. Cannot specify arbitrary initial conditions at $x = 0$; hence $x = 0$ is a singular point.

13. $y = 1 + x + \frac{x^2}{2!} + \cdots + \frac{x^n}{n!} + \cdots = e^x$

14. $y = 1 + \frac{x^2}{2} + \frac{x^4}{2 \cdot 4} + \frac{x^6}{2 \cdot 4 \cdot 6} + \cdots + \frac{x^{2n}}{2^n \cdot n!} + \cdots$

15. $y = 1 + x + x^2 + \cdots + x^n + \cdots = \frac{1}{1-x}$

16. $y = a_0\left(1 + x + \frac{x^2}{2!} + \cdots + \frac{x^n}{n!} + \cdots\right)$
$+ 2\left(\frac{x^3}{3!} + \frac{x^4}{4!} + \cdots + \frac{x^n}{n!} + \cdots\right)$
$= a_0 e^x + 2\left(e^x - 1 - x - \frac{x^2}{2}\right) = ce^x - 2 - 2x - x^2$

18. $1,\ 1 - 3x^2,\ 1 - 10x^2 + \frac{35}{3}x^4; \quad x,\ x - \frac{5}{3}x^3,\ x - \frac{14}{3}x^3 + \frac{21}{5}x^5$

19. a. $1,\ x,\ (3x^2 - 1)/2,\ (5x^3 - 3x)/2,$
$(35x^4 - 30x^2 + 3)/8,\ (63x^5 - 70x^3 + 15x)/8$

c. $P_1,\ 0; \quad P_2,\ \pm 0.57735; \quad P_3,\ 0,\ \pm 0.77460;$
$P_4,\ \pm 0.33998,\ \pm 0.86114; P_5,\ 0,\ \pm 0.53847, \pm 0.90618$

Section 5.4, page 218

1. $y = c_1x^{-1} + c_2x^{-2}$

2. $y = c_1|x+1|^{-1/2} + c_2|x+1|^{-3/2}$

3. $y = c_1x^2 + c_2x^2 \ln|x|$

4. $y = c_1x + c_2x \ln|x|$

5. $y = c_1|x|^{\left(-5+\sqrt{29}\right)/2} + c_2|x|^{\left(-5-\sqrt{29}\right)/2}$

6. $y = c_1|x|^{3/2} \cos\left(\frac{\sqrt{3}}{2} \ln|x|\right) + c_2|x|^{3/2} \sin\left(\frac{\sqrt{3}}{2} \ln|x|\right)$

7. $y = c_1x^3 + c_2x^3 \ln|x|$

8. $y = c_1(x-2)^{-2} \cos(2 \ln|x-2|) + c_2(x-2)^{-2} \sin(2 \ln|x-2|)$

9. $y = 2x^{3/2} - x^{-1}$

10. $y = 2x^{-1/2} \cos(2 \ln x) - x^{-1/2} \sin(2 \ln x)$

11. $y = 2x^2 - 7x^2 \ln|x|$

12. $x = 0$, regular

13. $x = 0$, regular; $x = 1$, irregular

14. $x = 0$, irregular; $x = 1$, regular

15. $x = 0$, irregular; $x = \pm 1$, regular

16. $x = 1$, regular; $x = -1$, irregular

17. $x = 0$, regular

18. $x = 1$, regular; $x = -2$, irregular

19. $x = 0, 3$, regular

20. $x = 0$, regular

21. $x = 0$, irregular

22. $x = 0, \pm n\pi$, regular

23. $x = 0$, irregular; $x = \pm n\pi$, regular

24. $\alpha < 1$

25. $\beta > 0$

26. $\gamma = 2$

27. a. $\alpha < 1$ and $\beta > 0$
b. $\alpha < 1$ and $\beta \geq 0$, or $\alpha = 1$ and $\beta > 0$
c. $\alpha > 1$ and $\beta > 0$
d. $\alpha > 1$ and $\beta \geq 0$, or $\alpha = 1$ and $\beta > 0$
e. $\alpha = 1$ and $\beta > 0$

30. $y = a_0\left(1 - \frac{x^2}{2 \cdot 5} + \frac{x^4}{2 \cdot 4 \cdot 5 \cdot 9} - \cdots\right)$

33. Irregular singular point

34. Regular singular point

35. Regular singular point

36. Irregular singular point

37. Irregular singular point

Section 5.5, page 223

1. b. $r(2r-1) = 0; \quad a_n = -\frac{a_{n-2}}{(n+r)[2(n+r)-1]};$
$r_1 = \frac{1}{2},\ r_2 = 0$

c. $y_1(x) = x^{1/2}\left(1 - \frac{x^2}{2 \cdot 5} + \frac{x^4}{2 \cdot 4 \cdot 5 \cdot 9} - \frac{x^6}{2 \cdot 4 \cdot 6 \cdot 5 \cdot 9 \cdot 13}\right.$
$\left. + \cdots + \frac{(-1)^n x^{2n}}{2^n n! 5 \cdot 9 \cdot 13 \cdots (4n+1)} + \cdots\right)$

d. $y_2(x) = 1 - \frac{x^2}{2 \cdot 3} + \frac{x^4}{2 \cdot 4 \cdot 3 \cdot 7} - \frac{x^6}{2 \cdot 4 \cdot 6 \cdot 3 \cdot 7 \cdot 11} + \cdots$
$+ \frac{(-1)^n x^{2n}}{2^n n! 3 \cdot 7 \cdot 11 \cdots (4n-1)} + \cdots$

2. b. $r^2 - \frac{1}{9} = 0; \quad a_n = -\frac{a_{n-2}}{(n+r)^2 - \frac{1}{9}}; \quad r_1 = \frac{1}{3},\ r_2 = -\frac{1}{3}$

c. $y_1(x) = x^{1/3}\left[1 - \frac{1}{1!\left(1+\frac{1}{3}\right)}\left(\frac{x}{2}\right)^2\right.$
$+ \frac{1}{2!\left(1+\frac{1}{3}\right)\left(2+\frac{1}{3}\right)}\left(\frac{x}{2}\right)^4 + \cdots$
$\left. + \frac{(-1)^m}{m!\left(1+\frac{1}{3}\right)\left(2+\frac{1}{3}\right)\cdots\left(m+\frac{1}{3}\right)}\left(\frac{x}{2}\right)^{2m} + \cdots\right]$

d. $y_2(x) = x^{-1/3}\left(1 - \frac{1}{1!(1-\frac{1}{3})}\left(\frac{x}{2}\right)^2 + \frac{1}{2!(1-\frac{1}{3})(2-\frac{1}{3})}\left(\frac{x}{2}\right)^4 + \cdots + \frac{(-1)^m}{m!(1-\frac{1}{3})(2-\frac{1}{3})\cdots(m-\frac{1}{3})}\left(\frac{x}{2}\right)^{2m} + \cdots\right)$

Hint: Let $n = 2m$ in the recurrence relation, $m = 1, 2, 3, \ldots$.

3. b. $r(r-1) = 0;\quad a_n = -\frac{a_{n-1}}{(n+r)(n+r-1)};$ $r_1 = 1, r_2 = 0$
c. $y_1(x) = x\left(1 - \frac{x}{1!2!} + \frac{x^2}{2!3!} + \cdots + \frac{(-1)^n}{n!(n+1)!}x^n + \cdots\right)$

4. b. $r^2 = 0;\quad a_n = \frac{a_{n-1}}{(n+r)^2};\quad r_1 = r_2 = 0$
c. $y_1(x) = 1 + \frac{x}{(1!)^2} + \frac{x^2}{(2!)^2} + \cdots + \frac{x^n}{(n!)^2} + \cdots$

5. b. $r^2 - 2 = 0;\quad a_n = -\frac{a_{n-1}}{(n+r)^2 - 2};\quad r_1 = \sqrt{2},\ r_2 = -\sqrt{2}$
c. $y_1(x) = x^{\sqrt{2}} \times \left(1 - \frac{x}{1(1+2\sqrt{2})} + \frac{x^2}{2!(1+2\sqrt{2})(2+2\sqrt{2})} + \cdots + \frac{(-1)^n}{n!(1+2\sqrt{2})(2+2\sqrt{2})\cdots(n+2\sqrt{2})}x^n + \cdots\right)$
d. $y_2(x) = x^{-\sqrt{2}} \times \left(1 - \frac{x}{1(1-2\sqrt{2})} + \frac{x^2}{2!(1-2\sqrt{2})(2-2\sqrt{2})} + \cdots + \frac{(-1)^n}{n!(1-2\sqrt{2})(2-2\sqrt{2})\cdots(n-2\sqrt{2})}x^n + \cdots\right)$

6. b. $r^2 = 0;\quad (n+r)a_n = a_{n-1};\quad r_1 = r_2 = 0$
c. $y_1(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} + \cdots = e^x$

7. a. $r^2 = 0;\quad r_1 = 0,\ r_2 = 0$
b. $y_1(x) = 1 + \frac{\alpha(\alpha+1)}{2 \cdot 1^2}(x-1) - \frac{\alpha(\alpha+1)(1\cdot 2 - \alpha(\alpha+1))}{(2\cdot 1^2)(2\cdot 2^2)}(x-1)^2 + \cdots + (-1)^{n+1} \times \frac{\alpha(\alpha+1)(1\cdot 2 - \alpha(\alpha+1))\cdots(n(n-1)-\alpha(\alpha+1))}{2^n(n!)^2} \times (x-1)^n + \cdots$

8. a. $r_1 = \frac{1}{2},\ r_2 = 0$ at both $x = \pm 1$
b. $y_1(x) = |x-1|^{1/2}\left(1 + \sum_{n=1}^{\infty} \frac{(-1)^n P_n}{2^n(2n+1)!}(x-1)^n\right)$ where $P_n = (1+2\alpha)\cdots(2n-1+2\alpha)(1-2\alpha)\cdots(2n-1-2\alpha)$, $y_2(x) = 1 + \sum_{n=1}^{\infty}\frac{(-1)^n Q_n}{n!1\cdot 3\cdot 5\cdots(2n-1)}(x-1)^n$ where $Q_n = (1+\alpha)\cdots(n-1+\alpha)(-\alpha)(1-\alpha)\cdots(n-1-\alpha)$,

9. b. $r^2 = 0;\quad r_1 = 0,\quad r_2 = 0;\quad a_n = \frac{(n-1-\lambda)a_{n-1}}{n^2}$
c. $y_1(x) = 1 + \frac{-\lambda}{(1!)^2}x + \frac{(-\lambda)(1-\lambda)}{(2!)^2}x^2 + \cdots + \frac{(-\lambda)(1-\lambda)\cdots(n-1-\lambda)}{(n!)^2}x^n + \cdots$
For $\lambda = n$, the coefficients of all terms past x^n are zero.

12. e. $((n-1)^2 - 1)b_n = -b_{n-2}$, it is impossible to determine b_2.

Section 5.6, page 229

1. a. $x = 0;$
b. $r(r-1) = 0;\quad r_1 = 1,\ r_2 = 0$

2. a. $x = 0;$
b. $r^2 - 3r + 2 = 0;\quad r_1 = 2,\ r_2 = 1$

3. None

4. a. $x = 0;$
b. $r\left(r - \frac{3}{4}\right) = 0;\quad r_1 = \frac{3}{4},\ r_2 = 0$
a. $x = -2;$
b. $r\left(r - \frac{5}{4}\right) = 0;\quad r_1 = \frac{5}{4},\ r_2 = 0$

5. a. $x = 0;$
b. $r^2 + 1 = 0;\quad r_1 = i,\ r_2 = -i$

6. a. $x = 1;$
b. $r^2 + r = 0;\quad r_1 = 0,\ r_2 = -1$

7. a. $x = -2;$
b. $r^2 - (5/4)r = 0;\quad r_1 = 5/4,\ r_2 = 0$

8. a. $x = 2;$
b. $r^2 - 2r = 0;\quad r_1 = 2,\ r_2 = 0$
a. $x = -2;$
b. $r^2 - 2r = 0;\quad r_1 = 2,\ r_2 = 0$

9. b. $r_1 = 0,\quad r_2 = 0$
c. $y_1(x) = 1 + x + \frac{1}{4}x^2 + \frac{1}{36}x^3 + \cdots,$
$y_2(x) = y_1(x)\ln x - 2x - \frac{3}{4}x^2 - \frac{11}{108}x^3 + \cdots$

10. b. $r_1 = 1,\ r_2 = 0$
c. $y_1(x) = x - 4x^2 + \frac{17}{3}x^3 - \frac{47}{12}x^4 + \cdots,$
$y_2(x) = -6y_1(x)\ln x + 1 - 33x^2 + \frac{449}{6}x^3 + \cdots$

11. b. $r_1 = 1,\ r_2 = 0$
c. $y_1(x) = x - \frac{1}{2}x^2 + \frac{1}{12}x^3 - \frac{1}{144}x^4 + \cdots,$
$y_2(x) = -y_1(x)\ln x + 1 - \frac{3}{4}x^2 + \frac{7}{36}x^3 - \frac{35}{1728}x^4 + \cdots$

12. b. $r_1 = 1,\ r_2 = -1$
c. $y_1(x) = x - \frac{1}{24}x^3 + \frac{1}{720}x^5 + \cdots,$
$y_2(x) = -\frac{1}{3}y_1(x)\ln x + x^{-1} - \frac{1}{90}x^3 + \cdots$

13. b. $r_1 = \frac{1}{2},\ r_2 = 0$
c. $y_1(x) = (x-1)^{1/2}\left(1 - \frac{3}{4}(x-1) + \frac{53}{480}(x-1)^2 + \cdots\right),$
d. $\rho = 1$

14. c. *Hint:* $(n-1)(n-2) + (1+\alpha+\beta)(n-1) + \alpha\beta = (n-1+\alpha)(n-1+\beta)$
d. *Hint:* $(n-\gamma)(n-1-\gamma) + (1+\alpha+\beta)(n-\gamma) + \alpha\beta = (n-\gamma+\alpha)(n-\gamma+\beta)$

Section 5.7, page 239

1. $y_1(x) = \sum_{n=0}^{\infty}\frac{(-1)^n x^n}{n!(n+1)!},$
$y_2(x) = -y_1(x)\ln x + \frac{1}{x}\left(1 - \sum_{n=1}^{\infty}\frac{H_n + H_{n-1}}{n!(n-1)!}(-1)^n x^n\right)$

2. $y_1(x) = \frac{1}{x}\sum_{n=0}^{\infty}\frac{(-1)^n x^n}{(n!)^2},$
$y_2(x) = y_1(x)\ln x - \frac{2}{x}\sum_{n=1}^{\infty}\frac{(-1)^n H_n}{(n!)^2}x^n$

3. $y_1(x) = \sum_{n=0}^{\infty}\frac{(-1)^n 2^n}{(n!)^2}x^n,$
$y_2(x) = y_1(x)\ln x - 2\sum_{n=1}^{\infty}\frac{(-1)^n 2^n H_n}{(n!)^2}x^n$

4.
$y_1(x) = x^{3/2}\left(1 + \sum_{m=1}^{\infty}\frac{(-1)^m}{m!(1+\frac{3}{2})(2+\frac{3}{2})\cdots(m+\frac{3}{2})}\left(\frac{x}{2}\right)^{2m}\right)$
$y_2(x) = x^{-3/2}\left(1 + \sum_{m=1}^{\infty}\frac{(-1)^m}{m!(1-\frac{3}{2})(2-\frac{3}{2})\cdots(m-\frac{3}{2})}\left(\frac{x}{2}\right)^{2m}\right)$
Hint: Let $n = 2m$ in the recurrence relation, $m = 1, 2, 3, \ldots$. For $r = -\frac{3}{2}$, $a_1 = 0$ and a_3 is arbitrary.

Chapter 6

Section 6.1, page 247

1. Piecewise continuous
2. Neither
3. Continuous
4. a. $F(s) = 1/s^2,\quad s > 0$
b. $F(s) = 2/s^3,\quad s > 0$
c. $F(s) = n!/s^{n+1},\quad s > 0$
5. $F(s) = \frac{s}{s^2 + a^2},\quad s > 0$
6. $F(s) = \frac{s}{s^2 - b^2},\quad s > |b|$
7. $F(s) = \frac{b}{s^2 - b^2},\quad s > |b|$
8. $F(s) = \frac{b}{s^2 + b^2},\quad s > 0$
9. $F(s) = \frac{s}{s^2 + b^2},\quad s > 0$
10. $F(s) = \frac{b}{(s-a)^2 + b^2},\quad s > a$
11. $F(s) = \frac{s-a}{(s-a)^2 + b^2},\quad s > a$
12. $F(s) = \frac{1}{(s-a)^2},\quad s > a$
13. $F(s) = \frac{2as}{(s^2 + a^2)^2},\quad s > 0$
14. $F(s) = \frac{n!}{(s-a)^{n+1}},\quad s > a$
15. $F(s) = \frac{2a(3s^2 - a^2)}{(s^2 + a^2)^3},\quad s > 0$
16. $F(s) = \frac{1 - e^{-\pi s}}{s}$
17. $F(s) = \frac{1 - e^{-s}}{s^2}$
18. $F(s) = \frac{1 - 2e^{-s} + e^{-2s}}{s^2}$
19. Converges
20. Converges
21. Diverges
23. d. $\Gamma(3/2) = \sqrt{\pi}/2;\quad \Gamma(11/2) = 945\sqrt{\pi}/32$

Section 6.2, page 255

1. $f(t) = \frac{3}{2}\sin(2t)$
2. $f(t) = 2t^2 e^t$

3. $f(t) = \frac{2}{5}e^t - \frac{2}{5}e^{-4t}$
4. $f(t) = 2e^{-t}\cos(2t)$
5. $f(t) = 2\cosh(2t) - \frac{3}{2}\sinh(2t)$
6. $f(t) = 3 - 2\sin(2t) + 5\cos(2t)$
7. $f(t) = -2e^{-2t}\cos t + 5e^{-2t}\sin t$
8. $y = \frac{1}{5}(e^{3t} + 4e^{-2t})$
9. $y = 2e^{-t} - e^{-2t}$
10. $y = e^t \sin t$
11. $y = 2e^t\cos\left(\sqrt{3}\,t\right) - \left(2/\sqrt{3}\right)e^t\sin\left(\sqrt{3}\,t\right)$
12. $y = 2e^{-t}\cos(2t) + \frac{1}{2}e^{-t}\sin(2t)$
13. $y = te^t - t^2e^t + \frac{2}{3}t^3e^t$
14. $y = \cosh t$
15. $y = ((\omega^2 - 5)\cos(\omega t) + \cos(2t))/(\omega^2 - 4)$
16. $y = \frac{1}{5}(e^{-t} - e^t\cos t + 7e^t\sin t)$
17. $Y(s) = \frac{s}{s^2 + 4} + \frac{1 - e^{-\pi s}}{s(s^2 + 4)}$
18. $Y(s) = \frac{1 - e^{-s}}{s^2(s^2 + 4)}$
19. $Y(s) = \frac{1 - 2e^{-s} + e^{-2s}}{s^2(s^2 + 1)}$
22. $F(s) = 1/(s-a)^2$
23. $F(s) = 2b(3s^2 - b^2)/(s^2 + b^2)^3$
24. $F(s) = n!/(s-a)^{n+1}$
25. $F(s) = 2b(s-a)/[(s-a)^2 + b^2]^2$
27. a. $Y' + s^2 Y = s$
b. $s^2 Y'' + 2sY' - (s^2 + \alpha(\alpha+1))Y = -1$

Section 6.3, page 262

5. b. $f(t) = -2u_3(t) + 4u_5(t) - u_7(t)$
6. b. $f(t) = 1 - 2u_1(t) + 2u_2(t) - 2u_3(t) + u_4(t)$
7. b. $f(t) = 1 + u_2(t)[e^{-(t-2)} - 1]$
8. b. $f(t) = t + u_2(t)(2-t) + u_5(t)(5-t) - u_7(t)(7-t)$
9. $F(s) = 2e^{-s}/s^3$
10. $F(s) = \frac{e^{-\pi s}}{s^2} - \frac{e^{-2\pi s}}{s^2}(1 + \pi s)$
11. $F(s) = \frac{1}{s}(e^{-s} + 2e^{-3s} - 6e^{-4s})$
12. $F(s) = s^{-2}\left((1-s)e^{-2s} - (1+s)e^{-3s}\right)$
13. $f(t) = t^3 e^{2t}$
14. $f(t) = \frac{1}{3}u_2(t)\left(e^{t-2} - e^{-2(t-2)}\right)$
15. $f(t) = 2u_2(t)e^{t-2}\cos(t-2)$
16. $f(t) = u_1(t) + u_2(t) - u_3(t) - u_4(t)$
18. $f(t) = 2(2t)^n$
19. $f(t) = \frac{1}{2}e^{-t/2}\cos t$
20. $f(t) = \frac{1}{6}e^{t/3}\left(e^{2t/3} - 1\right)$
21. $F(s) = s^{-1}\left(1 - e^{-s}\right),\quad s > 0$
22. $F(s) = s^{-1}\left(1 - e^{-s} + e^{-2s} - e^{-3s}\right),\quad s > 0$
23. $F(s) = \frac{1}{s}\sum_{n=0}^{\infty}(-1)^n e^{-ns} = \frac{1/s}{1 + e^{-s}},\quad s > 0$

25. $\mathcal{L}\{f(t)\} = \dfrac{1/s}{1+e^{-s}}, \quad s > 0$

26. $\mathcal{L}\{f(t)\} = \dfrac{1-e^{-s}}{s\left(1+e^{-s}\right)}, \quad s > 0$

27. $\mathcal{L}\{f(t)\} = \dfrac{1-(1+s)e^{-s}}{s^2\left(1-e^{-s}\right)}, \quad s > 0$

28. $\mathcal{L}\{f(t)\} = \dfrac{1+e^{-\pi s}}{(1+s^2)(1-e^{-\pi s})}, \quad s > 0$

29. a. $\mathcal{L}\{f(t)\} = s^{-1}(1-e^{-s}), \quad s > 0$
b. $\mathcal{L}\{g(t)\} = s^{-2}(1-e^{-s}), \quad s > 0$
c. $\mathcal{L}\{h(t)\} = s^{-2}(1-e^{-s})^2, \quad s > 0$

30. b. $\mathcal{L}\{p(t)\} = \dfrac{1-e^{-s}}{s^2(1+e^{-s})}, \quad s > 0$

Section 6.4, page 268

1. b. $y = 1 - \cos t + \sin t - u_{3\pi}(t)(1+\cos t)$

2. b. $y = e^{-t}\sin t + \frac{1}{2}u_{\pi}(t)(1+e^{-(t-\pi)}\cos t + e^{-(t-\pi)}\sin t)$
$-\frac{1}{2}u_{2\pi}(t)(1-e^{-(t-2\pi)}\cos t - e^{-(t-2\pi)}\sin t)$

3. b. $y = \frac{1}{6}(1-u_{2\pi}(t))(2\sin t - \sin(2t))$

4. b. $y = \frac{1}{2} + \frac{1}{2}e^{-2t} - e^{-t} - u_{10}(t)\left(\frac{1}{2} + \frac{1}{2}e^{-2(t-10)} - e^{-(t-10)}\right)$

5. b. $y = -\frac{16}{25} + \frac{4}{5}t + \frac{1}{25}e^{-t/2}(16\cos t - 12\sin t)$
$+ u_{\pi/2}(t)\left(\frac{16}{25} + \frac{2\pi}{5} - \frac{4}{5}t - \frac{1}{25}e^{\pi/4-t/2}(12\cos t + 16\sin t)\right)$

6. b. $y = h(t) + u_{\pi}(t)h(t-\pi)$,
$h(t) = \frac{4}{17}(-4\cos t + \sin t + 4e^{-t/2}\cos t + e^{-t/2}\sin t)$

7. b. $y = u_{\pi}(t)\left(\frac{1}{4} - \frac{1}{4}\cos(2t-2\pi)\right)$
$- u_{3\pi}(t)\left(\frac{1}{4} - \frac{1}{4}\cos(2t-6\pi)\right)$

8. b. $y = h(t) - u_{\pi}(t)h(t-\pi)$,
$h(t) = (3 - 4\cos t + \cos(2t))/12$

9. $f(t) = \frac{h}{k}(t-t_0)(u_{t_0}(t) - u_{t_0+k}(t)) + hu_{t_0+k}(t)$

10. $g(t) = \frac{h}{k}(t-t_0)\left(u_{t_0}(t) - u_{t_0+k}(t)\right)$
$-\frac{h}{k}(t-t_0-2k)\left(u_{t_0+k}(t) - u_{t_0+2k}(t)\right)$

11. b. $u(t) = 4ku_{3/2}(t)h\left(t-\frac{3}{2}\right) - 4ku_{5/2}(t)h\left(t-\frac{5}{2}\right)$,
$h(t) = \frac{1}{4} - \frac{\sqrt{7}}{84}e^{-t/8}\sin\left(\frac{3\sqrt{7}}{8}t\right) - \frac{1}{4}e^{-t/8}\cos\left(\frac{3\sqrt{7}}{8}t\right)$
d. $k = 2.51$
e. $\tau = 25.6773$

12. a. $k = 5$
b. $y = (u_5(t)h(t-5) - u_{5+k}(t)h(t-5-k))/k$,
$h(t) = \frac{1}{4}t - \frac{1}{8}\sin(2t)$

13. b. $y = 1 - \cos t + 2\sum_{k=1}^{n}(-1)^k u_{k\pi}(t)(1-\cos(t-k\pi))$

15. b. $y = 1 - \cos t + \sum_{k=1}^{n}(-1)^k u_{k\pi}(t)(1-\cos(t-k\pi))$

17. a. $y = 1 - \cos t + 2\sum_{k=1}^{n}(-1)^k u_{11k/4}(t)(1-\cos(t-11k/4))$

Section 6.5, page 273

1. a. $y = e^{-t}\cos t + e^{-t}\sin t + u_{\pi}(t)e^{-(t-\pi)}\sin(t-\pi)$

2. a. $y = \frac{1}{2}u_{\pi}(t)\sin(2(t-\pi)) - \frac{1}{2}u_{2\pi}(t)\sin(2(t-2\pi))$

3. a. $y = -\frac{1}{2}e^{-2t} + \frac{1}{2}e^{-t} + u_5(t)\left(-e^{-2(t-5)} + e^{-(t-5)}\right) + u_{10}(t)\left(\frac{1}{2} + \frac{1}{2}e^{-2(t-10)} - e^{-(t-10)}\right)$

4. a. $y = \frac{1}{4}\sin t - \frac{1}{4}\cos t + \frac{1}{4}e^{-t}\cos(\sqrt{2}\,t) + \frac{1}{\sqrt{2}}u_{3\pi}(t)e^{-(t-3\pi)}\sin\left(\sqrt{2}(t-3\pi)\right)$

5. a. $y = \sin t + u_{2\pi}(t)\sin(t-2\pi)$

6. a. $y = u_{\pi/4}(t)\sin(2(t-\pi/4))$

7. a. $y = \frac{1}{5}\cos t + \frac{2}{5}\sin t - \frac{1}{5}e^{-t}\cos t - \frac{3}{5}e^{-t}\sin t$
$+ u_{\pi/2}(t)e^{-(t-\pi/2)}\sin(t-\pi/2)$

8. a. $y = u_1(t)(\sinh(t-1) - \sin(t-1))/2$

9. a. $-e^{-T/4}\delta(t-5-T), \quad T = 8\pi/\sqrt{15}$

10. a. $y = \frac{4}{\sqrt{15}}u_1(t)e^{-(t-1)/4}\sin\left(\frac{\sqrt{15}}{4}(t-1)\right)$
b. $t_1 \cong 2.3613, \quad y_1 \cong 0.71153$
c. $y = \frac{8\sqrt{7}}{21}u_1(t)e^{-(t-1)/8}\sin\left(\frac{3\sqrt{7}}{8}(t-1)\right)$;
$t_1 \cong 2.4569, \quad y_1 \cong 0.83351$
d. $t_1 = 1 + \pi/2 \cong 2.5708, \quad y_1 = 1$

11. a. $k_1 \cong 2.8108$
b. $k_1 \cong 2.3995$
c. $k_1 = 2$

12. a. $\phi(t,k) = \frac{1}{2k}(u_{4-k}(t)h(t-4+k) - u_{4+k}(t)h(t-4-k))$,
$h(t) = 1 - \cos t$
b. $\phi_0(t) = u_4(t)\sin(t-4)$
c. Yes

13. b. $y = \sum_{k=1}^{20} u_{k\pi}(t)\sin(t-k\pi)$

14. b. $y = \sum_{k=1}^{20}(-1)^{k+1}u_{k\pi}(t)\sin(t-k\pi)$

15. b. $y = \sum_{k=1}^{15} u_{(2k-1)\pi}(t)\sin(t-(2k-1)\pi)$

16. b. $y = \sum_{k=1}^{40}(-1)^{k+1}u_{11k/4}(t)\sin(t-11k/4)$

17. b. $y = \frac{20}{\sqrt{399}}\sum_{k=1}^{20}(-1)^{k+1}u_{k\pi}(t)e^{-(t-k\pi)/20}$
$\times \sin\left(\frac{\sqrt{399}}{20}(t-k\pi)\right)$

18. b. $y = \frac{20}{\sqrt{399}}\sum_{k=1}^{15}u_{(2k-1)\pi}(t)e^{-(t-(2k-1)\pi)/20}$
$\times \sin\left(\frac{\sqrt{399}}{20}(t-(2k-1)\pi)\right)$

Section 6.6, page 279

3. $\sin t * \sin t = \frac{1}{2}(\sin t - t\cos t)$ is negative when $t = 2\pi$, for example.

4. $F(s) = \dfrac{2}{s^2(s^2+4)}$

5. $F(s) = \dfrac{1}{(s+1)(s^2+1)}$

6. $F(s) = \dfrac{s}{(s^2+1)^2}$

7. $f(t) = \frac{1}{6}\int_0^t (t-\tau)^3 \sin\tau\, d\tau$

8. $f(t) = \int_0^t e^{-(t-\tau)}\cos(2\tau)d\tau$

9. $f(t) = \frac{1}{2}\int_0^t (t-\tau)e^{-(t-\tau)}\sin(2\tau)d\tau$

10. c. $\int_0^1 u^m(1-u)^n\, du = \dfrac{\Gamma(m+1)\Gamma(n+1)}{\Gamma(m+n+2)}$

11. $y = \frac{1}{\omega}\sin(\omega t) + \frac{1}{\omega}\int_0^t \sin(\omega(t-\tau))g(\tau)\, d\tau$

12. $y = \frac{1}{8}\int_0^t e^{-(t-\tau)/2}\sin(2(t-\tau))g(\tau)\, d\tau$

13. $y = e^{-t/2}\cos t - \frac{1}{2}e^{-t/2}\sin t$
$+ \int_0^t e^{-(t-\tau)/2}\sin(t-\tau)(1-u_{\pi}(\tau))\, d\tau$

14. $y = 2e^{-t} - e^{-2t} + \int_0^t \left(e^{-(t-\tau)} - e^{-2(t-\tau)}\right)\cos(\alpha\tau)\, d\tau$

15. $y = \frac{4}{3}\cos t - \frac{1}{3}\cos(2t)$
$+ \frac{1}{6}\int_0^t (2\sin(t-\tau) - \sin(2(t-\tau)))g(\tau)\, d\tau$

16. $\Phi(s) = \dfrac{F(s)}{1+K(s)}$

17. a. $\phi(t) = \frac{1}{3}(4\sin(2t) - 2\sin t)$

18. a. $\phi(t) = \cos t$
b. $\phi''(t) + \phi(t) = 0, \quad \phi(0) = 1, \quad \phi'(0) = 0$

19. a. $\phi(t) = \left(1 - 2t + t^2\right)e^{-t}$
b. $\phi''(t) + 2\phi'(t) + \phi(t) = 2e^{-t}, \quad \phi(0) = 1, \quad \phi'(0) = -3$

20. a. $\phi(t) = \frac{1}{3}e^{-t} - \frac{1}{3}e^{t/2}\cos\left(\frac{\sqrt{3}}{2}t\right) + \frac{1}{\sqrt{3}}e^{t/2}\sin\left(\frac{\sqrt{3}}{2}t\right)$
b. $\phi'''(t) + \phi(t) = 0, \quad \phi(0) = 0, \quad \phi'(0) = 0, \quad \phi''(0) = 1$

21. a. $\phi(t) = \cos t$
b. $\phi^{(4)}(t) - \phi(t) = 0, \quad \phi(0) = 1, \quad \phi'(0) = 0,$
$\phi''(0) = -1, \quad \phi'''(0) = 0$

Chapter 7

Section 7.1, page 284

1. $x_1' = x_2, \quad x_2' = -2x_1 - 0.5x_2$

2. $x_1' = x_2, \quad x_2' = -(1 - 0.25t^{-2})x_1 - t^{-1}x_2$

3. $x_1' = x_2, \quad x_2' = x_3, \quad x_3' = x_4, \quad x_4' = x_1$

4. $x_1' = x_2, \quad x_2' = -4x_1 - 0.25x_2 + 2\cos(3t), \quad x_1(0) = 1,$
$x_2(0) = -2$

5. $x_1' = x_2, \quad x_2' = -q(t)x_1 - p(t)x_2 + g(t),$
$x_1(0) = u_0, \quad x_2(0) = u_0'$

6. c. $x_1 = c_1e^{-t} + c_2e^{-3t}$
d. $x_2 = c_1e^{-t} - c_2e^{-3t}$

7. a. $x_1'' - x_1' - 2x_1 = 0$
b. $x_1 = \frac{11}{3}e^{2t} - \frac{2}{3}e^{-t}, \quad x_2 = \frac{11}{6}e^{2t} - \frac{4}{3}e^{-t}$
c. Graph is asymptotic to the line $x_1 = 2x_2$ in the first quadrant.

8. a. $x_1'' + 4x_1 = 0$
b. $x_1 = 3\cos(2t) + 4\sin(2t), \quad x_2 = -3\sin(2t) + 4\cos(2t)$
c. Graph is a circle, center at origin, radius 5, traversed clockwise.

9. a. $x_1'' + x_1' + 4.25x_1 = 0$
b. $x_1 = -2e^{-t/2}\cos(2t) + 2e^{-t/2}\sin(2t)$,
$x_2 = 2e^{-t/2}\cos(2t) + 2e^{-t/2}\sin(2t)$
c. Graph is a clockwise spiral, approaching the origin.

10. $LRCI'' + LI' + RI = 0$

15. $y_1' = y_3, \quad y_2' = y_4, \quad m_1y_3' = -(k_1+k_2)y_1 + k_2y_2 + F_1(t),$
$m_2y_4' = k_2y_1 - (k_2+k_3)y_2 + F_2(t)$

19. a. $Q_1' = \frac{3}{2} - \frac{1}{10}Q_1 + \frac{3}{40}Q_2, \quad Q_1(0) = 25,$
$Q_2' = 3 + \frac{1}{10}Q_1 - \frac{1}{5}Q_2, \quad Q_2(0) = 15$
b. $Q_1^E = 42, \quad Q_2^E = 36$
c. $x_1' = -\frac{1}{10}x_1 + \frac{3}{40}x_2, \quad x_1(0) = -17,$
$x_2' = \frac{1}{10}x_1 - \frac{1}{5}x_2, \quad x_2(0) = -21$

20. a. $Q_1' = 3q_1 - \frac{1}{15}Q_1 + \frac{1}{100}Q_2, \quad Q_1(0) = Q_1^0,$
$Q_2' = q_2 + \frac{1}{30}Q_1 - \frac{3}{100}Q_2, \quad Q_2(0) = Q_2^0$
b. $Q_1^E = 6(9q_1 + q_2), \quad Q_2^E = 20(3q_1 + 2q_2)$
c. No
d. $\frac{10}{9} \le Q_2^E/Q_1^E \le \frac{20}{3}$

Section 7.2, page 293

1. a. $\begin{pmatrix} 6 & -6 & 3 \\ 5 & 9 & -2 \\ 2 & 3 & 8 \end{pmatrix}$
b. $\begin{pmatrix} -15 & 6 & -12 \\ 7 & -18 & -1 \\ -26 & -3 & -5 \end{pmatrix}$
c. $\begin{pmatrix} 6 & -12 & 3 \\ 4 & 3 & 7 \\ 9 & 12 & 0 \end{pmatrix}$
d. $\begin{pmatrix} -8 & -9 & 11 \\ 14 & 12 & -5 \\ 5 & -8 & 5 \end{pmatrix}$

2. a. $\begin{pmatrix} 1-i & -7+2i \\ -1+2i & 2+3i \end{pmatrix}$
b. $\begin{pmatrix} 3+4i & 6i \\ 11+6i & 6-5i \end{pmatrix}$
c. $\begin{pmatrix} -3+5i & 7+5i \\ 2+i & 7+2i \end{pmatrix}$
d. $\begin{pmatrix} 8+7i & 4-4i \\ 6-4i & -4 \end{pmatrix}$

3. a. $\begin{pmatrix} -2 & 1 & 2 \\ 1 & 0 & -1 \\ 2 & -3 & 1 \end{pmatrix}$
b. $\begin{pmatrix} 1 & 3 & -2 \\ 2 & -1 & 1 \\ 3 & -1 & 0 \end{pmatrix}$
c, d. $\begin{pmatrix} -1 & 4 & 0 \\ 3 & -1 & 0 \\ 5 & -4 & 1 \end{pmatrix}$

4. a. $\begin{pmatrix} 3-2i & 2-i \\ 1+i & -2+3i \end{pmatrix}$

b. $\begin{pmatrix} 3+2i & 1-i \\ 2+i & -2-3i \end{pmatrix}$

c. $\begin{pmatrix} 3+2i & 2+i \\ 1-i & -2-3i \end{pmatrix}$

5. a. $\begin{pmatrix} 7 & -11 & -3 \\ 11 & 20 & 17 \\ -4 & 3 & -12 \end{pmatrix}$

b. $\begin{pmatrix} 5 & 0 & -1 \\ 2 & 7 & 4 \\ -1 & 1 & 4 \end{pmatrix}$

c. $\begin{pmatrix} 6 & -8 & -11 \\ 9 & 15 & 6 \\ -5 & -1 & 5 \end{pmatrix}$

7. a. $4i$

b. $12-8i$

c. $2+2i$

d. 16

8. $\begin{pmatrix} \frac{3}{11} & -\frac{4}{11} \\ \frac{2}{11} & \frac{1}{11} \end{pmatrix}$

9. $\begin{pmatrix} \frac{1}{6} & \frac{1}{12} \\ -\frac{1}{2} & \frac{1}{4} \end{pmatrix}$

10. $\begin{pmatrix} 1 & -3 & 2 \\ -3 & 3 & -1 \\ 2 & -1 & 0 \end{pmatrix}$

11. Singular

12. $\begin{pmatrix} \frac{1}{2} & -\frac{1}{4} & \frac{1}{8} \\ 0 & \frac{1}{2} & -\frac{1}{4} \\ 0 & 0 & \frac{1}{2} \end{pmatrix}$

13. Singular

14. $\begin{pmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{pmatrix}$

16. a. $\begin{pmatrix} 7e^t & 5e^{-t} & 10e^{2t} \\ -e^t & 7e^{-t} & 2e^{2t} \\ 8e^t & 0 & -e^{2t} \end{pmatrix}$

b. $\begin{pmatrix} 2e^{2t}-2+3e^{3t} & 1+4e^{-2t}-e^t & 3e^{3t}+2e^t-e^{4t} \\ 4e^{2t}-1-3e^{3t} & 2+2e^{-2t}+e^t & 6e^{3t}+e^t+e^{4t} \\ -2e^{2t}-3+6e^{3t} & -1+6e^{-2t}-2e^t & -3e^{3t}+3e^t-2e^{4t} \end{pmatrix}$

c. $\begin{pmatrix} e^t & -2e^{-t} & 2e^{2t} \\ 2e^t & -e^{-t} & -2e^{2t} \\ -e^t & -3e^{-t} & 4e^{2t} \end{pmatrix}$

d. $(e-1)\begin{pmatrix} 1 & 2e^{-1} & \frac{1}{2}(e+1) \\ 2 & e^{-1} & -\frac{1}{2}(e+1) \\ -1 & 3e^{-1} & e+1 \end{pmatrix}$

Section 7.3, page 303

1. $x_1=-\frac{1}{3},\quad x_2=\frac{7}{3},\quad x_3=-\frac{1}{3}$

2. No solution

3. $x_1=-c,\quad x_2=c+1,\quad x_3=c$, where c is arbitrary

4. $x_1=c,\quad x_2=-c,\quad x_3=-c$, where c is arbitrary

5. $x_1=0,\quad x_2=0,\quad x_3=0$

6. Linearly independent

7. $\mathbf{x}^{(1)}-5\mathbf{x}^{(2)}+2\mathbf{x}^{(3)}=\mathbf{0}$

8. Linearly independent

9. $\mathbf{x}^{(1)}+\mathbf{x}^{(2)}-\mathbf{x}^{(4)}=\mathbf{0}$

11. $3\mathbf{x}^{(1)}(t)-6\mathbf{x}^{(2)}(t)+\mathbf{x}^{(3)}(t)=\mathbf{0}$

12. Linearly independent

14. $\lambda_1=2,\ \mathbf{x}^{(1)}=\begin{pmatrix}1\\3\end{pmatrix};\quad \lambda_2=4,\ \mathbf{x}^{(2)}=\begin{pmatrix}1\\1\end{pmatrix}$

15. $\lambda_1=1+2i,\ \mathbf{x}^{(1)}=\begin{pmatrix}1\\1-i\end{pmatrix};\quad \lambda_2=1-2i,\ \mathbf{x}^{(2)}=\begin{pmatrix}1\\1+i\end{pmatrix}$

16. $\lambda_1=-3,\ \mathbf{x}^{(1)}=\begin{pmatrix}1\\-1\end{pmatrix};\quad \lambda_2=-1,\ \mathbf{x}^{(2)}=\begin{pmatrix}1\\1\end{pmatrix}$

17. $\lambda_1=2,\ \mathbf{x}^{(1)}=\begin{pmatrix}\sqrt{3}\\1\end{pmatrix};\quad \lambda_2=-2,\ \mathbf{x}^{(2)}=\begin{pmatrix}1\\-\sqrt{3}\end{pmatrix}$

18. $\lambda_1=1,\ \mathbf{x}^{(1)}=\begin{pmatrix}2\\-3\\2\end{pmatrix};\quad \lambda_2=1+2i,\ \mathbf{x}^{(2)}=\begin{pmatrix}0\\1\\-i\end{pmatrix};$

$\lambda_3=1-2i,\ \mathbf{x}^{(3)}=\begin{pmatrix}0\\1\\i\end{pmatrix}$

19. $\lambda_1=1,\ \mathbf{x}^{(1)}=\begin{pmatrix}1\\0\\-1\end{pmatrix};\quad \lambda_2=2,\ \mathbf{x}^{(2)}=\begin{pmatrix}-2\\1\\0\end{pmatrix};$

$\lambda_3=3,\ \mathbf{x}^{(3)}=\begin{pmatrix}0\\1\\-1\end{pmatrix}$

20. $\lambda_1=1,\ \mathbf{x}^{(1)}=\begin{pmatrix}2\\-2\\-1\end{pmatrix};\quad \lambda_2=2,\ \mathbf{x}^{(2)}=\begin{pmatrix}2\\1\\2\end{pmatrix};$

$\lambda_3=-1,\ \mathbf{x}^{(3)}=\begin{pmatrix}1\\2\\-2\end{pmatrix}$

Section 7.4, page 308

1. d. $\mathbf{x}(t)=\frac{1}{2}\mathbf{x}^{(1)}+\frac{1}{2}\mathbf{x}^{(2)}=\begin{pmatrix}\frac{1}{2}e^t+\frac{1}{2}e^{-t}\\ \frac{1}{2}e^t+\frac{3}{2}e^{-t}\end{pmatrix},$

e. 2.

2. d. $\mathbf{x}(t)=-\frac{1}{5}\mathbf{x}^{(1)}+\frac{6}{5}\mathbf{x}^{(2)}=\begin{pmatrix}-\frac{1}{5}e^{-3t}+\frac{6}{5}e^{2t}\\ \frac{4}{5}e^{-3t}+\frac{6}{5}e^{2t}\end{pmatrix}$

e. $5e^{-t}$.

3. d. $\mathbf{x}(t)=\frac{1}{5}\mathbf{x}^{(1)}-\frac{8}{5}\mathbf{x}^{(2)}=\begin{pmatrix}\cos t-8\sin t\\ 2\cos t-3\sin t\end{pmatrix},$

e. -5.

4. d. $\mathbf{x}(t)=\frac{1}{2}\mathbf{x}^{(1)}+0\mathbf{x}^{(2)}=\begin{pmatrix}1\\2\end{pmatrix},$

e. 2.

5. d. $\mathbf{x}(t)=\frac{1}{4}\mathbf{x}^{(1)}+\mathbf{x}^{(2)}=\begin{pmatrix}\frac{t}{4}+t^{-1}\\ \frac{t}{4}+3t^{-1}\end{pmatrix},$

e. 2.

6. d. $\mathbf{x}(t)=2\mathbf{x}^{(1)}+0\mathbf{x}^{(2)}=\begin{pmatrix}2t^{-1}\\4t^{-1}\end{pmatrix},$

e. $-3t$.

8. c. $W(t)=c\exp\left(\int (p_{11}(t)+p_{22}(t))dt\right)$

12. a. $W(t)=t^2$

b. $\mathbf{x}^{(1)}$ and $\mathbf{x}^{(2)}$ are linearly independent at each point except $t=0$; they are linearly independent on every interval.

c. At least one coefficient must be discontinuous at $t=0$.

d. $\mathbf{x}'=\begin{pmatrix}0 & 1\\ -2t^{-2} & 2t^{-1}\end{pmatrix}\mathbf{x}$

13. a. $W(t)=t(t-2)e^t$

b. $\mathbf{x}^{(1)}$ and $\mathbf{x}^{(2)}$ are linearly independent at each point except $t=0$ and $t=2$; they are linearly independent on every interval.

c. There must be at least one discontinuous coefficient at $t=0$ and $t=2$.

d. $\mathbf{x}'=\begin{pmatrix}0 & 1\\ \frac{2-2t}{t^2-2t} & \frac{t^2-2}{t^2-2t}\end{pmatrix}\mathbf{x}$

Section 7.5, page 318

1. a. $\mathbf{x}=c_1\begin{pmatrix}1\\2\end{pmatrix}e^{-t}+c_2\begin{pmatrix}2\\1\end{pmatrix}e^{2t}$

2. a. $\mathbf{x}=c_1\begin{pmatrix}1\\1\end{pmatrix}e^{-t}+c_2\begin{pmatrix}2\\3\end{pmatrix}e^{-2t}$

3. a. $\mathbf{x}=c_1\begin{pmatrix}1\\1\end{pmatrix}e^{t}+c_2\begin{pmatrix}1\\3\end{pmatrix}e^{-t}$

4. a. $\mathbf{x}=c_1\begin{pmatrix}1\\-1\end{pmatrix}e^{t/2}+c_2\begin{pmatrix}1\\1\end{pmatrix}e^{2t}$

5. a. $\mathbf{x}=c_1\begin{pmatrix}3\\4\end{pmatrix}+c_2\begin{pmatrix}1\\2\end{pmatrix}e^{-2t}$

6. a. $\mathbf{x}=c_1\begin{pmatrix}-2\\1\end{pmatrix}+c_2\begin{pmatrix}-3\\1\end{pmatrix}e^{t}$

7. $\mathbf{x}=c_1\begin{pmatrix}1\\1\\1\end{pmatrix}e^{4t}+c_2\begin{pmatrix}1\\-2\\1\end{pmatrix}e^{t}+c_3\begin{pmatrix}1\\0\\-1\end{pmatrix}e^{-t}$

8. $\mathbf{x}=c_1\begin{pmatrix}1\\-4\\1\end{pmatrix}e^{-t}+c_2\begin{pmatrix}1\\0\\-1\end{pmatrix}e^{-t}+c_3\begin{pmatrix}2\\1\\2\end{pmatrix}e^{8t}$

9. $\mathbf{x}=c_1\begin{pmatrix}1\\-4\\-1\end{pmatrix}e^{t}+c_2\begin{pmatrix}1\\-1\\-1\end{pmatrix}e^{-2t}+c_3\begin{pmatrix}1\\2\\1\end{pmatrix}e^{3t}$

10. $\mathbf{x}=-\frac{3}{2}\begin{pmatrix}1\\3\end{pmatrix}e^{2t}+\frac{7}{2}\begin{pmatrix}1\\1\end{pmatrix}e^{4t}$

11. $\mathbf{x}=\frac{1}{2}\begin{pmatrix}1\\1\end{pmatrix}e^{-t}+\frac{1}{2}\begin{pmatrix}1\\5\end{pmatrix}e^{3t}$

12. $\mathbf{x}=6\begin{pmatrix}1\\2\\-1\end{pmatrix}e^{t}+3\begin{pmatrix}1\\-2\\1\end{pmatrix}e^{-t}-\begin{pmatrix}2\\1\\-8\end{pmatrix}e^{4t}$

14. $\mathbf{x}=c_1\begin{pmatrix}1\\1\end{pmatrix}t+c_2\begin{pmatrix}1\\3\end{pmatrix}t^{-1}$

15. $\mathbf{x}=c_1\begin{pmatrix}1\\3\end{pmatrix}t^2+c_2\begin{pmatrix}1\\1\end{pmatrix}t^4$

16. $\mathbf{x}=c_1\begin{pmatrix}3\\4\end{pmatrix}+c_2\begin{pmatrix}1\\2\end{pmatrix}t^{-2}$

21. a. $x_1'=x_2,\quad x_2'=-(c/a)x_1-(b/a)x_2$

22. a. $\mathbf{x}=-\frac{55}{8}\begin{pmatrix}3\\2\end{pmatrix}e^{-t/20}+\frac{29}{8}\begin{pmatrix}1\\-2\end{pmatrix}e^{-t/4}$

c. $T\cong 74.39$

23. a. $\mathbf{x}=c_1\begin{pmatrix}-\sqrt{2}\\1\end{pmatrix}e^{\left(-2+\sqrt{2}\right)t/2}+c_2\begin{pmatrix}\sqrt{2}\\1\end{pmatrix}e^{\left(-2-\sqrt{2}\right)t/2};$

$r_{1,2}=(-2\pm\sqrt{2})/2;$ node

b. $\mathbf{x}=c_1\begin{pmatrix}-1\\\sqrt{2}\end{pmatrix}e^{\left(-1+\sqrt{2}\right)t}+c_2\begin{pmatrix}1\\\sqrt{2}\end{pmatrix}e^{\left(-1-\sqrt{2}\right)t};$

$r_{1,2}=-1\pm\sqrt{2}$; saddle point

c. $r_{1,2}=-1\pm\sqrt{\alpha};\ \alpha=1$

24. a. $\begin{pmatrix}I\\V\end{pmatrix}=c_1\begin{pmatrix}1\\3\end{pmatrix}e^{-2t}+c_2\begin{pmatrix}1\\1\end{pmatrix}e^{-t}$

25. a. $\left(\frac{1}{CR_2}-\frac{R_1}{L}\right)^2-\frac{4}{CL}>0$

Section 7.6, page 327

1. a. $\mathbf{x}=c_1e^{-t}\begin{pmatrix}2\cos(2t)\\ \sin(2t)\end{pmatrix}+c_2e^{-t}\begin{pmatrix}-2\sin(2t)\\ \cos(2t)\end{pmatrix}$

2. a. $\mathbf{x}=c_1\begin{pmatrix}5\cos t\\ 2\cos t+\sin t\end{pmatrix}+c_2\begin{pmatrix}5\sin t\\ -\cos t+2\sin t\end{pmatrix}$

3. a. $\mathbf{x}=c_1e^{-t}\begin{pmatrix}\cos t\\ 2\cos t+\sin t\end{pmatrix}+c_2e^{-t}\begin{pmatrix}\sin t\\ -\cos t+2\sin t\end{pmatrix}$

4. a. $\mathbf{x}=c_1\begin{pmatrix}-2\cos 3t\\ \cos 3t+3\sin 3t\end{pmatrix}+c_2\begin{pmatrix}-2\sin 3t\\ \sin 3t-3\cos 3t\end{pmatrix}$

5. $\mathbf{x}=c_1\begin{pmatrix}2\\-3\\2\end{pmatrix}e^{t}+c_2e^{t}\begin{pmatrix}0\\ \cos(2t)\\ \sin(2t)\end{pmatrix}+c_3e^{t}\begin{pmatrix}0\\ \sin(2t)\\ -\cos(2t)\end{pmatrix}$

6. $\mathbf{x}=c_1\begin{pmatrix}2\\-2\\1\end{pmatrix}e^{-2t}+c_2e^{-t}\begin{pmatrix}-\sqrt{2}\sin(\sqrt{2}\,t)\\ \cos(\sqrt{2}\,t)\\ -\cos(\sqrt{2}\,t)-\sqrt{2}\sin(\sqrt{2}\,t)\end{pmatrix}+$

$c_3e^{-t}\begin{pmatrix}\sqrt{2}\cos(\sqrt{2}\,t)\\ \sin(\sqrt{2}\,t)\\ \sqrt{2}\cos(\sqrt{2}\,t)-\sin(\sqrt{2}\,t)\end{pmatrix}$

7. $\mathbf{x}=e^{-t}\begin{pmatrix}\cos t-3\sin t\\ \cos t-\sin t\end{pmatrix}$

8. $\mathbf{x}=e^{-2t}\begin{pmatrix}\cos t-5\sin t\\ -2\cos t-3\sin t\end{pmatrix}$

9. a. $r=-\frac{1}{4}\pm i$

10. a. $r=\frac{1}{5}\pm i$

11. a. $r=\alpha\pm i$

b. $\alpha=0$

12. a. $r=\left(\alpha\pm\sqrt{\alpha^2-20}\right)/2$

b. $\alpha=-\sqrt{20},\ 0,\ \sqrt{20}$

13. a. $r=\frac{5}{4}\pm\frac{1}{2}\sqrt{3\alpha}$

b. $\alpha=0,\ 25/12$

14. a. $r=-1\pm\sqrt{-\alpha}$

b. $\alpha=-1,\ 0$

15. a. $r = -1 \pm \sqrt{25 + 8\alpha}$

b. $\alpha = -25/8,\ -3$

16. $\mathbf{x} = c_1 t^{-1}\begin{pmatrix} \cos\left(\sqrt{2}\ln t\right) \\ \sqrt{2}\sin\left(\sqrt{2}\ln t\right) \end{pmatrix} + c_2 t^{-1}\begin{pmatrix} \sin\left(\sqrt{2}\ln t\right) \\ -\sqrt{2}\cos\left(\sqrt{2}\ln t\right) \end{pmatrix}$

17. $\mathbf{x} = c_1\begin{pmatrix} 5\cos(\ln t) \\ 2\cos(\ln t) + \sin(\ln t) \end{pmatrix} + c_2\begin{pmatrix} 5\sin(\ln t) \\ -\cos(\ln t) + 2\sin(\ln t) \end{pmatrix}$

18. a. $r = -\frac{1}{4} \pm i,\ -\frac{1}{4}$

19. a. $r = -\frac{1}{4} \pm i,\ \frac{1}{10}$

20. b. $\begin{pmatrix} I \\ V \end{pmatrix} = c_1 e^{-t/2}\begin{pmatrix} \cos(t/2) \\ 4\sin(t/2) \end{pmatrix} + c_2 e^{-t/2}\begin{pmatrix} \sin(t/2) \\ -4\cos(t/2) \end{pmatrix}$

c. Use $c_1 = 2,\quad c_2 = -\frac{3}{4}$ in answer to part (b).

d. $\lim_{t\to\infty} I(t) = \lim_{t\to\infty} V(t) = 0$; no

21. b. $\begin{pmatrix} I \\ V \end{pmatrix} = c_1 e^{-t}\begin{pmatrix} \cos t \\ -\cos t - \sin t \end{pmatrix} + c_2 e^{-t}\begin{pmatrix} \sin t \\ -\sin t + \cos t \end{pmatrix}$

c. Use $c_1 = 2$ and $c_2 = 3$ in answer to part (b).

d. $\lim_{t\to\infty} I(t) = \lim_{t\to\infty} V(t) = 0$; no

23. b. $r = \pm i\sqrt{k/m}$

d. $|r|$ is the natural frequency.

24. c. $r_1^2 = -1,\quad \xi^{(1)} = \begin{pmatrix} 3 \\ 2 \end{pmatrix};\quad r_2^2 = -4,\quad \xi^{(2)} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$

d. $x_1 = 3c_1\cos t + 3c_2\sin t + 3c_3\cos(2t) + 3c_4\sin(2t),$
$x_2 = 2c_1\cos t + 2c_2\sin t - 4c_3\cos(2t) - 4c_4\sin(2t)$

e. $x_1' = -3c_1\sin t + 3c_2\cos t - 6c_3\sin(2t) + 6c_4\cos(2t),$
$x_2' = -2c_1\sin t + 2c_2\cos t + 8c_3\sin(2t) - 8c_4\cos(2t)$

25. a. $\mathbf{A} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -4 & 3 & 0 & 0 \\ 9/4 & -13/4 & 0 & 0 \end{pmatrix}$

b. $r_1 = i,\ \xi^{(1)} = \begin{pmatrix} 1 \\ 1 \\ i \\ i \end{pmatrix};\quad r_2 = -i,\ \xi^{(2)} = \begin{pmatrix} 1 \\ 1 \\ -i \\ -i \end{pmatrix};$

$r_3 = \frac{5}{2}i,\ \xi^{(3)} = \begin{pmatrix} 4 \\ -3 \\ 10i \\ -\frac{15}{2}i \end{pmatrix};\quad r_4 = -\frac{5}{2}i,\ \xi^{(4)} = \begin{pmatrix} 4 \\ -3 \\ -10i \\ \frac{15}{2}i \end{pmatrix}$

c. $\mathbf{y} = c_1\begin{pmatrix} \cos t \\ \cos t \\ -\sin t \\ -\sin t \end{pmatrix} + c_2\begin{pmatrix} \sin t \\ \sin t \\ \cos t \\ \cos t \end{pmatrix}$
$+ c_3\begin{pmatrix} 4\cos\left(\frac{5}{2}t\right) \\ -3\cos\left(\frac{5}{2}t\right) \\ -10\sin\left(\frac{5}{2}t\right) \\ \frac{15}{2}\sin\left(\frac{5}{2}t\right) \end{pmatrix} + c_4\begin{pmatrix} 4\sin\left(\frac{5}{2}t\right) \\ -3\sin\left(\frac{5}{2}t\right) \\ 10\left(\frac{5}{2}t\right) \\ -\frac{15}{2}\cos\left(\frac{5}{2}t\right) \end{pmatrix}$

e. $c_1 = \frac{10}{7},\quad c_2 = 0,\quad c_3 = \frac{1}{7},\quad c_4 = 0.$ period $= 4\pi$.

Section 7.7, page 336

1. b. $\Phi(t) = \begin{pmatrix} -\frac{1}{3}e^{-t} + \frac{4}{3}e^{2t} & \frac{2}{3}e^{-t} - \frac{2}{3}e^{2t} \\ -\frac{2}{3}e^{-t} + \frac{2}{3}e^{2t} & \frac{4}{3}e^{-t} - \frac{1}{3}e^{2t} \end{pmatrix}$

2. b. $\Phi(t) = \begin{pmatrix} \frac{1}{2}e^{-t/2} + \frac{1}{2}e^{-t} & e^{-t/2} - e^{-t} \\ \frac{1}{4}e^{-t/2} - \frac{1}{4}e^{-t} & \frac{1}{2}e^{-t/2} + \frac{1}{2}e^{-t} \end{pmatrix}$

3. b. $\Phi(t) = \begin{pmatrix} \cos t + 2\sin t & -5\sin t \\ \sin t & \cos t - 2\sin t \end{pmatrix}$

4. b. $\Phi(t) = \begin{pmatrix} e^{-t}\cos(2t) & -2e^{-t}\sin(2t) \\ \frac{1}{2}e^{-t}\sin(2t) & e^{-t}\cos(2t) \end{pmatrix}$

5. b. $\Phi(t) = \begin{pmatrix} -\frac{1}{2}e^{2t} + \frac{3}{2}e^{4t} & \frac{1}{2}e^{2t} - \frac{1}{2}e^{4t} \\ -\frac{3}{2}e^{2t} + \frac{3}{2}e^{4t} & \frac{3}{2}e^{2t} - \frac{1}{2}e^{4t} \end{pmatrix}$

6. b. $\Phi(t) = \begin{pmatrix} e^{-t}\cos t + 2e^{-t}\sin t & -e^{-t}\sin t \\ 5e^{-t}\sin t & e^{-t}\cos t - 2e^{-t}\sin t \end{pmatrix}$

7. b. $\Phi(t) =$
$\begin{pmatrix} -2e^{-2t} + 3e^{-t} & -e^{-2t} + e^{-t} & -e^{-2t} + e^{-t} \\ \frac{5}{2}e^{-2t} - 4e^{-t} + \frac{3}{2}e^{2t} & \frac{5}{4}e^{-2t} - \frac{4}{3}e^{-t} + \frac{13}{12}e^{2t} & \frac{5}{4}e^{-2t} - \frac{4}{3}e^{-t} + \frac{1}{12}e^{2t} \\ \frac{7}{2}e^{-2t} - 2e^{-t} - \frac{3}{2}e^{2t} & \frac{7}{4}e^{-2t} - \frac{2}{3}e^{-t} - \frac{13}{12}e^{2t} & \frac{7}{4}e^{-2t} - \frac{2}{3}e^{-t} - \frac{1}{12}e^{2t} \end{pmatrix}$

8. b. $\Phi(t) =$
$\begin{pmatrix} \frac{1}{6}e^{t} + \frac{1}{3}e^{-2t} + \frac{1}{2}e^{3t} & -\frac{1}{3}e^{t} + \frac{1}{3}e^{-2t} & \frac{1}{2}e^{t} - e^{-2t} + \frac{1}{2}e^{3t} \\ -\frac{2}{3}e^{t} - \frac{1}{3}e^{-2t} + e^{3t} & \frac{4}{3}e^{t} - \frac{1}{3}e^{-2t} & -2e^{t} + e^{-2t} + e^{3t} \\ -\frac{1}{6}e^{t} - \frac{1}{3}e^{-2t} + \frac{1}{2}e^{3t} & \frac{1}{3}e^{t} - \frac{1}{3}e^{-2t} & -\frac{1}{2}e^{t} + e^{-2t} + \frac{1}{2}e^{3t} \end{pmatrix}$

9. $\mathbf{x} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}e^{-t}\cos(2t) + \begin{pmatrix} -2 \\ 3/2 \end{pmatrix}e^{-t}\sin(2t)$

14. c. $\mathbf{x} = \begin{pmatrix} u_0 \\ v_0 \end{pmatrix}\cos(\omega t) + \begin{pmatrix} v_0 \\ -\omega^2 u_0 \end{pmatrix}\frac{\sin(\omega t)}{\omega}$

Section 7.8, page 343

1. a. $\mathbf{x} = c_1\begin{pmatrix} 2 \\ 1 \end{pmatrix}e^{t} + c_2\left(\begin{pmatrix} 2 \\ 1 \end{pmatrix}te^{t} + \begin{pmatrix} 1 \\ 0 \end{pmatrix}e^{t}\right)$

2. a. $\mathbf{x} = c_1\begin{pmatrix} 1 \\ 2 \end{pmatrix} + c_2\left(\begin{pmatrix} 1 \\ 2 \end{pmatrix}t - \begin{pmatrix} 0 \\ \frac{1}{2} \end{pmatrix}\right)$

3. a. $\mathbf{x} = c_1\begin{pmatrix} 2 \\ 1 \end{pmatrix}e^{-t} + c_2\left(\begin{pmatrix} 2 \\ 1 \end{pmatrix}te^{-t} + \begin{pmatrix} 0 \\ 2 \end{pmatrix}e^{-t}\right)$

4. $\mathbf{x} = c_1\begin{pmatrix} -3 \\ 4 \\ 2 \end{pmatrix}e^{-t} + c_2\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}e^{2t} + c_3\left(\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}te^{2t} + \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}e^{2t}\right)$

5. $\mathbf{x} = c_1\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}e^{2t} + c_2\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}e^{-t} + c_3\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}e^{-t}$

6. a. $\mathbf{x} = \begin{pmatrix} 3 + 4t \\ 2 + 4t \end{pmatrix}e^{-3t}$

7. a. $\mathbf{x} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}e^{-t} - 6\begin{pmatrix} 1 \\ 1 \end{pmatrix}te^{-t}$

8. a. $\mathbf{x} = 2\begin{pmatrix} 1 \\ 2 \end{pmatrix} + 14\begin{pmatrix} 3 \\ -1 \end{pmatrix}t$

9. a. $\mathbf{x} = \begin{pmatrix} -1 \\ 2 \\ -33 \end{pmatrix}e^{t} + 4\begin{pmatrix} 0 \\ 1 \\ -6 \end{pmatrix}te^{t} + 3\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}e^{2t}$

10. a. $\mathbf{x} = \frac{4}{3}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}e^{-t/2} + \frac{1}{3}\begin{pmatrix} 2 \\ 5 \\ -7 \end{pmatrix}e^{-7t/2}$

11. $\mathbf{x} = c_1\begin{pmatrix} 2 \\ 1 \end{pmatrix}t + c_2\left(\begin{pmatrix} 2 \\ 1 \end{pmatrix}t\ln t + \begin{pmatrix} 1 \\ 0 \end{pmatrix}t\right)$

12. $\mathbf{x} = c_1\begin{pmatrix} 1 \\ 1 \end{pmatrix}t^{-3} + c_2\left(\begin{pmatrix} 1 \\ 1 \end{pmatrix}t^{-3}\ln t - \begin{pmatrix} 0 \\ \frac{1}{4} \end{pmatrix}t^{-3}\right)$

14. b. $\begin{pmatrix} I \\ V \end{pmatrix} = -\begin{pmatrix} 1 \\ -2 \end{pmatrix}e^{-t/2} + \left(\begin{pmatrix} 1 \\ -2 \end{pmatrix}te^{-t/2} + \begin{pmatrix} 2 \\ 0 \end{pmatrix}e^{-t/2}\right)$

15. d. $\xi = -\xi^{(1)}$

e. $\xi = -(k_1 + k_2)\xi^{(1)},\quad k_1 + k_2 \neq 0$

17. a. $\mathbf{x}^{(1)}(t) = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}e^{2t}$

c. $\mathbf{x}^{(2)}(t) = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}te^{2t} + \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}e^{2t}$

d. $\mathbf{x}^{(3)}(t) = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}\frac{t^2}{2}e^{2t} + \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}te^{2t} + \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix}e^{2t}$

e. $\Psi(t) = e^{2t}\begin{pmatrix} 0 & 1 & t + 2 \\ 1 & t + 1 & t^2/2 + t \\ -1 & -t & -t^2/2 + 3 \end{pmatrix}$

f. $\mathbf{T} = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 1 & 0 \\ -1 & 0 & 3 \end{pmatrix},\quad \mathbf{T}^{-1} = \begin{pmatrix} -3 & 3 & 2 \\ 3 & -2 & -2 \\ -1 & 1 & 1 \end{pmatrix},$

$\mathbf{J} = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{pmatrix}$

18. a. $\mathbf{x}^{(1)}(t) = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}e^{t},\quad \mathbf{x}^{(2)}(t) = \begin{pmatrix} 0 \\ 2 \\ -3 \end{pmatrix}e^{t}$

b. $\mathbf{x}^{(3)}(t) = \begin{pmatrix} -2 \\ -4 \\ 2 \end{pmatrix}te^{t} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}e^{t}$

e. $\Psi(t) = e^{t}\begin{pmatrix} 1 & 0 & -2t \\ 0 & 2 & -4t \\ 2 & -3 & 2t + 1 \end{pmatrix}$ or $e^{t}\begin{pmatrix} 1 & -2 & -2t \\ 0 & -4 & -4t \\ 2 & 2 & 2t + 1 \end{pmatrix}$

f. $\mathbf{T} = \begin{pmatrix} 1 & -2 & 0 \\ 0 & -4 & 0 \\ 2 & 2 & 1 \end{pmatrix},\quad \mathbf{T}^{-1} = \begin{pmatrix} 1 & -1/2 & 0 \\ 0 & -1/4 & 0 \\ -2 & 3/2 & 1 \end{pmatrix},$

$\mathbf{J} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$

19. a. $\mathbf{J}^2 = \begin{pmatrix} \lambda^2 & 2\lambda \\ 0 & \lambda^2 \end{pmatrix},\quad \mathbf{J}^3 = \begin{pmatrix} \lambda^3 & 3\lambda^2 \\ 0 & \lambda^3 \end{pmatrix},\quad \mathbf{J}^4 = \begin{pmatrix} \lambda^4 & 4\lambda^3 \\ 0 & \lambda^4 \end{pmatrix}$

c. $\exp(\mathbf{J}t) = e^{\lambda t}\begin{pmatrix} 1 & t \\ 0 & 1 \end{pmatrix},$

d. $\mathbf{x} = \exp(\mathbf{J}t)\mathbf{x}^0$

20. c. $\exp(\mathbf{J}t) = e^{\lambda t}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & t \\ 0 & 0 & 1 \end{pmatrix}$

21. c. $\exp(\mathbf{J}t) = e^{\lambda t}\begin{pmatrix} 1 & t & t^2/2 \\ 0 & 1 & t \\ 0 & 0 & 1 \end{pmatrix}$

Section 7.9, page 351

1. $\mathbf{x} = c_1\begin{pmatrix} 1 \\ 1 \end{pmatrix}e^{t} + c_2\begin{pmatrix} 1 \\ 3 \end{pmatrix}e^{-t} + \frac{3}{2}\begin{pmatrix} 1 \\ 1 \end{pmatrix}te^{t} - \frac{1}{4}\begin{pmatrix} 1 \\ 3 \end{pmatrix}e^{t}$
$+ \begin{pmatrix} 1 \\ 2 \end{pmatrix}t - \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

2. $\mathbf{x} = c_1\begin{pmatrix} 5\cos t \\ 2\cos t + \sin t \end{pmatrix} + c_2\begin{pmatrix} 5\sin t \\ -\cos t + 2\sin t \end{pmatrix}$
$+ \begin{pmatrix} 2 \\ 1 \end{pmatrix}t\cos t - \begin{pmatrix} 1 \\ 0 \end{pmatrix}t\sin t - \begin{pmatrix} 1 \\ 1 \end{pmatrix}\cos t$

3. $\mathbf{x} = c_1\begin{pmatrix} 1 \\ -4 \end{pmatrix}e^{-3t} + c_2\begin{pmatrix} 1 \\ 1 \end{pmatrix}e^{2t} - \begin{pmatrix} 0 \\ 1 \end{pmatrix}e^{-2t} + \frac{1}{2}\begin{pmatrix} 1 \\ 0 \end{pmatrix}e^{t}$

4. $\mathbf{x} = c_1\begin{pmatrix} 1 \\ 2 \end{pmatrix} + c_2\left(\begin{pmatrix} 1 \\ 2 \end{pmatrix}t - \frac{1}{2}\begin{pmatrix} 0 \\ 1 \end{pmatrix}\right)$
$-2\begin{pmatrix} 1 \\ 2 \end{pmatrix}\ln t + \begin{pmatrix} 2 \\ 5 \end{pmatrix}t^{-1} - \begin{pmatrix} \frac{1}{2} \\ 0 \end{pmatrix}t^{-2}$

5. $\mathbf{x} = c_1\begin{pmatrix} 1 \\ 2 \end{pmatrix}e^{3t} + c_2\begin{pmatrix} 1 \\ -2 \end{pmatrix}e^{-t} + \frac{1}{4}\begin{pmatrix} 1 \\ -8 \end{pmatrix}e^{t}$

6. $\mathbf{x} = c_1\begin{pmatrix} 1 \\ 1 \end{pmatrix}e^{t} + c_2\begin{pmatrix} 1 \\ 3 \end{pmatrix}e^{-t} + \begin{pmatrix} 1 \\ 0 \end{pmatrix}e^{t} + 2\begin{pmatrix} 1 \\ 1 \end{pmatrix}te^{t}$

7. $\mathbf{x} = c_1\begin{pmatrix} 1 \\ \sqrt{2} \end{pmatrix}e^{-t} + c_2\begin{pmatrix} -\sqrt{2} \\ 1 \end{pmatrix}e^{-4t} - \frac{1}{3}\begin{pmatrix} \sqrt{2} - 1 \\ 2 - \sqrt{2} \end{pmatrix}te^{-t} +$
$\frac{1}{9}\begin{pmatrix} 2 + \sqrt{2} \\ -1 - \sqrt{2} \end{pmatrix}e^{-t}$

8. $\mathbf{x} = c_1\begin{pmatrix} 5\cos t \\ 2\cos t + \sin t \end{pmatrix} + c_2\begin{pmatrix} 5\sin t \\ -\cos t + 2\sin t \end{pmatrix} +$
$\begin{pmatrix} 0 \\ 1/2 \end{pmatrix}t\cos t - \begin{pmatrix} 5/2 \\ 1 \end{pmatrix}t\sin t - \begin{pmatrix} 5/2 \\ 1 \end{pmatrix}\cos t$

9. a. $\Psi(t) = \begin{pmatrix} e^{-t/2}\cos\left(\frac{t}{2}\right) & e^{-t/2}\sin\left(\frac{t}{2}\right) \\ 4e^{-t/2}\sin\left(\frac{t}{2}\right) & -4e^{-t/2}\cos\left(\frac{t}{2}\right) \end{pmatrix}$

b. $\mathbf{x} = e^{-t/2}\begin{pmatrix} \sin\left(\frac{t}{2}\right) \\ 4 - 4\cos\left(\frac{t}{2}\right) \end{pmatrix}$

10. $\mathbf{x} = c_1\begin{pmatrix} 1 \\ 1 \end{pmatrix}t + c_2\begin{pmatrix} 1 \\ 3 \end{pmatrix}t^{-1} - \begin{pmatrix} 2 \\ 3 \end{pmatrix}$
$+ \frac{1}{2}\begin{pmatrix} 1 \\ 3 \end{pmatrix}t - \begin{pmatrix} 1 \\ 1 \end{pmatrix}t\ln t - \frac{1}{3}\begin{pmatrix} 4 \\ 3 \end{pmatrix}t^2$

11. $\mathbf{x} = c_1\begin{pmatrix} 2 \\ 1 \end{pmatrix}t^2 + c_2\begin{pmatrix} 1 \\ 2 \end{pmatrix}t^{-1} + \begin{pmatrix} 3 \\ 2 \end{pmatrix}t + \frac{1}{10}\begin{pmatrix} -2 \\ 1 \end{pmatrix}t^4 - \frac{1}{2}\begin{pmatrix} 2 \\ 3 \end{pmatrix}$

13. a. $\mathbf{x}'(t) = \begin{pmatrix} 0 & 1 \\ -6 & 5 \end{pmatrix}\mathbf{x} + \begin{pmatrix} 0 \\ 2e^{t} \end{pmatrix}.$

14. **a.** $\mathbf{x}(t) = \begin{pmatrix} 0 & 1 \\ -\dfrac{t+2}{t^2} & \dfrac{t+2}{t} \end{pmatrix}\mathbf{x} + \begin{pmatrix} 0 \\ 2t \end{pmatrix}.$

15. **a.** $\mathbf{x}(t) = \begin{pmatrix} 0 & 1 \\ -q(t) & -p(t) \end{pmatrix}\mathbf{x} + \begin{pmatrix} 0 \\ g(t) \end{pmatrix}.$

17. $(3(\alpha_1 - \alpha_2) - 4)/6 = c_1, \quad (\alpha_1 + \alpha_2 + 3)/2 = c_2$

Chapter 8

Section 8.1, page 361

3. **a.** 1.59980, 1.29288, 1.07242, 0.930175
b. 1.61124, 1.31361, 1.10012, 0.962552
c. 1.64337, 1.37164, 1.17763, 1.05334
d. 1.63301, 1.35295, 1.15267, 1.02407

4. **a.** 1.2025, 1.41603, 1.64289, 1.88590
b. 1.20388, 1.41936, 1.64896, 1.89572
c. 1.20864, 1.43104, 1.67042, 1.93076
d. 1.20693, 1.42683, 1.66265, 1.91802

5. **a.** 1.10244, 1.21426, 1.33484, 1.46399
b. 1.10365, 1.21656, 1.33817, 1.46832
c. 1.10720, 1.22333, 1.34797, 1.48110
d. 1.10603, 1.22110, 1.34473, 1.47688

6. **a.** 0.509239, 0.522187, 0.539023, 0.559936
b. 0.509701, 0.523155, 0.540550, 0.562089
c. 0.511127, 0.526155, 0.545306, 0.568822
d. 0.510645, 0.525138, 0.543690, 0.566529

7. **a.** −0.920498, −0.857538, −0.808030, −0.770038
b. −0.922575, −0.860923, −0.812300, −0.774965
c. −0.928059, −0.870054, −0.824021, −0.788686
d. −0.926341, −0.867163, −0.820279, −0.784275

8. **a.** 2.90330, 7.53999, 19.4292, 50.5614
b. 2.93506, 7.70957, 20.1081, 52.9779
c. 3.03951, 8.28137, 22.4562, 61.5496
d. 3.00306, 8.07933, 21.6163, 58.4462

9. **a.** 0.891830, 1.25225, 2.37818, 4.07257
b. 0.908902, 1.26872, 2.39336, 4.08799
c. 0.958565, 1.31786, 2.43924, 4.13474
d. 0.942261, 1.30153, 2.42389, 4.11908

10. **a.** 1.60729, 2.46830, 3.72167, 5.45963
b. 1.60996, 2.47460, 3.73356, 5.47774
c. 1.61792, 2.49356, 3.76940, 5.53223
d. 1.61528, 2.48723, 3.75742, 5.51404

11. **a.** −1.45865, −0.217545, 1.05715, 1.41487
b. −1.45322, −0.180813, 1.05903, 1.41244
c. −1.43600, −0.0681657, 1.06489, 1.40575
d. −1.44190, −0.105737, 1.06290, 1.40789

12. **a.** 0.587987, 0.791589, 1.14743, 1.70973
b. 0.589440, 0.795758, 1.15693, 1.72955
c. 0.593901, 0.808716, 1.18687, 1.79291
d. 0.592396, 0.804319, 1.17664, 1.77111

13. 1.595, 2.4636

14. $e_{n+1} = (2\phi(\bar{t}_n) - 1)h^2, \quad |e_{n+1}| \le \left(1 + 2\max_{0\le t\le 1}|\phi(t)|\right)h^2,$
$e_{n+1} = e^{2\bar{t}_n}h^2, \quad |e_1| \le 0.012, \quad |e_4| \le 0.022$

15. $e_{n+1} = (2\phi(\bar{t}_n) - \bar{t}_n)h^2, \quad |e_{n+1}| \le \left(1 + 2\max_{0\le t\le 1}|\phi(t)|\right)h^2,$
$e_{n+1} = 2e^{2\bar{t}_n}h^2, \quad |e_1| \le 0.024, \quad |e_4| \le 0.045$

16. $e_{n+1} = \left(19 - 15\bar{t}_n\phi\left(\bar{t}_n\right)^{-1/2}\right)h^2/4$

17. $e_{n+1} = \left(1 + \left(\bar{t}_n + \phi\left(\bar{t}_n\right)\right)^{-1/2}\right)h^2/4$

18. $e_{n+1} = \left(2 - \left(\phi\left(\bar{t}_n\right) + 2\bar{t}_n^2\right)\exp\left(-\bar{t}_n\phi\left(\bar{t}_n\right)\right)\right.$
$\left. - \bar{t}_n\exp\left(-2\bar{t}_n\phi\left(\bar{t}_n\right)\right)\right)h^2/2$

19. **a** 1.2, 1.0, 1.2
b $\phi(t) = 1 + (1/5\pi)\sin(5\pi t)$
d 1.1, 1.1, 1.0, 1.0
e $h < 1/\sqrt{50\pi} \cong 0.08$

21. $e_{n+1} = -\frac{1}{2}\phi''\left(\bar{t}_n\right)h^2$

22. **a.** 1.55, 2.34, 3.46, 5.07
b. 1.20, 1.39, 1.57, 1.74
c. 1.20, 1.42, 1.65, 1.90

23. **a.** 0
b. 60
c. −92.16

24. $0.224 \ne 0.225$

Section 8.2, page 366

2. **a.** 1.19512, 1.38120, 1.55909, 1.72956
b. 1.19515, 1.38125, 1.55916, 1.72965
c. 1.19516, 1.38126, 1.55918, 1.72967

3. **a.** 1.20526, 1.42273, 1.65511, 1.90570
b. 1.20533, 1.42290, 1.65542, 1.90621
c. 1.20534, 1.42294, 1.65550, 1.90634

4. **a.** 1.10483, 1.21882, 1.34146, 1.47263
b. 1.10484, 1.21884, 1.34147, 1.47262
c. 1.10484, 1.21884, 1.34147, 1.47262

5. **a.** 0.510164, 0.524126, 0.542083, 0.564251
b. 0.510168, 0.524135, 0.542100, 0.564277
c. 0.510169, 0.524137, 0.542104, 0.564284

6. **a.** −0.924650, −0.864338, −0.816642, −0.780008
b. −0.924550, −0.864177, −0.816442, −0.779781
c. −0.924525, −0.864138, −0.816393, −0.779725

7. **a.** 2.96719, 7.88313, 20.8114, 55.5106
b. 2.96800, 7.88755, 20.8294, 55.5758

8. **a.** 0.926139, 1.28558, 2.40898, 4.10386
b. 0.925815, 1.28525, 2.40869, 4.10359

9. **a.** 3.96217, 5.10887, 6.43134, 7.92332
b. 3.96218, 5.10889, 6.43138, 7.92337

10. **a.** 1.61263, 2.48097, 3.74556, 5.49595
b. 1.61263, 2.48092, 3.74550, 5.49589

11. **a.** 0.590897, 0.799950, 1.16653, 1.74969
b. 0.590906, 0.799988, 1.16663, 1.74992

13. **a.** $e_{n+1} = (38h^3/3)\exp(4\bar{t}_n)$,
$|e_{n+1}| \le 37,758.8h^3$ on
$0 \le t \le 2$,
c. $|e_1| \le 0.00193389$

14. **a.** $e_{n+1} = (2h^3/3)\exp(2\bar{t}_n)$,
$|e_{n+1}| \le 4.92604h^3$ on
$0 \le t \le 1$,
b. $|e_1| \le 0.000814269$

15. **a.** $e_{n+1} = (4h^3/3)\exp(2\bar{t}_n)$,
$|e_{n+1}| \le 9.85207h^3$ on
$0 \le t \le 1$,
b. $|e_1| \le 0.00162854$

16. $h \cong 0.036$

17. $h \cong 0.023$

18. $h \cong 0.081$

19. $h \cong 0.117$

21. 1.19512, 1.38120, 1.55909, 1.72956

22. 1.62268, 1.33435, 1.12789, 0.995130

23. 1.20526, 1.42273, 1.65511, 1.90570

24. 1.10485, 1.21886, 1.34149, 1.47264

Section 8.3, page 370

2. **a.** 1.19516, 1.38127, 1.55918, 1.72968
b. 1.19516, 1.38127, 1.55918, 1.72968

3. **a.** 1.62231, 1.33362, 1.12686, 0.993839
b. 1.62230, 1.33362, 1.12685, 0.993826

4. **a.** 1.10484, 1.21884, 1.34147, 1.47262
b. 1.10484, 1.21884, 1.34147, 1.47262

5. **a.** 0.510170, 0.524138, 0.542105, 0.564286
b. 0.520169, 0.524138, 0.542105, 0.564286

6. **a.** −0.924517, −0.864125, −0.816377, −0.779706
b. −0.924517, −0.864125, −0.816377, −0.779706

7. **a.** 2.96825, 7.88889, 20.8349, 55.5957
b. 2.96828, 7.88904, 20.8355, 55.5980

8. **a.** 0.925725, 1.28516, 2.40860, 4.10350
b. 0.925711, 1.28515, 2.40860, 4.10350

9. **a.** 3.96219, 5.10890, 6.43139, 7.92338
b. 3.96219, 5.10890, 6.43139, 7.92338

10. **a.** 1.61262, 2.48091, 3.74548, 5.49587
b. 1.61262, 2.48091, 3.74548, 5.49587

11. **a.** 0.590909, 0.800000, 1.166667, 1.75000
b. 0.590909, 0.800000, 1.166667, 1.75000

Section 8.4, page 375

1. **a.** 1.7296801, 1.8934697
b. 1.7296802, 1.8934698
c. 1.7296805, 1.8934711

2. **a.** 0.993852, 0.925764
b. 0.993846, 0.925746
c. 0.993869, 0.925837

3. **a.** 1.4726173, 1.6126215
b. 1.4726189, 1.6126231
c. 1.4726199, 1.6126256

4. **a.** 0.56428577, 0.59090918
b. 0.56428581, 0.59090923
c. 0.56428588, 0.59090952

5. **a.** −0.779693, −0.753135
b. −0.779692, −0.753137
c. −0.779680, −0.753089

6. **a.** 2.96828, 7.88907, 20.8356, 55.5984
b. 2.96829, 7.88909, 20.8357, 55.5986
c. 2.96831, 7.88926, 20.8364, 55.6015

7. **a.** 0.9257133, 1.285148, 2.408595, 4.103495
b. 0.9257124, 1.285148, 2.408595, 4.103495
c. 0.9257248, 1.285158, 2.408594, 4.103493

8. **a.** 3.962186, 5.108903, 6.431390, 7.923385
b. 3.962186, 5.108903, 6.431390, 7.923385
c. 3.962186, 5.108903, 6.431390, 7.923385

9. **a.** 1.612622, 2.480909, 3.745479, 5.495872
b. 1.612622, 2.480909, 3.745479, 5.495873
c. 1.612623, 2.480905, 3.745473, 5.495869

10. **a.** 0.5909091, 0.8000000, 1.166667, 1.750000
b. 0.5909091, 0.8000000, 1.166667, 1.750000
c. 0.5909092, 0.8000002, 1.166667, 1.750001

Section 8.5, page 378

1. **a.** 1.26, 0.76; 1.7714, 1.4824; 2.58991, 2.3703; 3.82374, 3.60413; 5.64246, 5.38885
b. 1.32493, 0.758933; 1.93679, 1.57919; 2.93414, 2.66099; 4.48318, 4.22639; 6.84236, 6.56452
c. 1.32489, 0.759516; 1.9369, 1.57999; 2.93459, 2.66201; 4.48422, 4.22784; 6.8444, 6.56684

2. **a.** 0.582, 1.18; 0.117969, 1.27344; −0.336912, 1.27382; −0.730007, 1.18572; −1.02134, 1.02371
b. 0.568451, 1.15775; 0.109776, 1.22556; −0.32208, 1.20347; −0.681296, 1.10162; −0.937852, 0.937852
c. 0.56845, 1.15775; 0.109773, 1.22557; −0.322081, 1.20347; −0.681291, 1.10161; −0.937841, 0.93784

3. **a.** −0.198, 0.618; −0.378796, 0.28329; −0.51932, −0.0321025; −0.594324, −0.326801; −0.588278, −0.57545
b. −0.196904, 0.630936; −0.372643, 0.298888; −0.501302, −0.0111429; −0.561270, −0.288943; −0.547053, −0.508303
c. −0.196935, 0.630939; −0.372687, 0.298866; −0.501345, −0.0112184; −0.561292, −0.28907; −0.547031, −0.508427

4. **a.** 2.96225, 1.34538; 2.34119, 1.67121; 1.90236, 1.97158; 1.56602, 2.23895; 1.29768, 2.46732
b. 3.06339, 1.34858; 2.44497, 1.68638; 1.9911, 2.00036; 1.63818, 2.27981; 1.3555, 2.5175
c. 3.06314, 1.34899; 2.44465, 1.68699; 1.99075, 2.00107; 1.63781, 2.28057; 1.35514, 2.51827

5. **a.** 1.42386, 2.18957; 1.82234, 2.36791; 2.21728, 2.53329; 2.61118, 2.68763; 2.9955, 2.83354
b. 1.41513, 2.18699; 1.81208, 2.36233; 2.20635, 2.5258; 2.59826, 2.6794; 2.97806, 2.82487
c. 1.41513, 2.18699; 1.81209, 2.36233; 2.20635, 2.52581; 2.59826, 2.67941; 2.97806, 2.82488

6. For $h = 0.05$ and 0.025: $x = 10.227$, $y = -4.9294$; these results agree with the exact solution to five digits.

7. 1.543, 0.0707503; 1.14743, −1.3885

8. 1.99521, −0.662442

Section 8.6, page 386

1. **b.** $\phi_2(t) - \phi_1(t) = 0.001e^t \to \infty$ as $t \to \infty$

2. **b.** $\phi_1(t) = \ln(e^t/(2 - e^t))$; $\phi_2(t) = \ln(1/(1 - t))$

3. **a., b.** $h = 0.00025$ is sufficient. **c.** $h = 0.005$ is sufficient.

4. **a.** $y = 4e^{-10t} + t^2/4$
c. Runge–Kutta is stable for $h = 0.25$ but unstable for $h = 0.3$.
d. $h = 5/13 \cong 0.384615$ is small enough.

5. **a.** $y = t$

6. **a.** $y = t^2$